DS-GA.3001 Embodied Learning and Vision

Mengye Ren

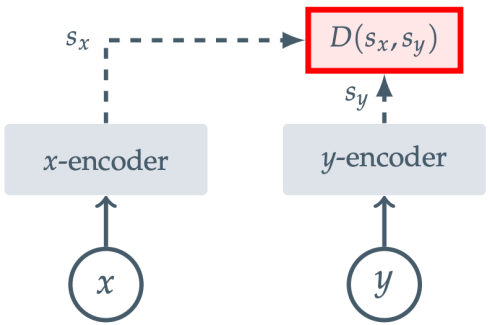
NYU

Spring 2026

elvcourse.org

Joint Embedding Predictive Architecture (JEPA)

- Predict what changes from one view to another.



JEA

Assran et al. Self-Supervised Learning from Images with a Joint-Embedding Predictive Architecture. ICCV 2023.

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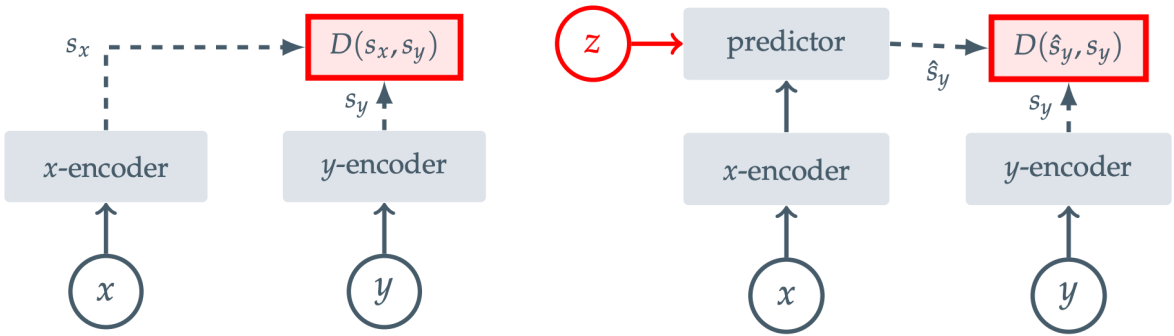

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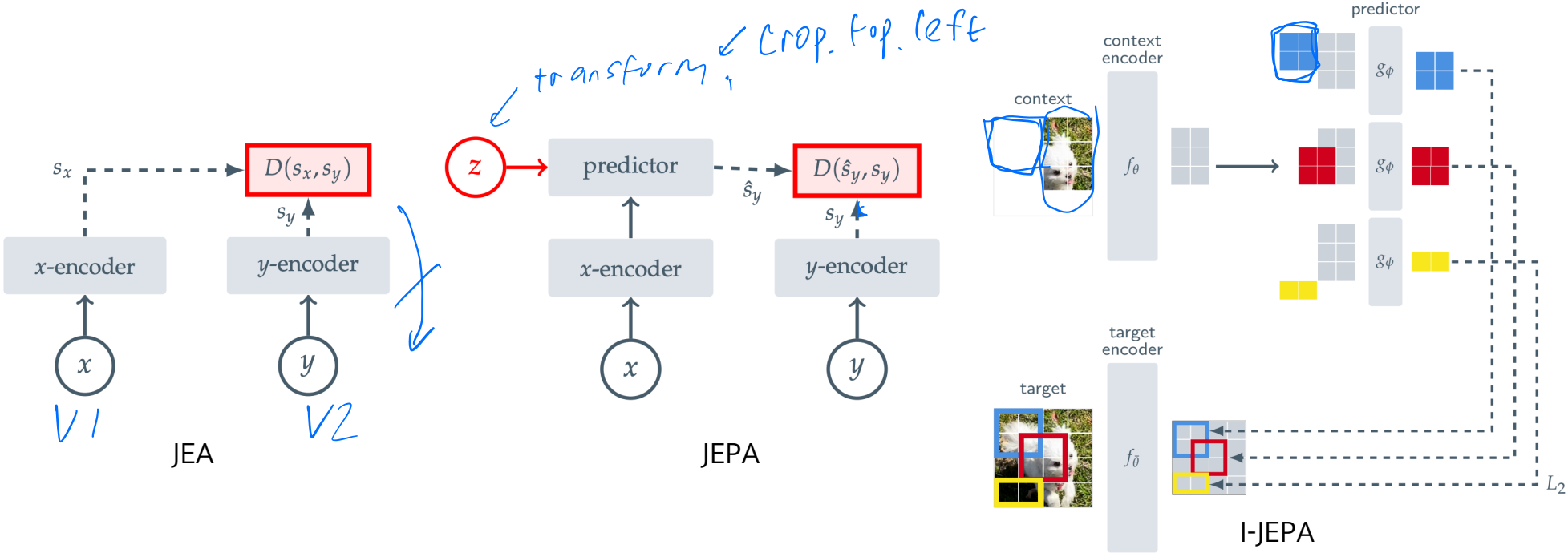

Assran et al. Self-Supervised Learning from Images with a Joint-Embedding Predictive Architecture. ICCV 2023.

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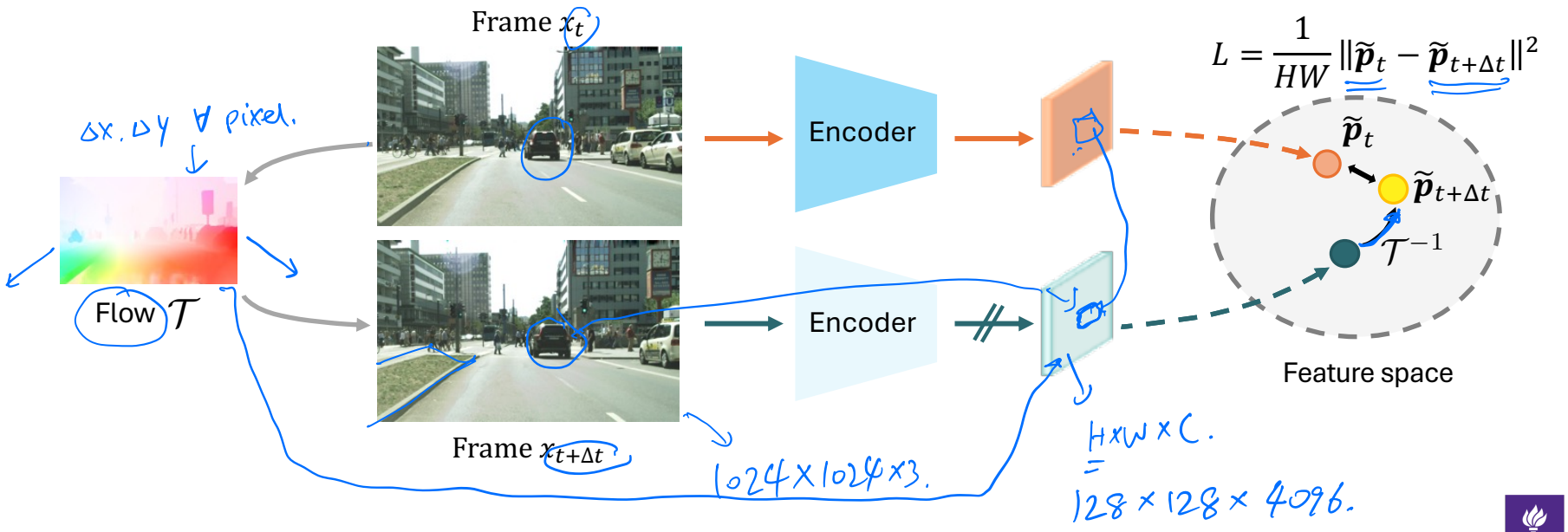
Joint Embedding Prediction with Dense Motion

- Can we use adjacent video frames as self-supervision?
- Objects move densely throughout the image.

motion equivariance.



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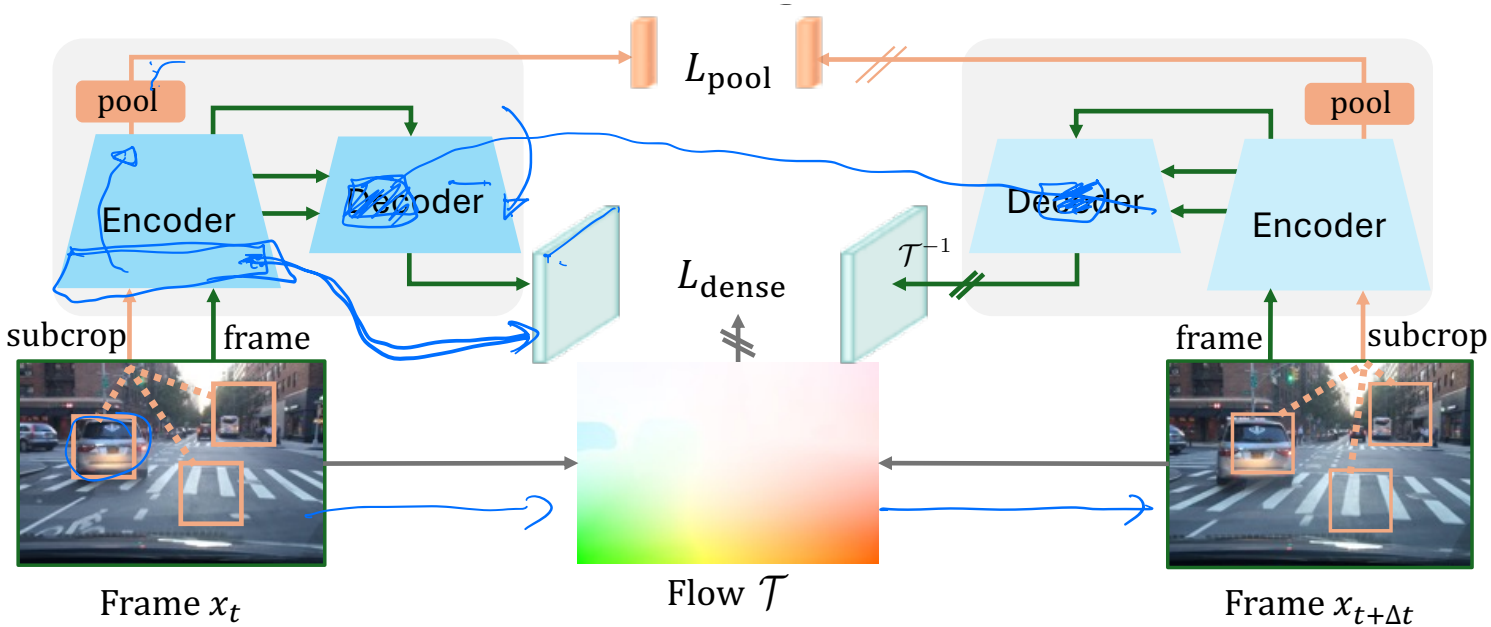
Joint Embedding Prediction with Dense Motion

- Perform SSL in multiple scales (small objects vs. big regions).



Wang et al. PooDLe: Pooled and Dense Self-Supervised Learning from Naturalistic Videos. ICLR 2025.

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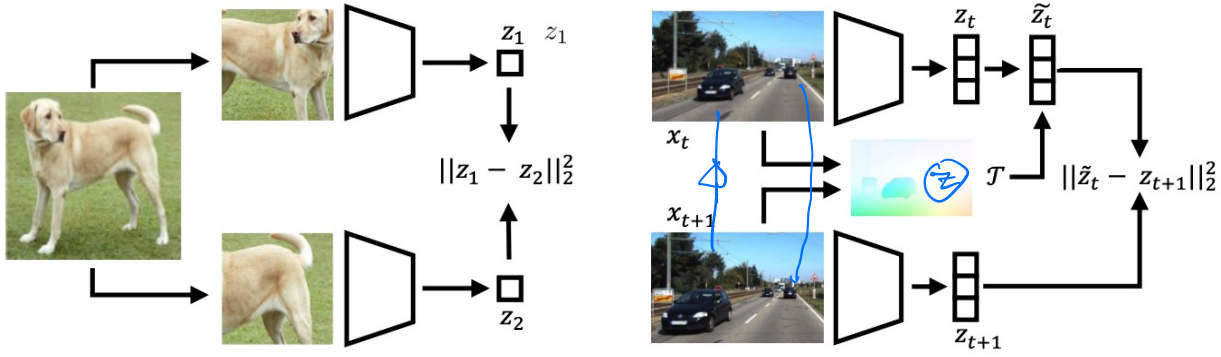
Joint Motion and Semantic Learning

- We either create or assume some external motion / action information to send to predictors.
- Additional motion needs to be inferred.



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JEPA w/ dense motion

Hoang et al. Midway Network: Learning Representations for Recognition and Motion from Latent Dynamics.

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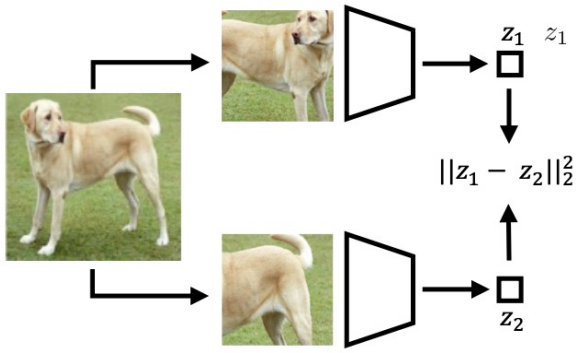
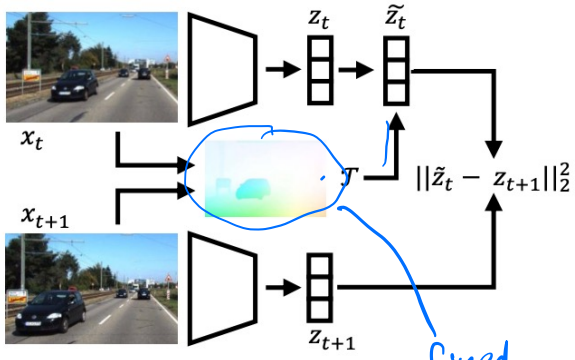
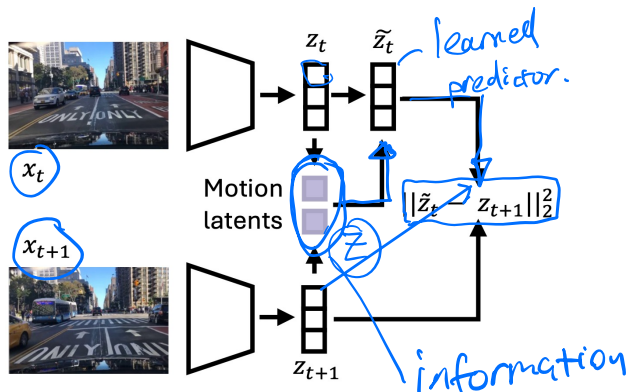


JEA



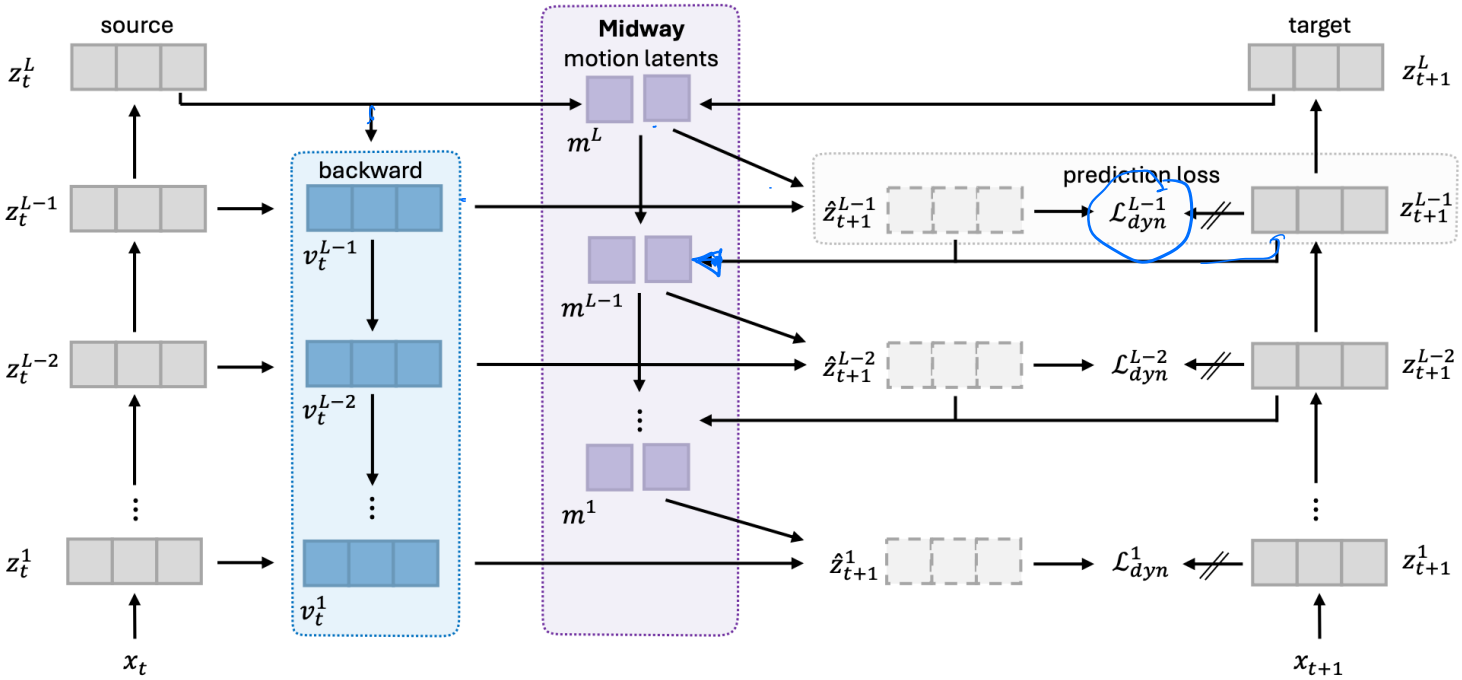
JEPA w/ dense motion



JEPA w/ inferred motion

Midway Networks

- Using backward refinement from Optical Flow networks.



Hoang et al. Midway Network: Learning Representations for Recognition and Motion from Latent Dynamics. ICLR 2026.

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Midway Networks

- Use perturbation to visualize the motion learned from motion tokens.

Hoang et al. Midway Network: Learning Representations for Recognition and Motion from Latent Dynamics. ICLR 2026.

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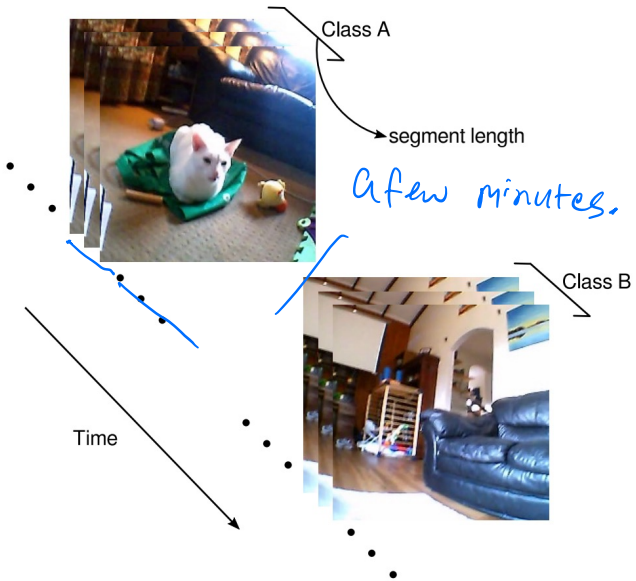
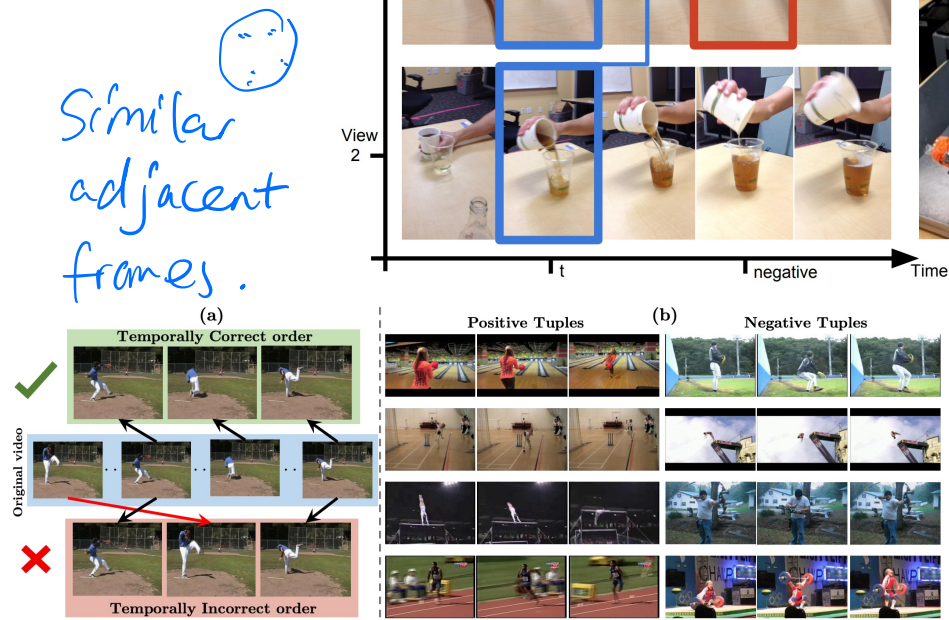
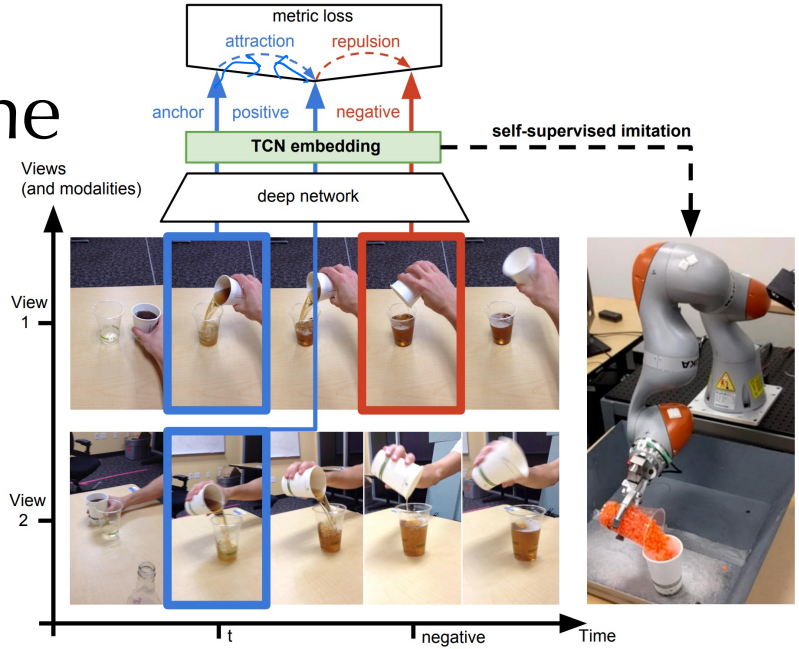
SSL with Time

- Use time as an additional source of supervision.







Misra et al. Shuffle and Learn: Unsupervised Learning using Temporal Order Verification. ECCV 2016.
Sermanet et al. Time-Contrastive Networks: Self-Supervised Learning from Video. ICRA 2018.
Orhan et al. Self-Supervised Learning through the Eyes of a Child. NeurIPS 2020.

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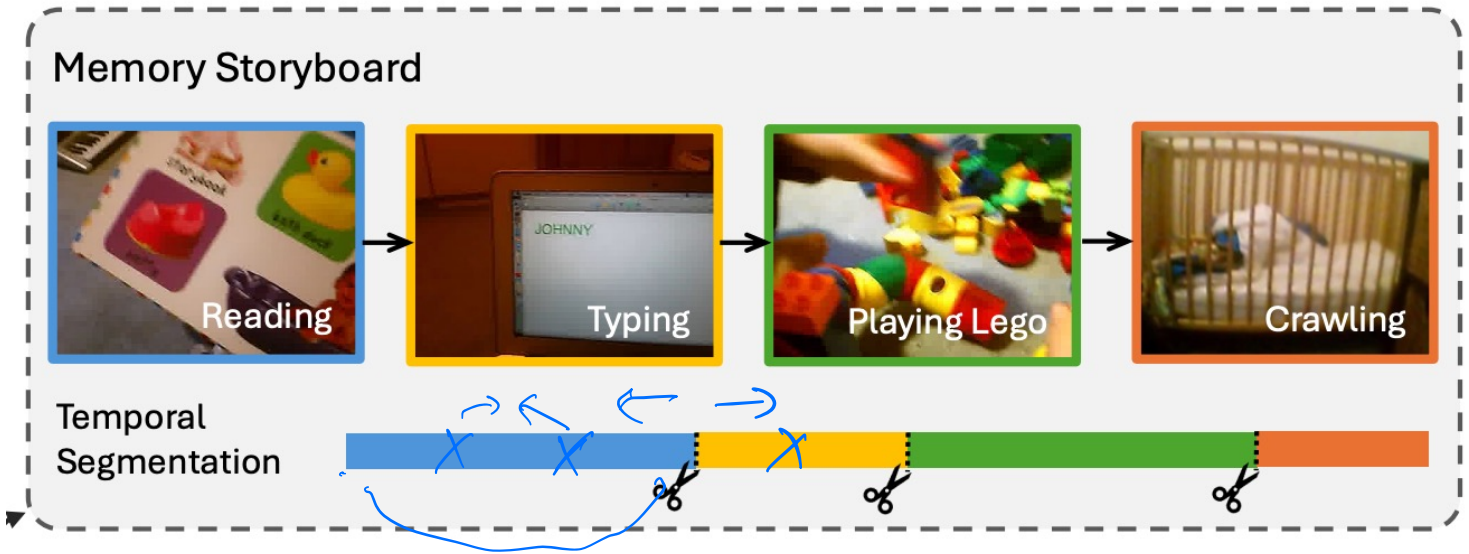
SSL with Time

- We can segment videos into meaningful events.
- Leverage the spatiotemporal continuity structure.



Yang & Ren. Memory Storyboard: Leveraging Temporal Segmentation for Streaming Self-Supervised Learning from Egocentric Videos. CoLLAs 2025.

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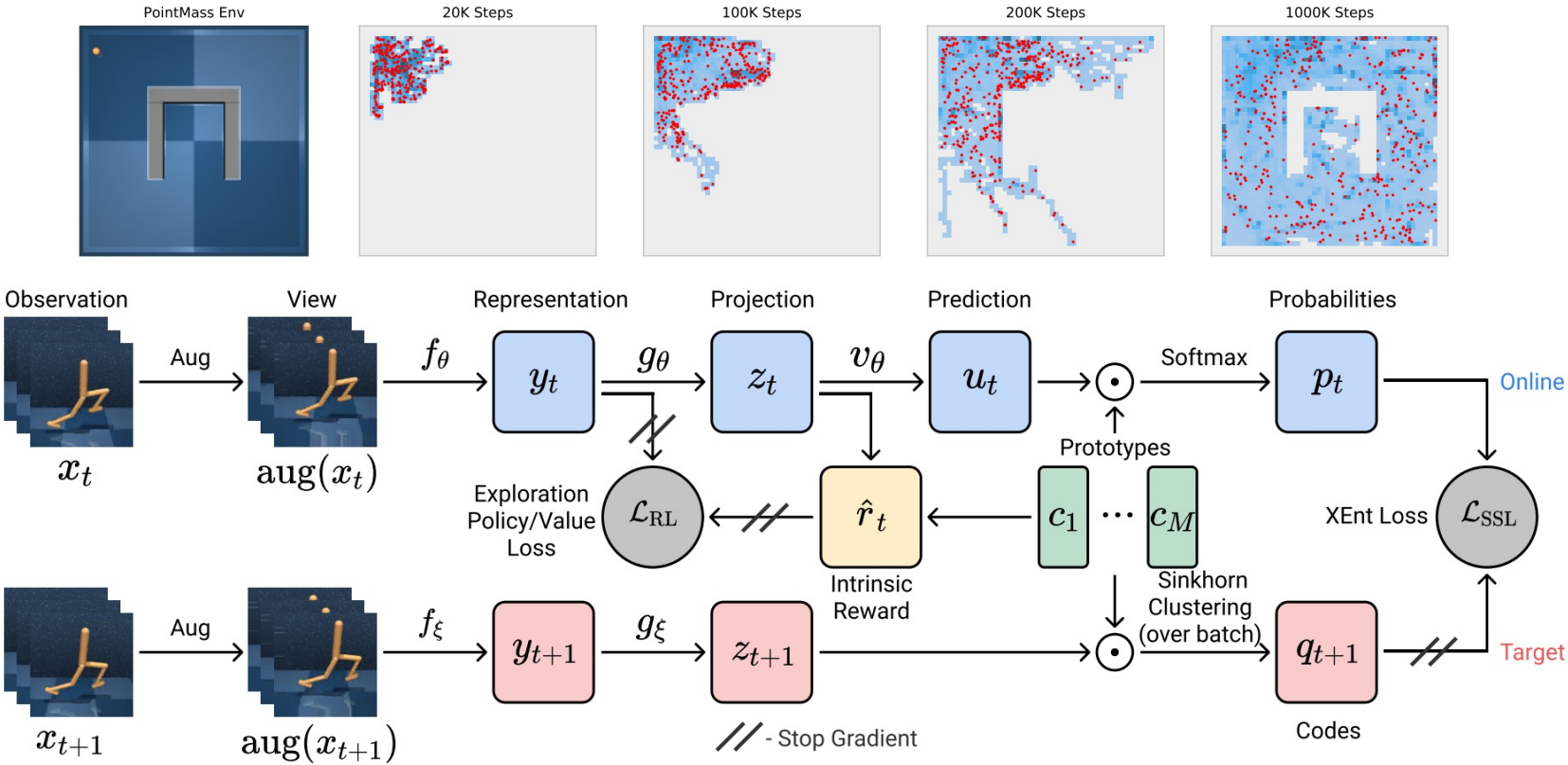
SSL for Visual Control



Yarats. Reinforcement Learning with Prototypical Representations. ICML 2021.

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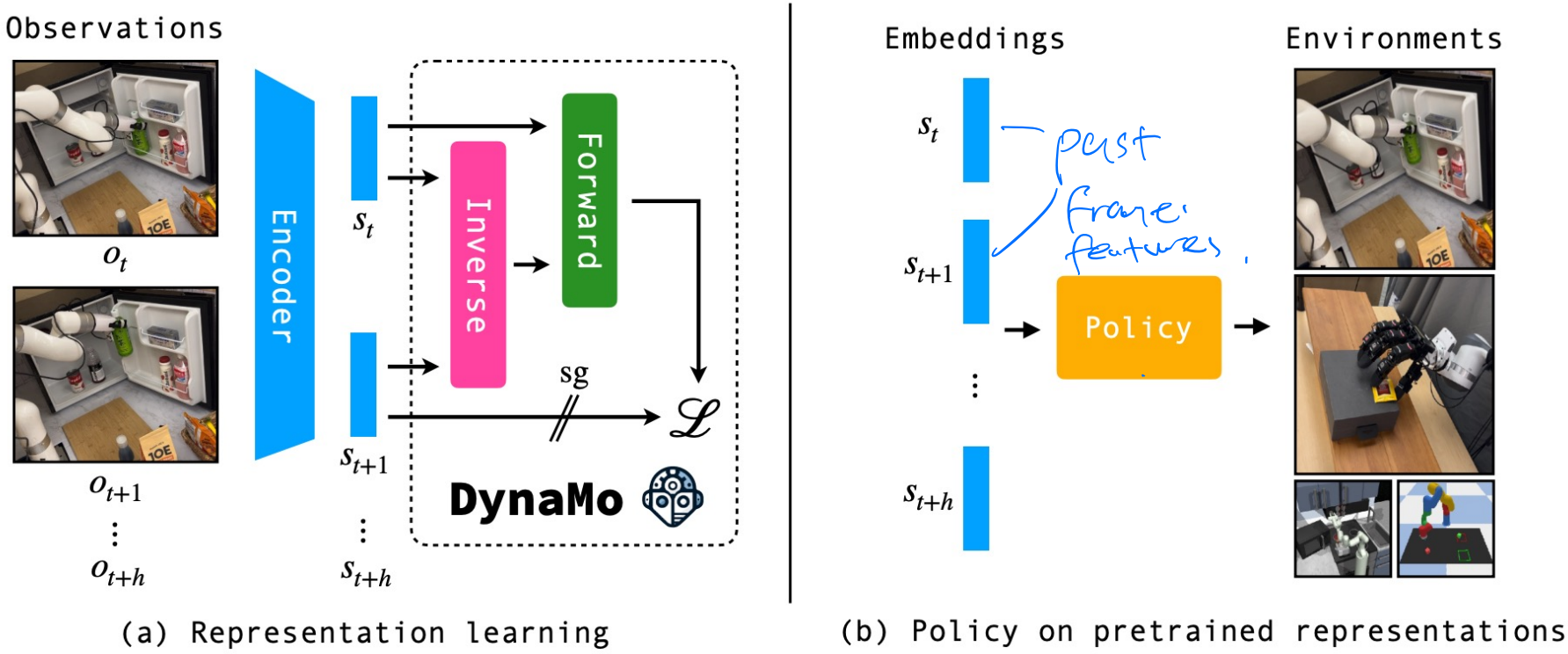

Cui et al. DynaMo: In-Domain Dynamics Pretraining for Visuo-Motor Control. NeurIPS 2024.

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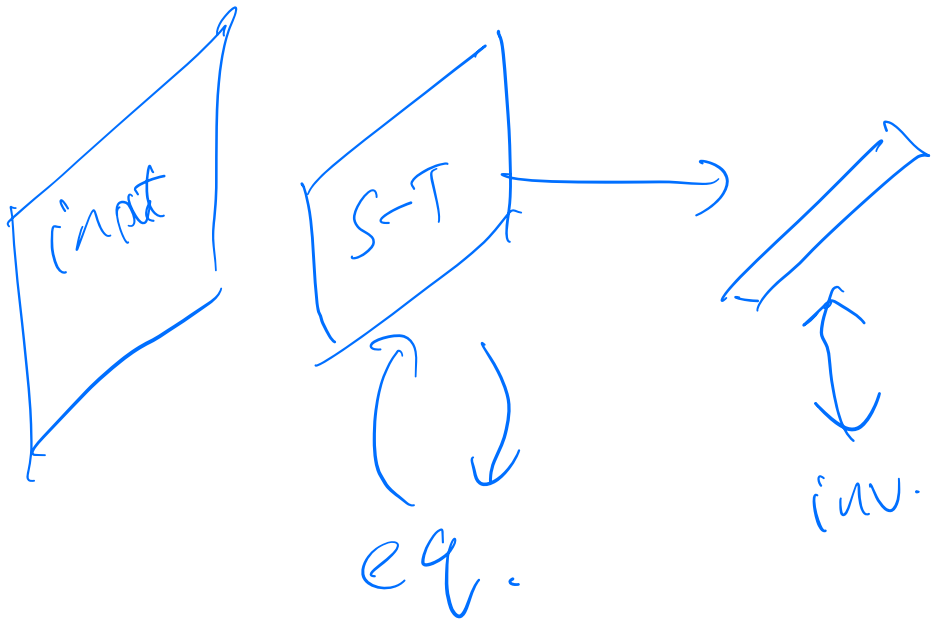
Summary

- Representation learning leverage the information in unlabeled data.
- Inductive biases matter
  - spatial-time invariance
  - motion equivariance



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Summary

- Representation learning leverage the information in unlabeled data.
- Inductive biases matter
  - spatial-time invariance
  - motion equivariance
- A foundation for sensorimotor learning – pretrained representations as a starter for policy learning.

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Emergent Attention, Object Discovery

- SSL representations show awareness of object classes and instance identities.

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Emergent Attention, Object Discovery

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- Why does attention show awareness of objects?

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Emergent Attention, Object Discovery

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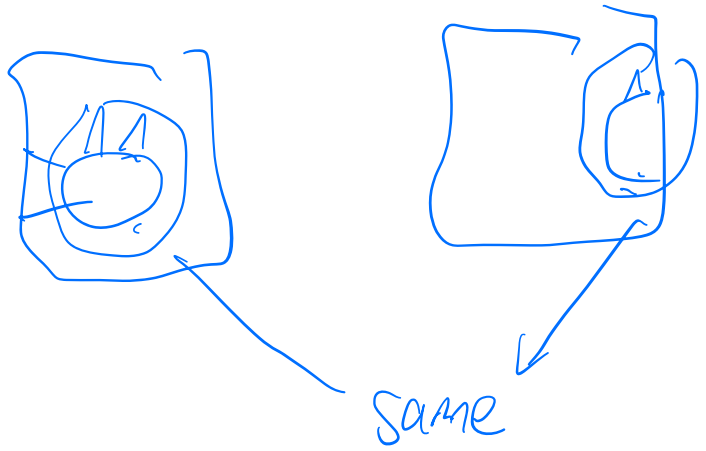
- Why does attention show awareness of objects?
- The network is encouraged to associate different parts of the objects together in order to identify whether two inputs belong to the same image or not.



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Emergent Attention, Object Discovery

- SSL representations show awareness of object classes and instance identities.
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- Attending to semantically similar parts facilitates the process.

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Emergent Attention, Object Discovery

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- The network is encouraged to associate different parts of the objects together in order to identify whether two inputs belong to the same image or not.
- Attending to semantically similar parts facilitates the process.
- The network is a hierarchical information processing pipeline – Lower layers integrate more granular and smaller neighborhood.

Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.

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Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.
- Then use these masks as pseudo labels and supervise the network to predict these low-quality masks.

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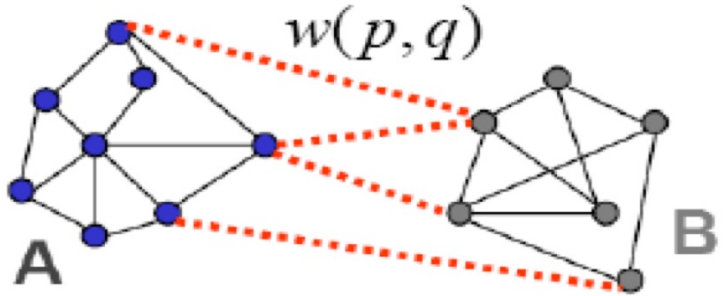
Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.
- Then use these masks as pseudo labels and supervise the network to predict these low-quality masks.
- Question: how do we come up with masks? What loss is used to supervise the network?

Graph Cut

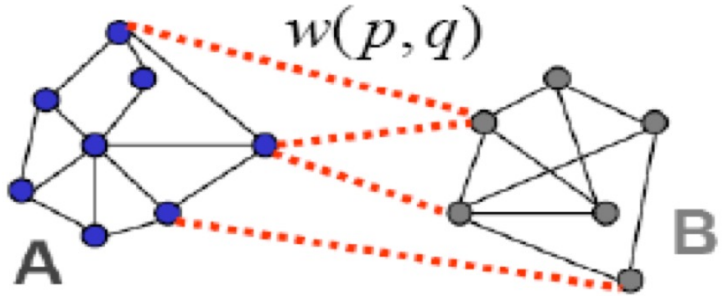
- Segmentation is essentially a clustering problem.



$$cut\,(A, B) = \sum_{p \in A, q \in B} w(p, q)$$

Graph Cut

- Segmentation is essentially a clustering problem.
- We can transform the clustering problem with the graph cut problem.



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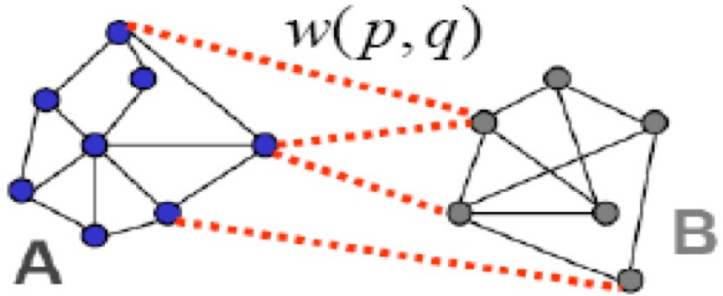
- Pixel = node.



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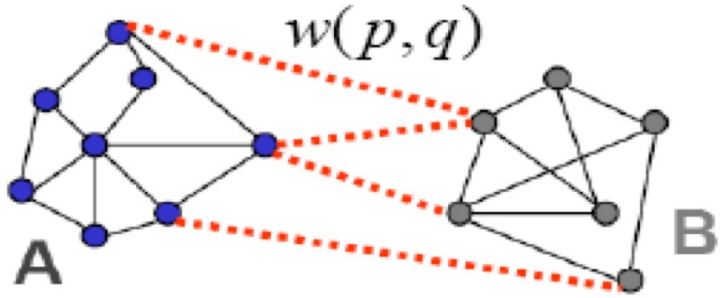

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Graph Cut

- Segmentation is essentially a clustering problem.
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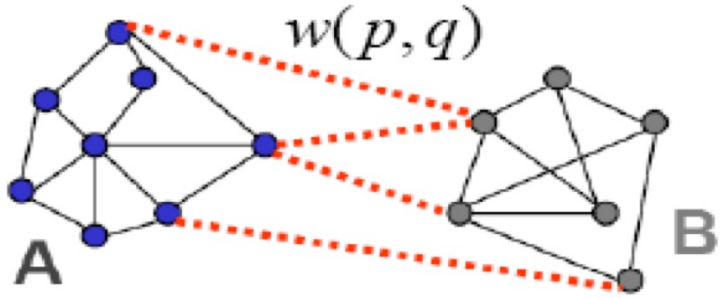
- Pixel = node.
- Affinity between the two pixels = edge value (flow).
- Objective: Cut the graph into disconnected components with a minimum sum of edge values.



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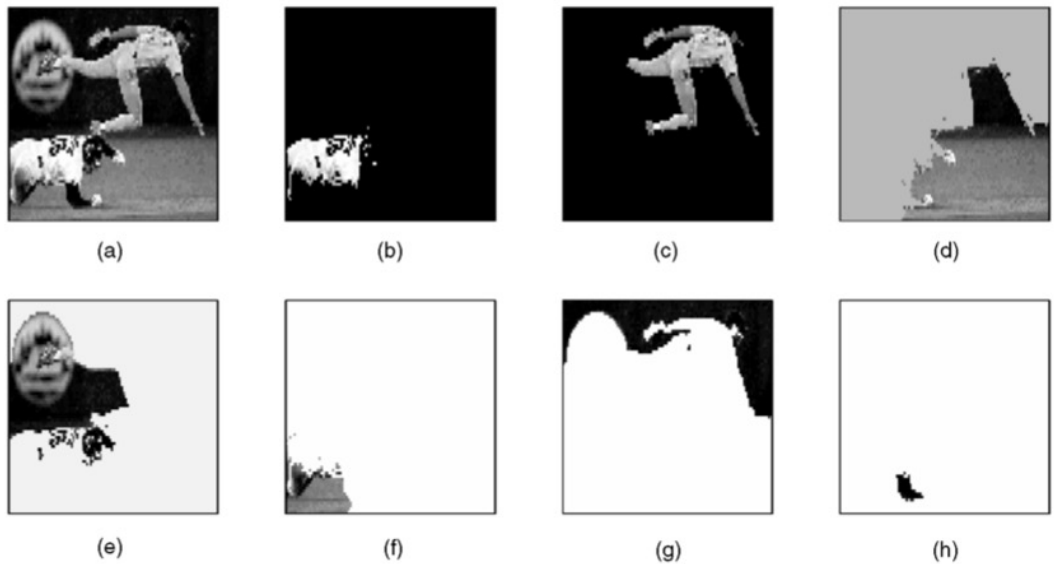
NCut

- Sort the eigenvectors from the smallest to the largest.



Shi and Malik. Normalized Cuts and Image Segmentation. TPAMI 2000.

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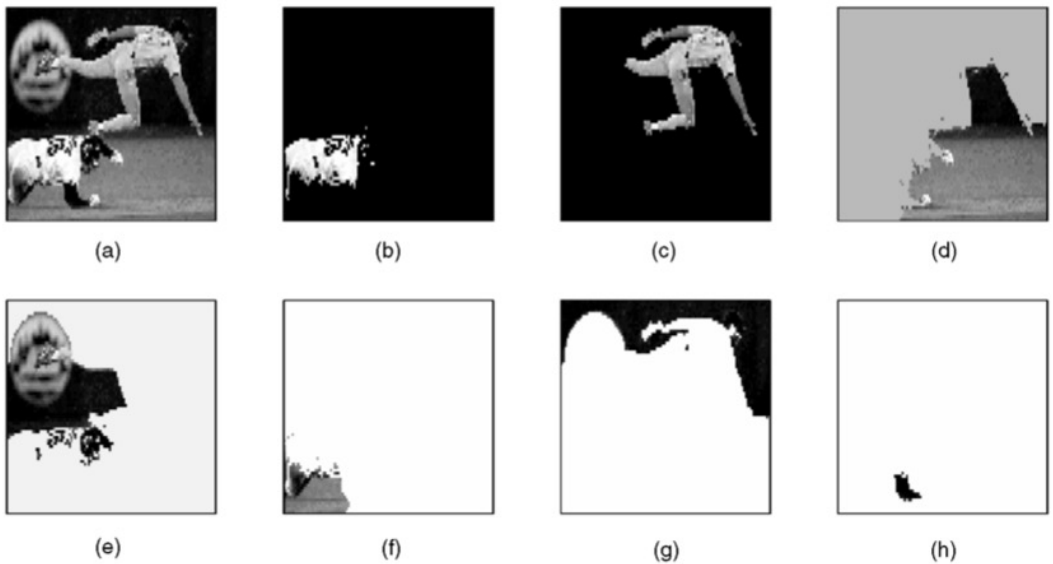
NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.



Shi and Malik. Normalized Cuts and Image Segmentation. TPAMI 2000.

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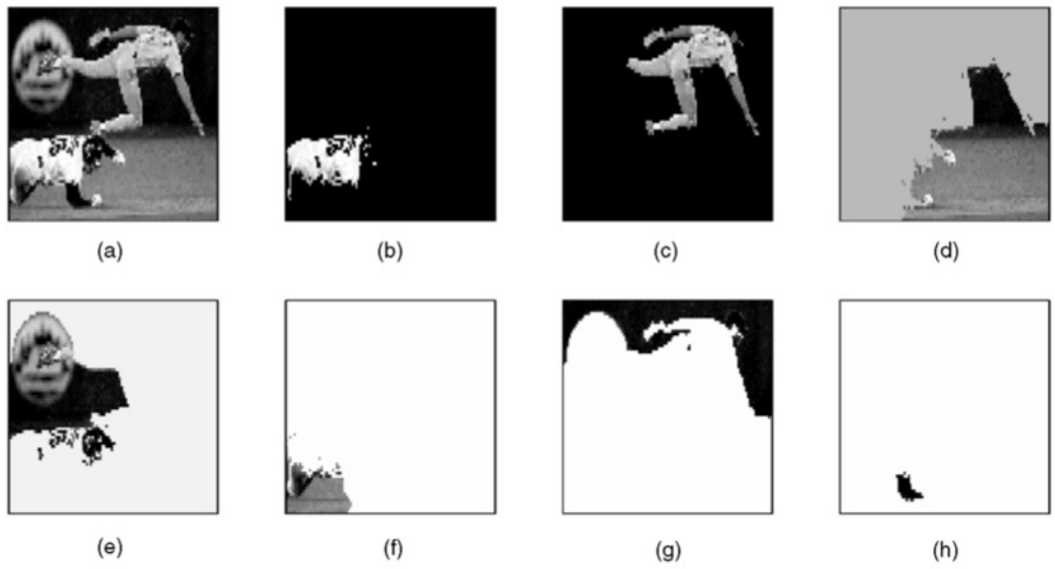
NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.
- Now, instead of segmenting pixels, we can directly segment semantically meaningful representations from self-supervision.



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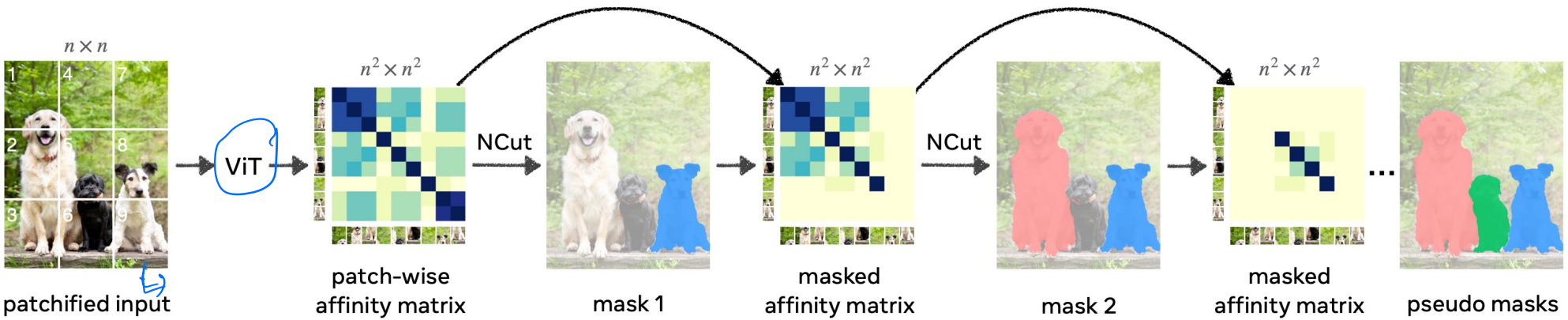
MaskCut

- Use a pretrained DINO ViT network.



Wang et al. Self-supervised transformers for unsupervised object discovery using normalized cut. CVPR 2022.
Wang et al. Cut and Learn for Unsupervised Object Detection and Instance Segmentation. CVPR 2023.

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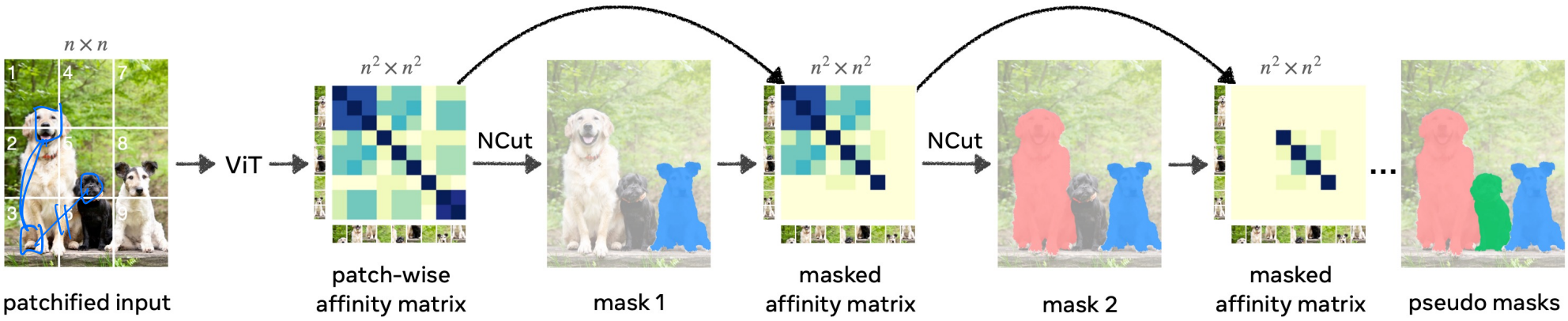
MaskCut

- Use a pretrained DINO ViT network.
- Use the "key" features from the last attention layer: $W_{ij} = \frac{K_i K_j}{\|K_i\|_2 \|K_j\|_2}$



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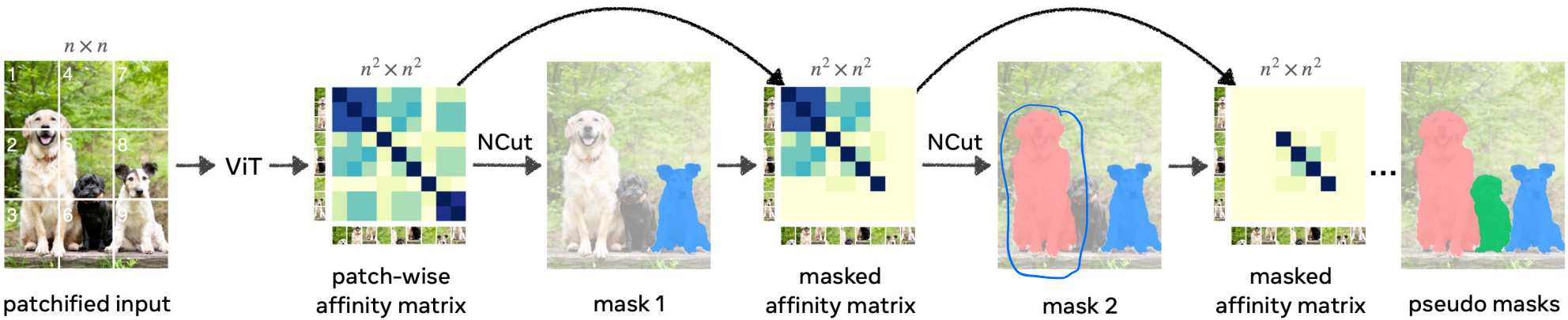
- Iterative NCut on the pairwise matrix by masking out the regions from previous stages.

Graph cut.



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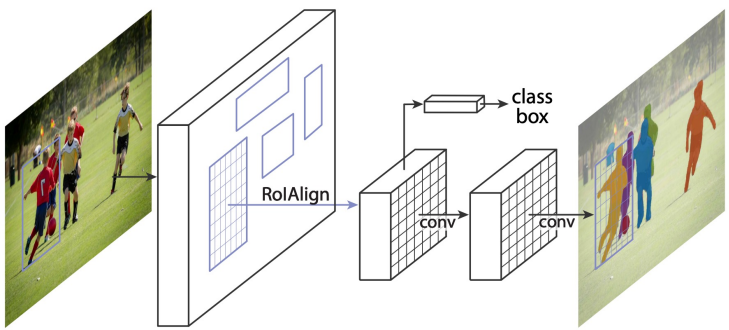
Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.



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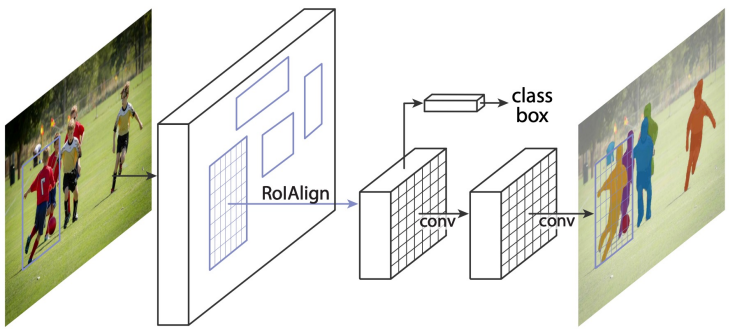

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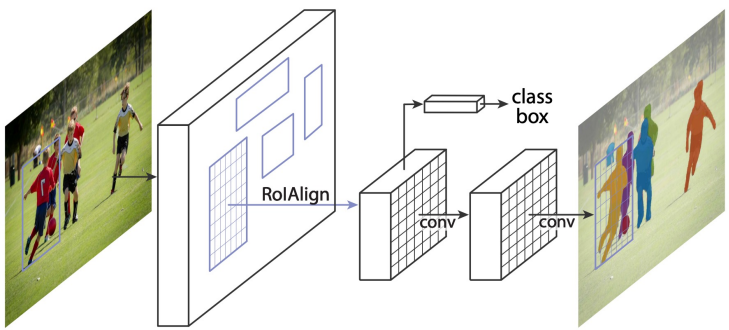
Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.
- Select the predictions with the highest confidence score and use them as labels.
- Neural networks can learn from the noisy labels and output smoother predictions.



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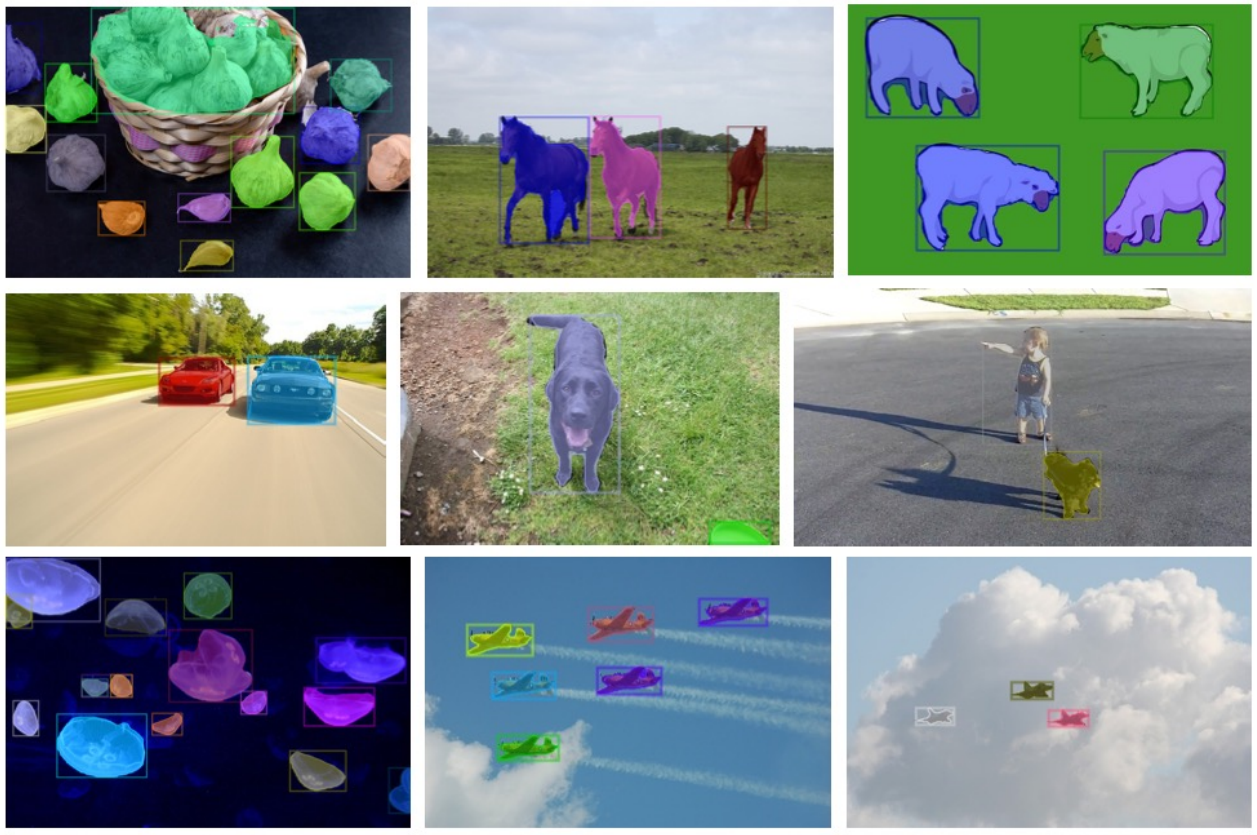
More Visualization



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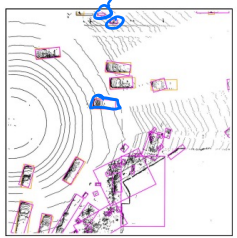
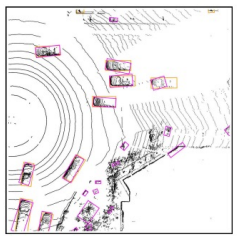
Pseudo Labels in 3D

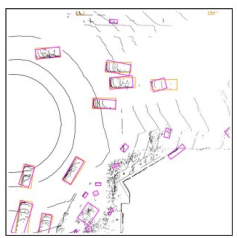
1 Point clustering pseudo-labels



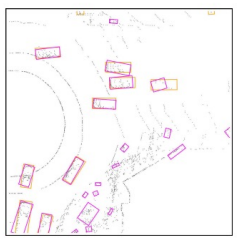
2 Filter out temporally inconsistent tracks



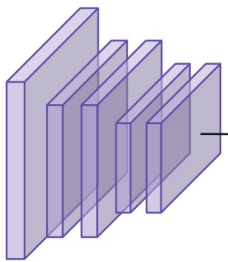
3 Randomly drop lidar beams



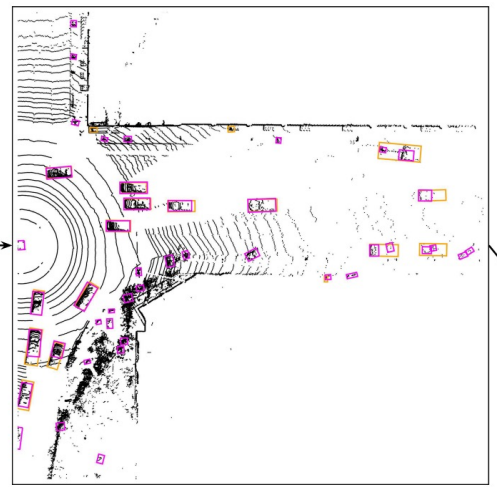
4 Randomly drop spherical rows/cols



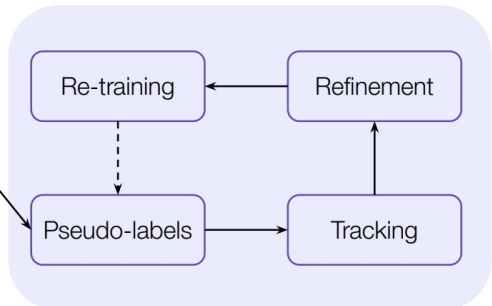
5 Train CNN in short range



6 Zero-shot generalization to long-range



7 Self-training in long-range



Zhang et al. Towards Unsupervised Object Detection from LiDAR Point Clouds. CVPR 2023.

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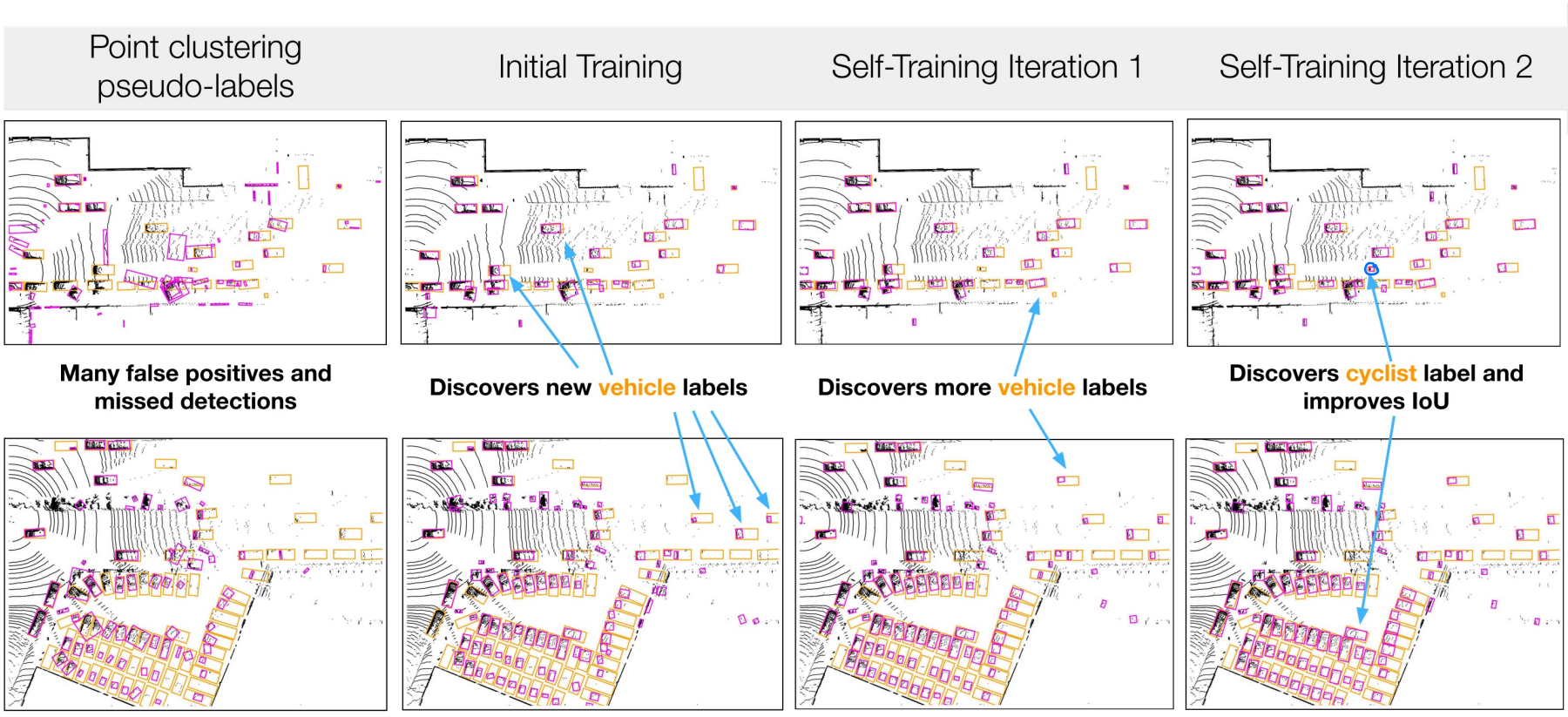
Iterative Refinement of Pseudo Labels



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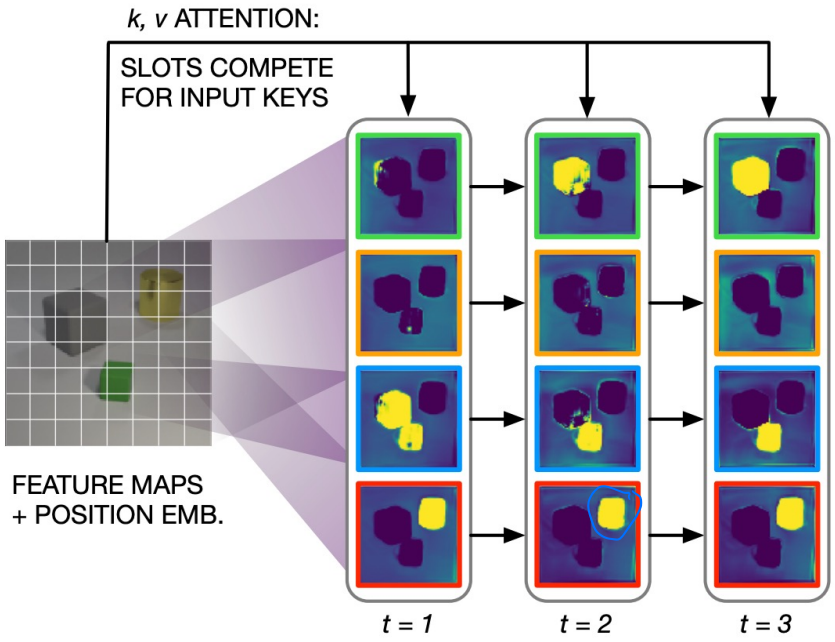
Slot Attention Networks

- Can we learn clustering as an end-to-end operation?



(a) Slot Attention module.

Locatello et al. Object-Centric Learning with Slot Attention. NeurIPS 2020.

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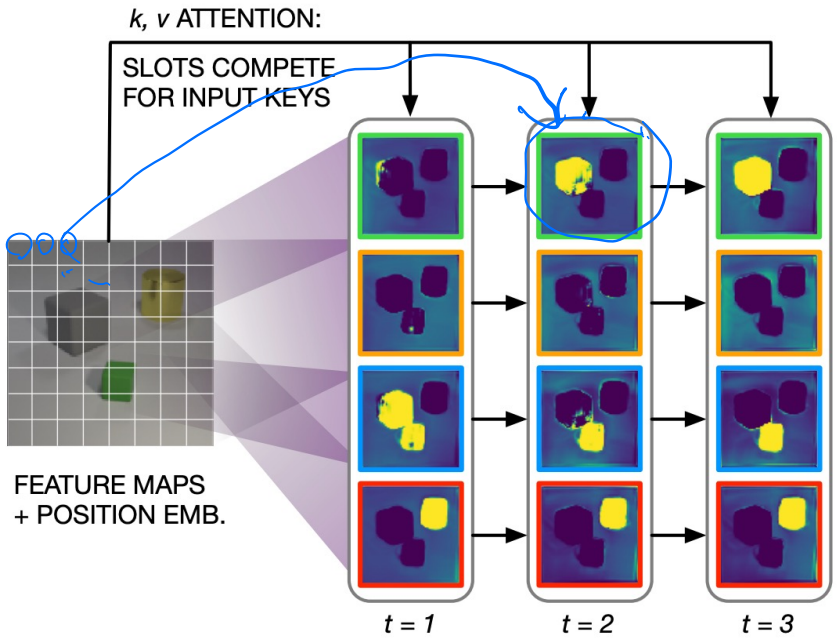

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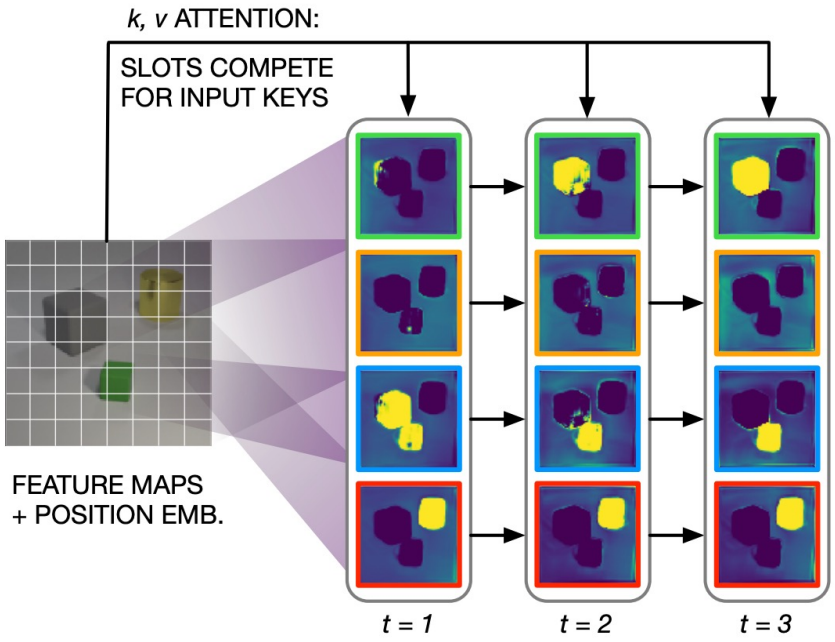

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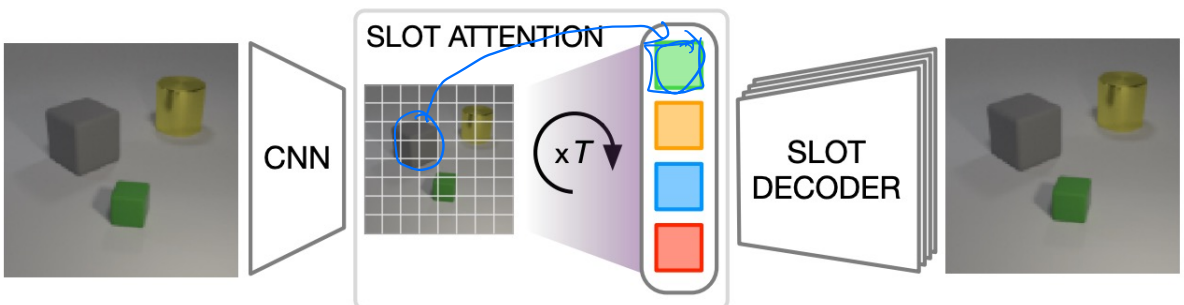

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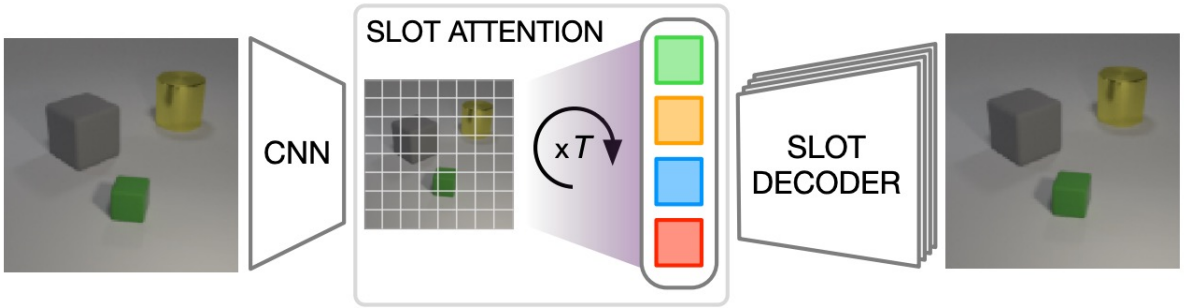
total objects.



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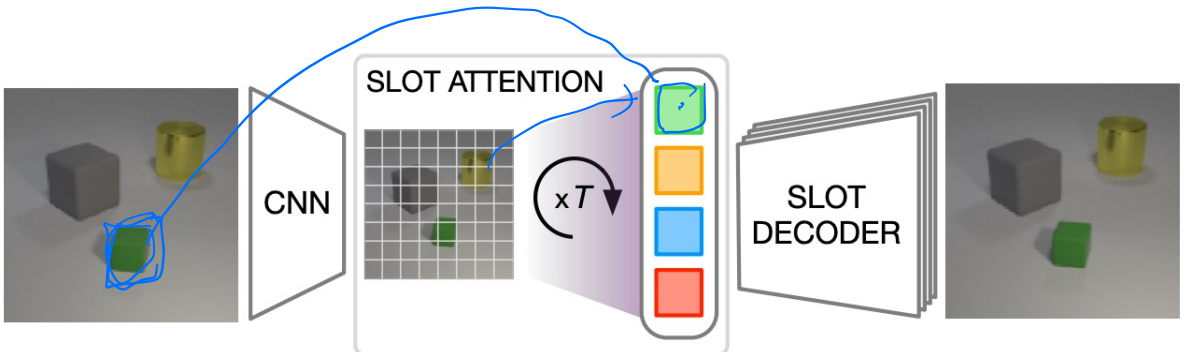

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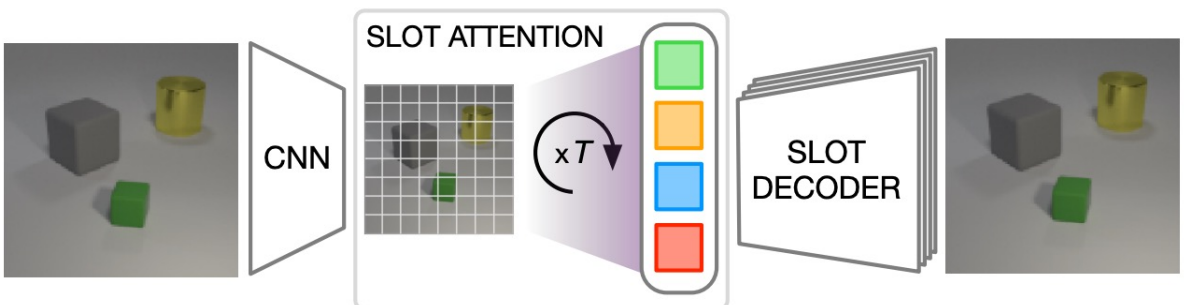

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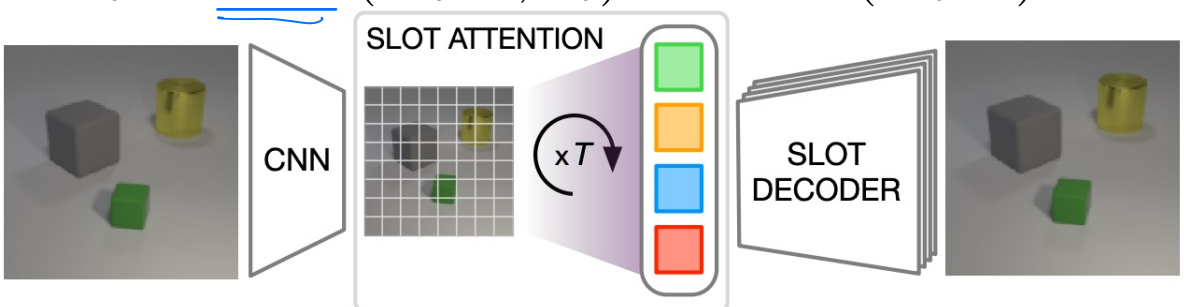

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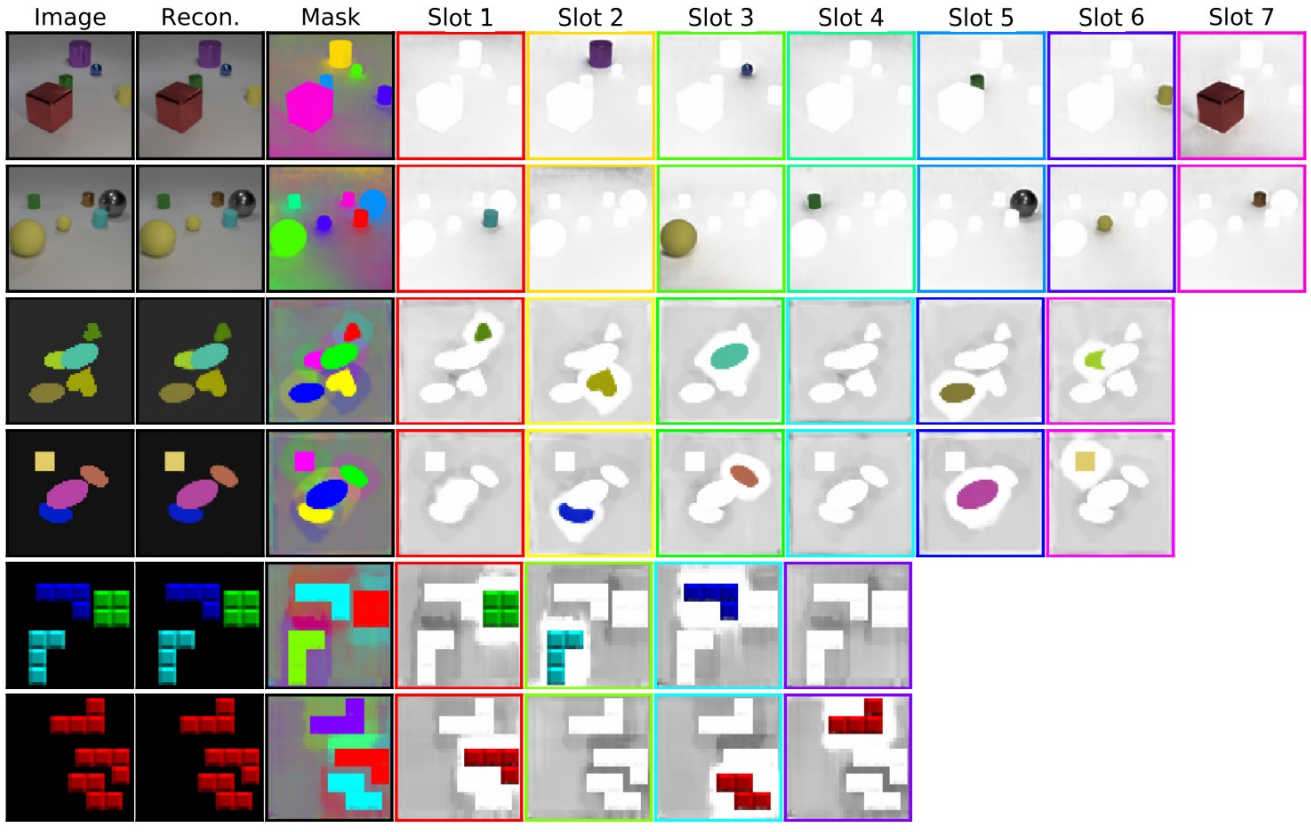

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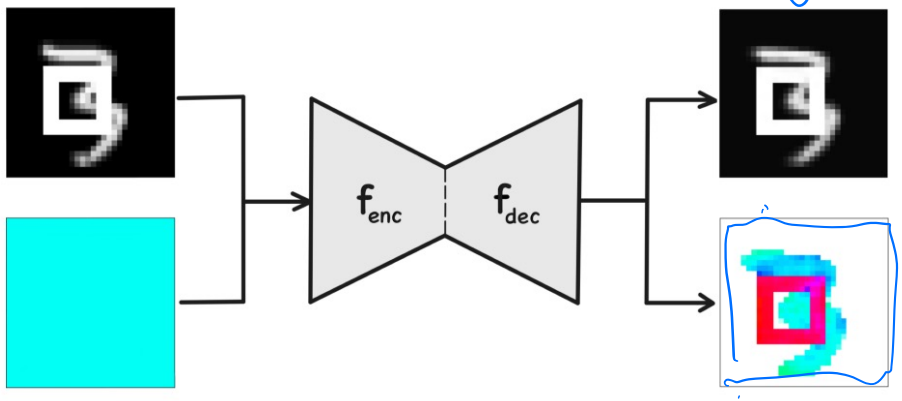
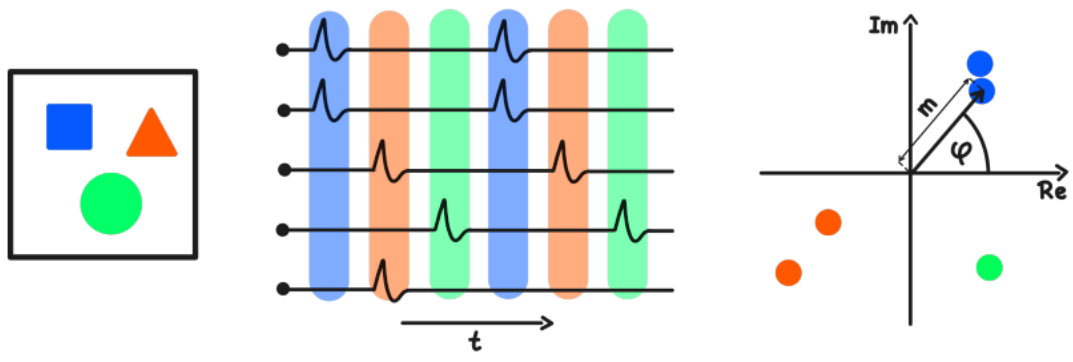
Complex-Valued Autoencoders (CAEs)

- The complex number can represent magnitude and phase: $z = m \cdot e^{i\varphi} \in \mathbb{C}$.





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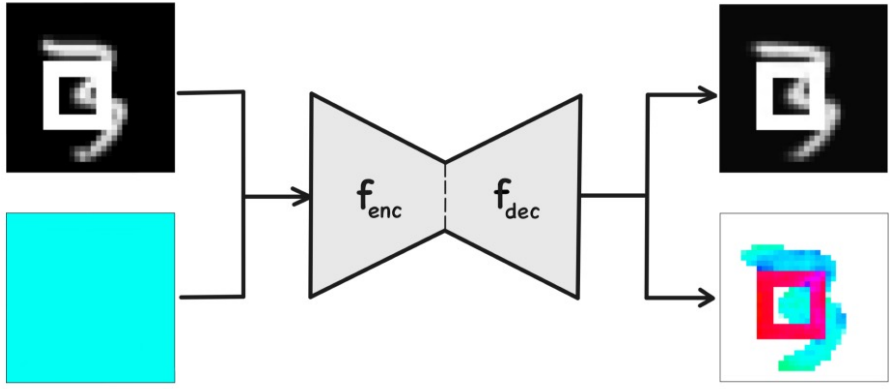

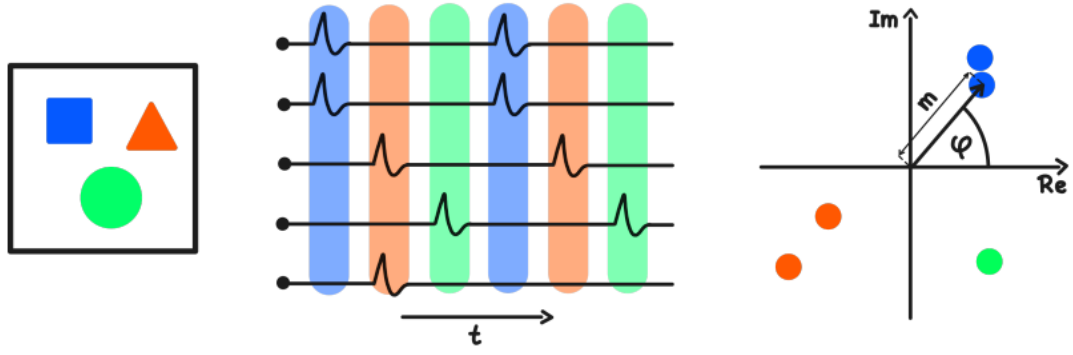

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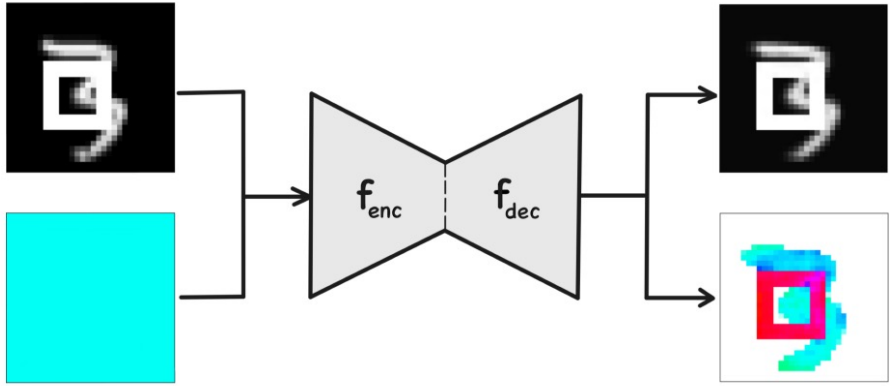

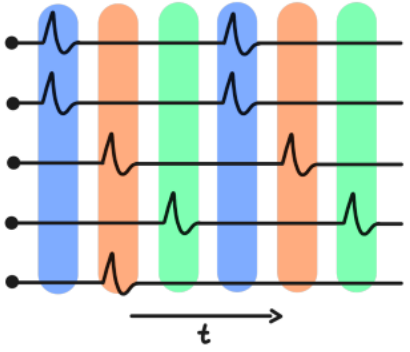

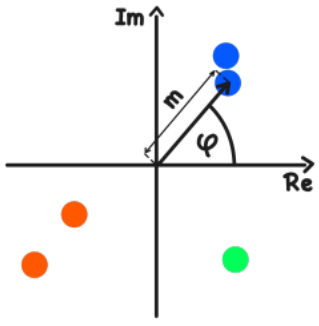

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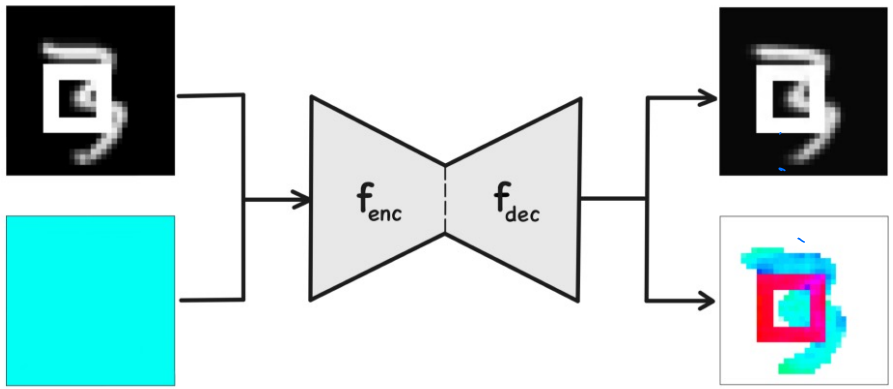

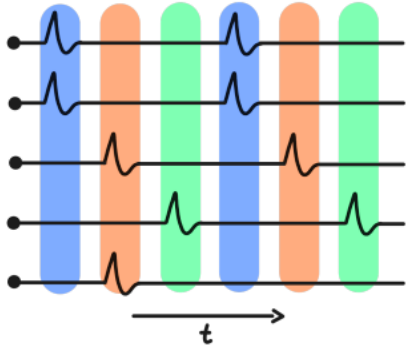

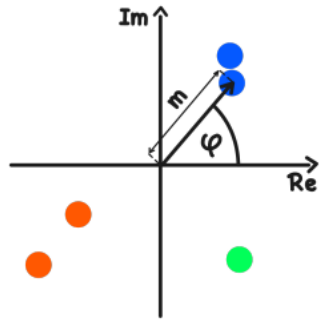

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CAE: More Details

- Apply weights separately to real and imaginary:

$$\boldsymbol{\psi} = f_{\mathbf{w}}(\mathbf{z}) = f_{\mathbf{w}}(\mathrm{Re}(\mathbf{z})) + f_{\mathbf{w}}(\mathrm{Im}(\mathbf{z})) \cdot i \quad \in \mathbb{C}_{d_{\mathrm{out}}}$$

Shared network.

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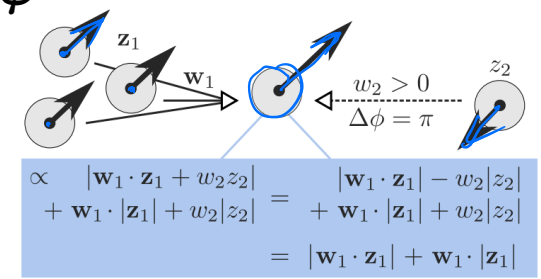
- Gating:

$$\boldsymbol{\chi} = f_{\mathbf{w}}(|\mathbf{z}|) + \boldsymbol{b}_{\boldsymbol{m}} \in \mathbb{R}^{d_{\text{out}}}$$

$$\boldsymbol{m}_{\mathbf{z}} = 0.5 \cdot \boldsymbol{m}_{\boldsymbol{\psi}} + 0.5 \cdot \boldsymbol{\chi} \in \mathbb{R}^{d_{\text{out}}}$$



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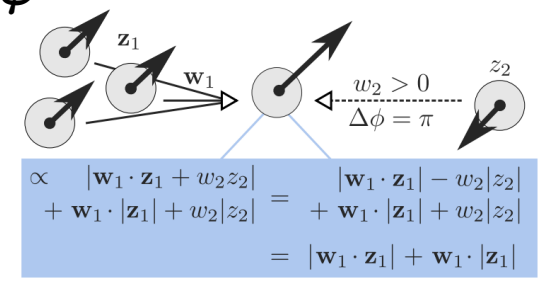

- Activation: $\mathbf{z}' = \mathrm{ReLU}(\mathrm{BatchNorm}(\boldsymbol{m}_{\mathbf{z}})) \cdot e^{i\boldsymbol{\varphi}_{\boldsymbol{\psi}}} \quad \in \mathbb{C}^{d_{\text{out}}}$

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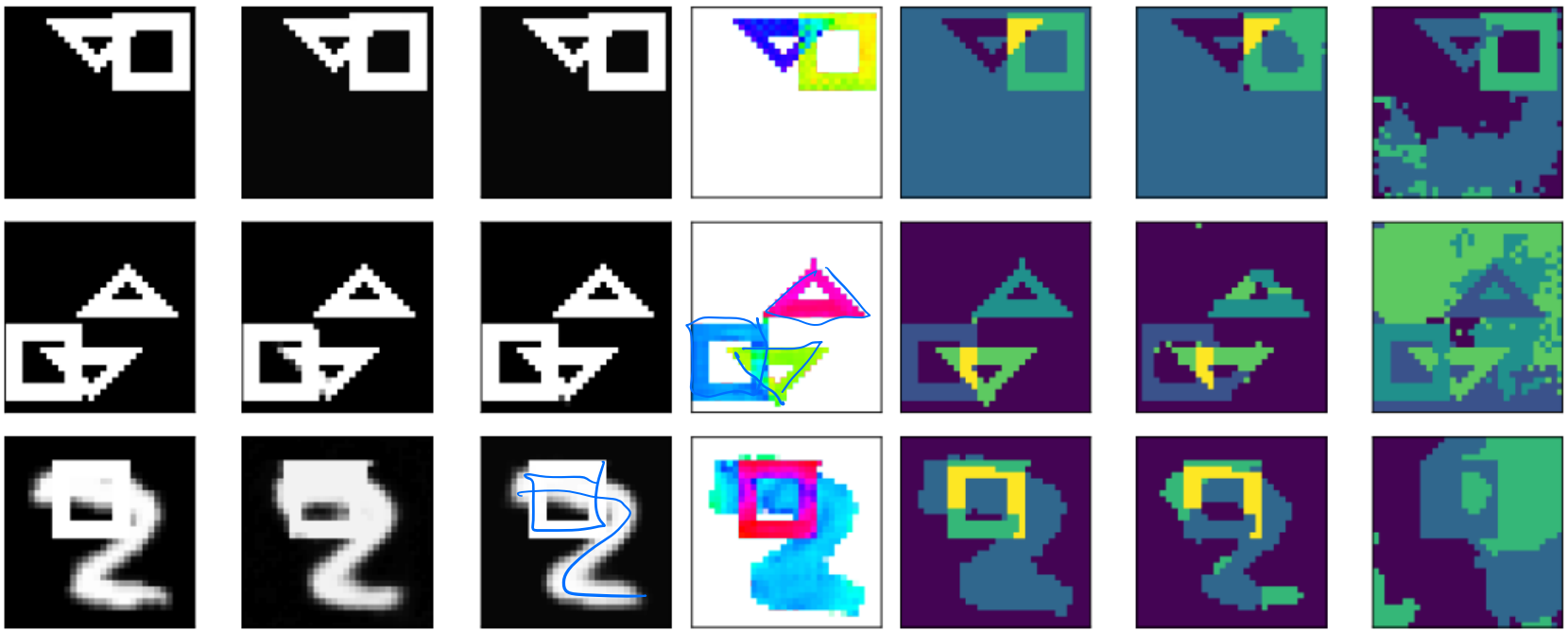

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  - Possible to train from scratch!
- What do we make use of the discovered objects? Is it better to keep the awareness in the latent space?

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Module 4:
World Models and End-to-End Planning

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World Models: Predicting Future

3D, latent, → predict future ← → planning.

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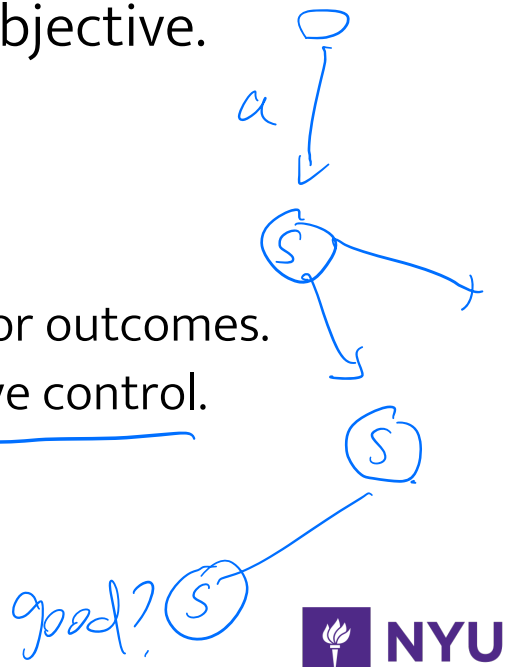
  - Can help planning by predicting at test time. Model predictive control.



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Model-Predictive Control (MPC)

- A classic example of world models.
- Analytical forms, complete knowledge of the dynamical system.

Rawlings et al. Model Predictive Control: Theory, Computation, and Design.
https://engineering.purdue.edu/~zak/Second_ed/MPC_handout.pdf

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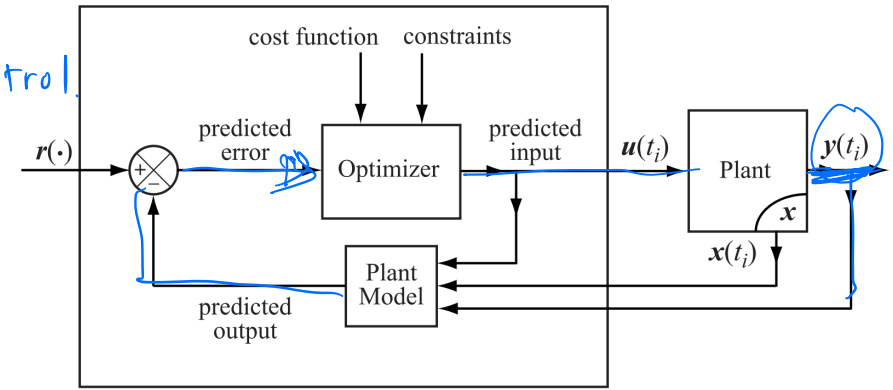

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Linear Form:

$$\frac{dx}{dt} = A\mathbf{x} + B\mathbf{u}$$
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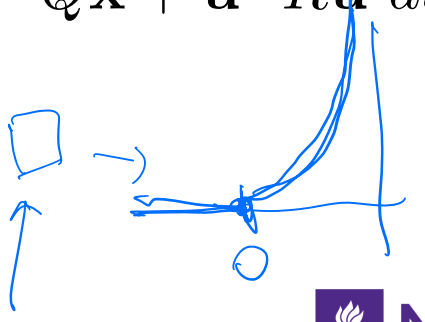
Quadratic Form (LQR):

$$J = \int_0^\infty \mathbf{x}^\top Q\mathbf{x} + \mathbf{u}^\top R\mathbf{u}\, dt.$$



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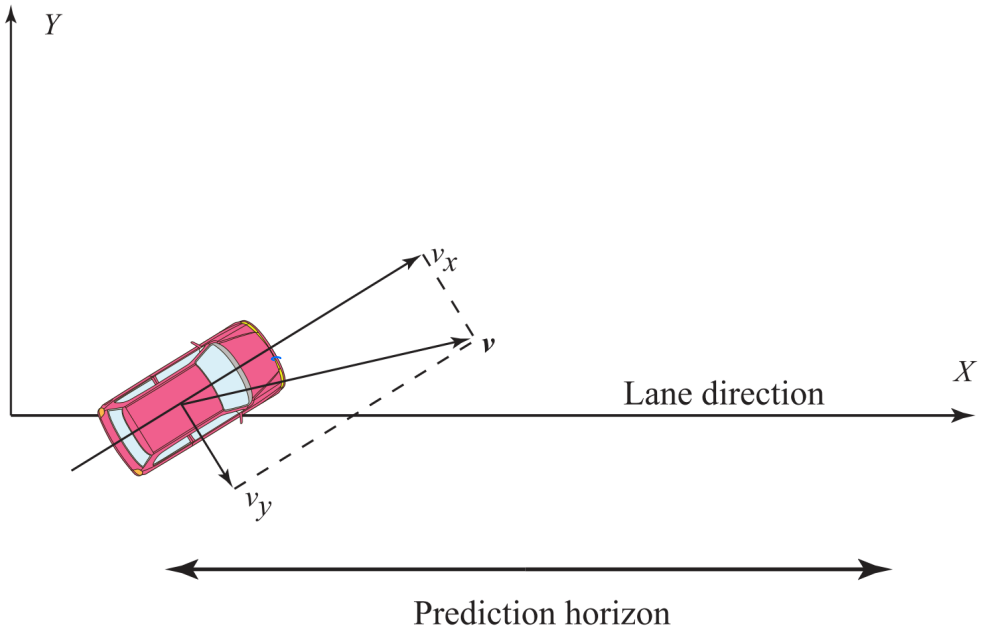
$$J(\mathbf{x}(0), \mathbf{u})$$

Form (LQR):

$$\mathbf{x}^\top Q\mathbf{x} + \mathbf{u}^\top R\mathbf{u}\ dt.$$



- Example: Cars: x: position velocity angle angular velocity; u: jerk and angular accel.;

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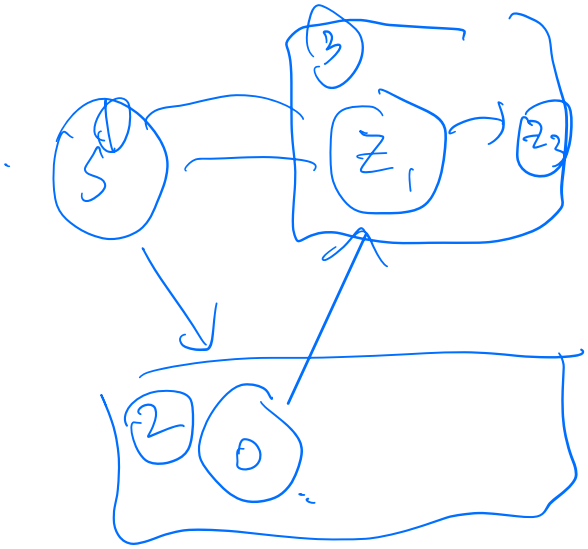
- Latent
  - Generate future latent states
  - May go outside of the latent manifold



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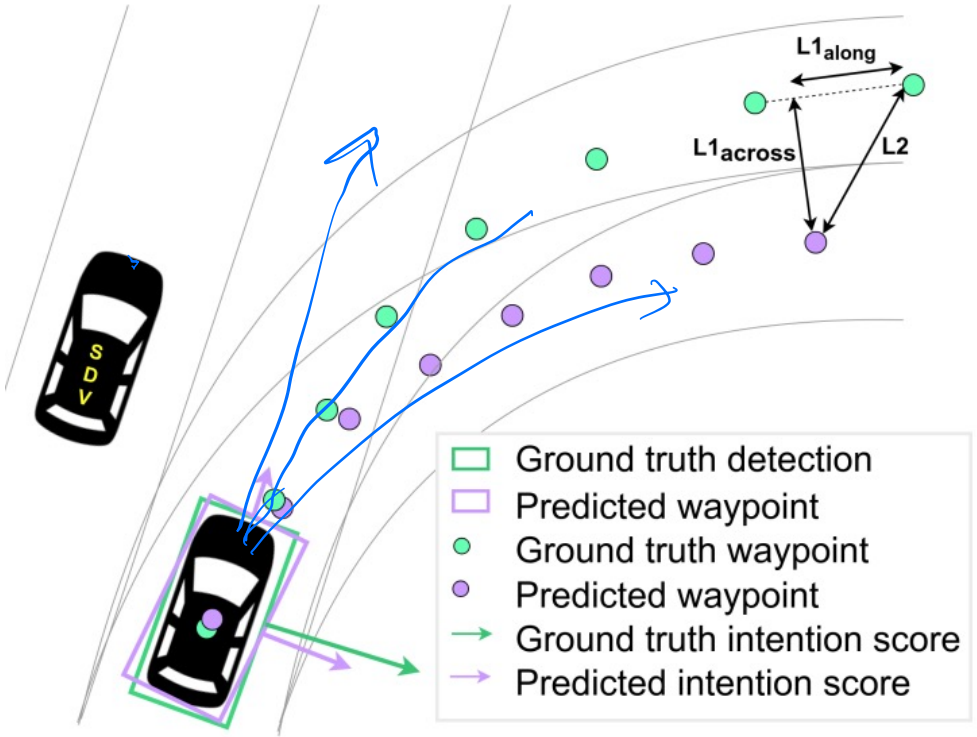
Trajectory Prediction as Object Detection

- We also need to predict external dynamic objects.



Casas et al. IntentNet: Learning to Predict Intention from Raw Sensor Data. CoRL 2018.

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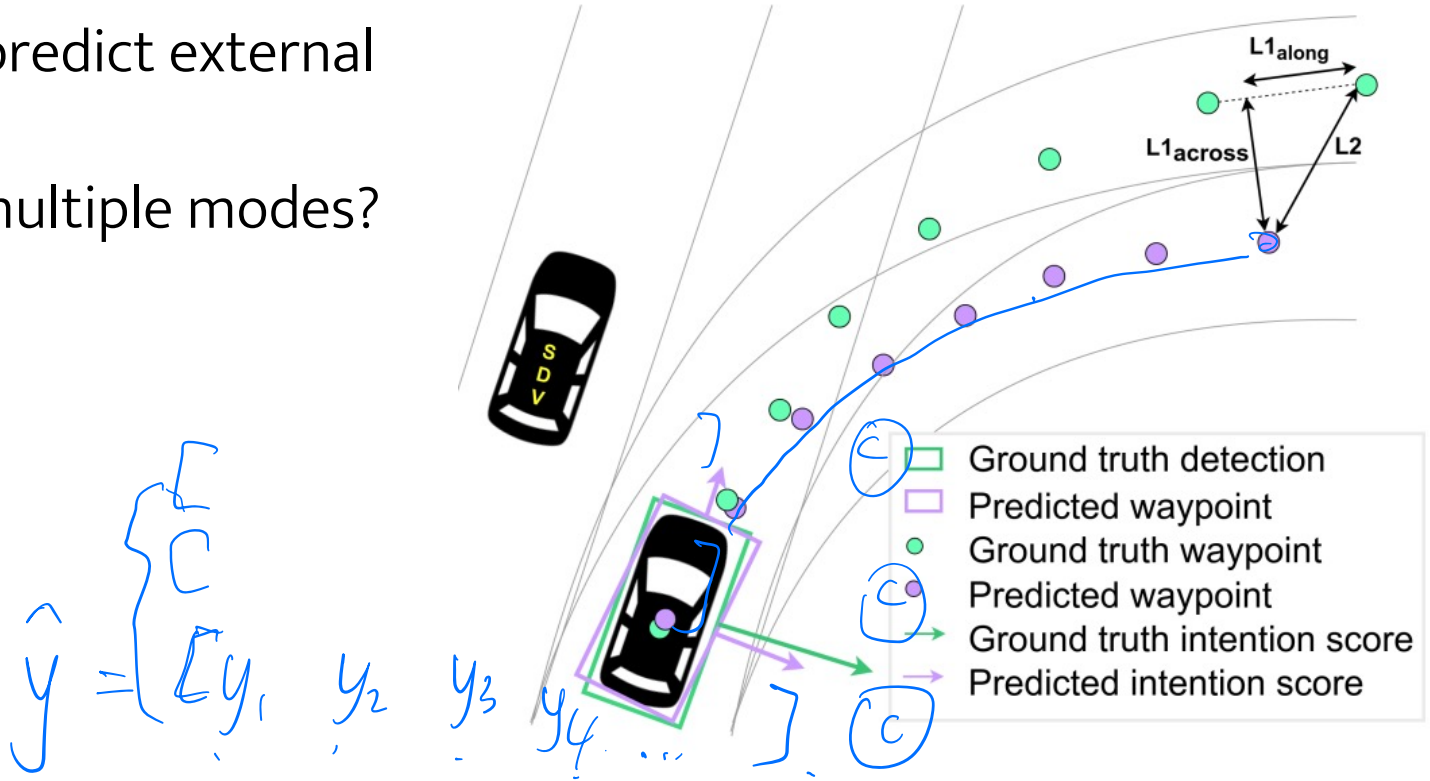
- We also need to predict external dynamic objects.
- How to capture multiple modes?



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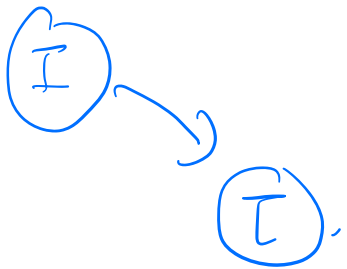

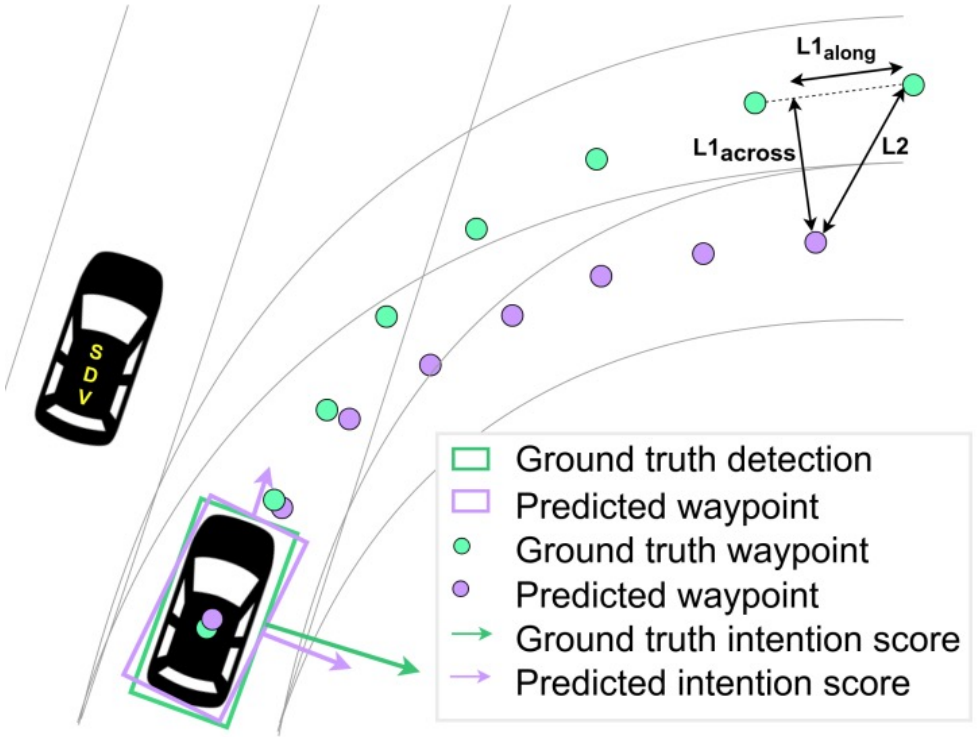

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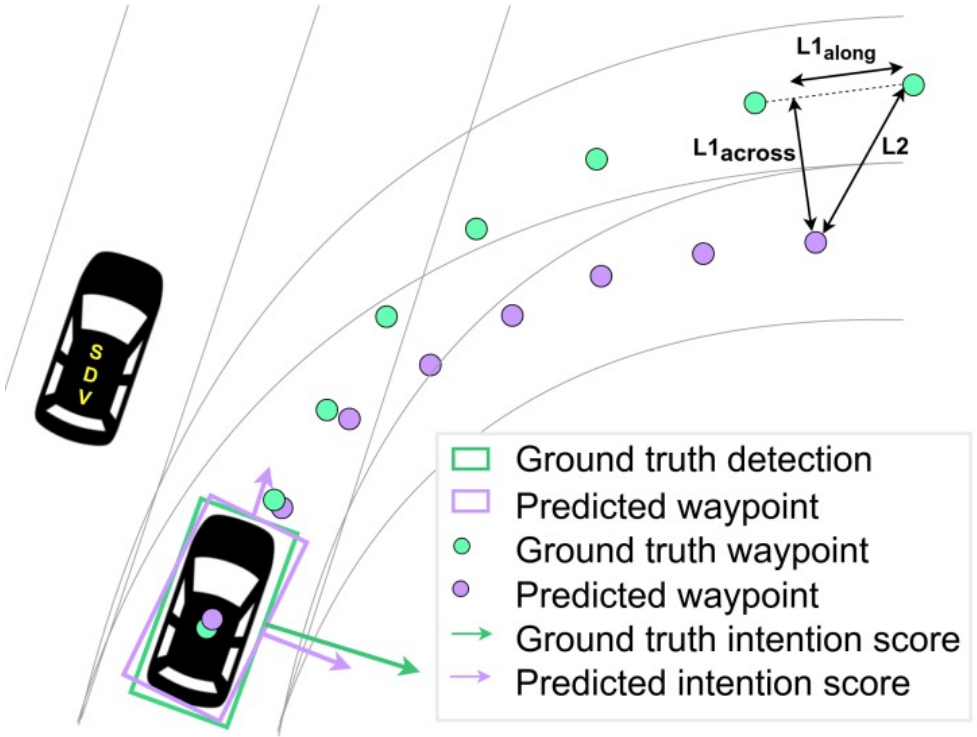

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- We also need to predict external dynamic objects.
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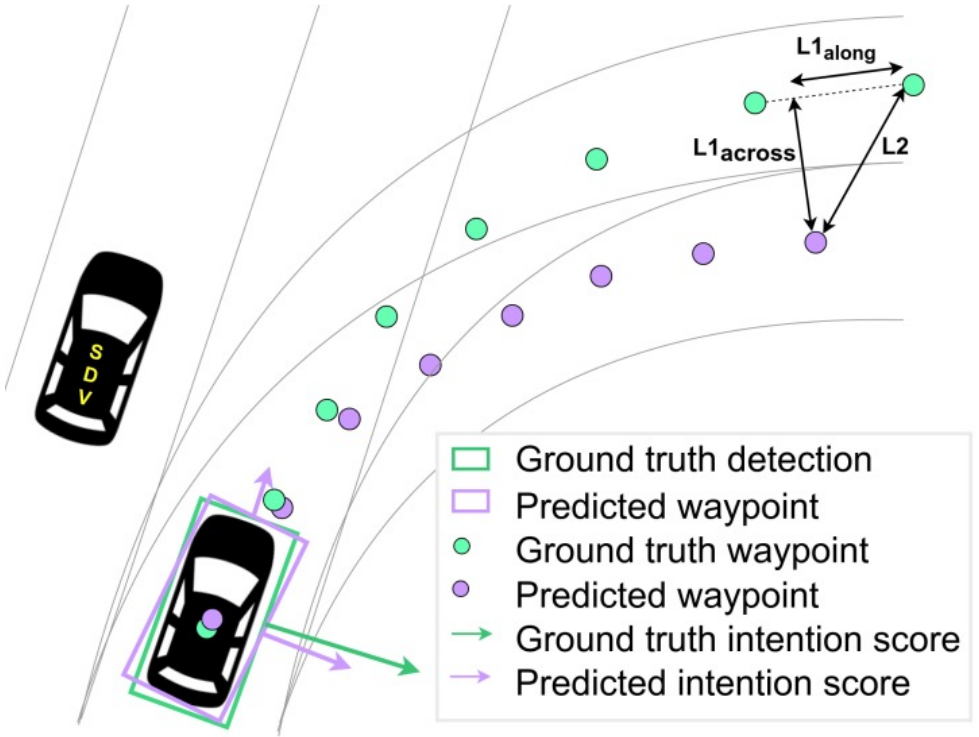
- Output multiple trajectories
- Requires high-level action labels.



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Semantic Occupancy Volume Prediction

- 4-D volume: H, W, T, C (class) $o_{i,j}^{t,c}$

num. timesteps.

Perceive, Predict, and Plan: Safe Motion Planning Through Interpretable Semantic Representations. ECCV 2020.

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Semantic Occupancy Volume Prediction

- 4-D volume: H, W, T, C (class) $o_{i,j}^{t,c}$
- Doesn't grow with the increasing number of actors.

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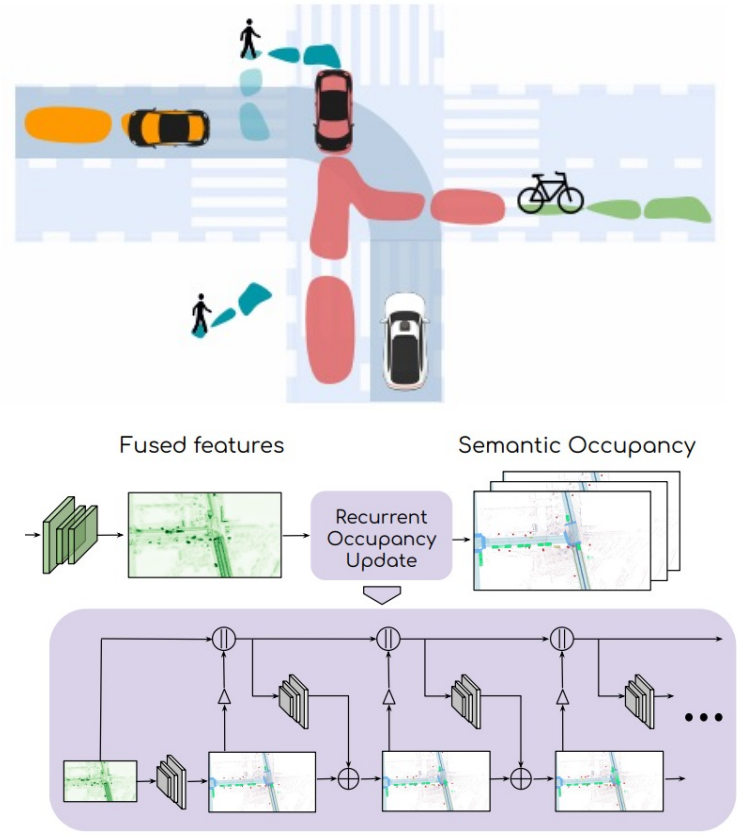
Semantic Occupancy Volume Prediction

- 4-D volume: H, W, T, C (class) $o_{i,j}^{t,c}$
- Doesn't grow with the increasing number of actors.
- Recurrent occupancy updates for further into the future.

$$l^{t,c} = l^{t-1,c} + \mathcal{U}_\theta^t(f_{\text{occ}}, l^{0:t-1,c})$$

↳ render Cost volume.



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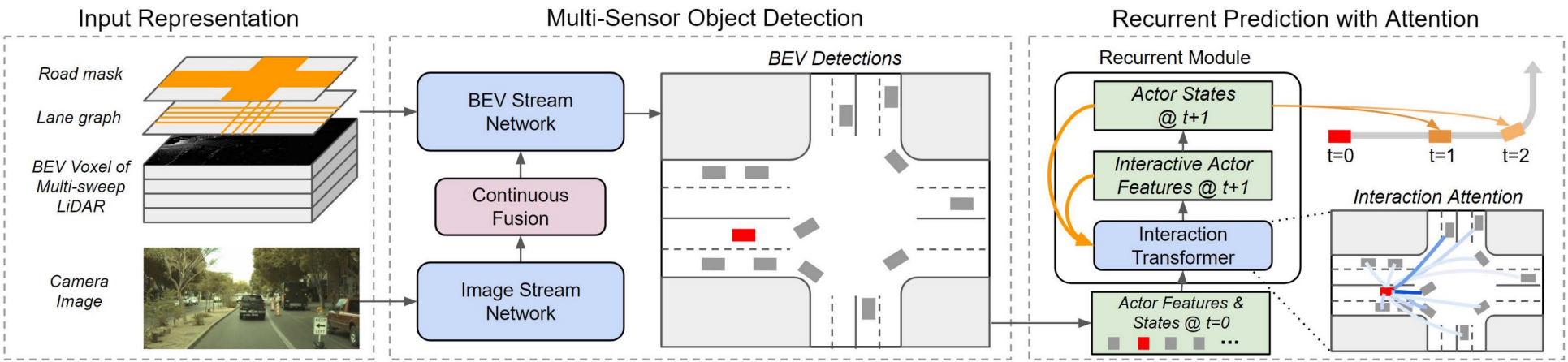
Multi-Agents Joint Prediction

- Joint predict future trajectories by attending to other actors.



Li et al. End-to-end Contextual Perception and Prediction with Interaction Transformer. IROS 2020.

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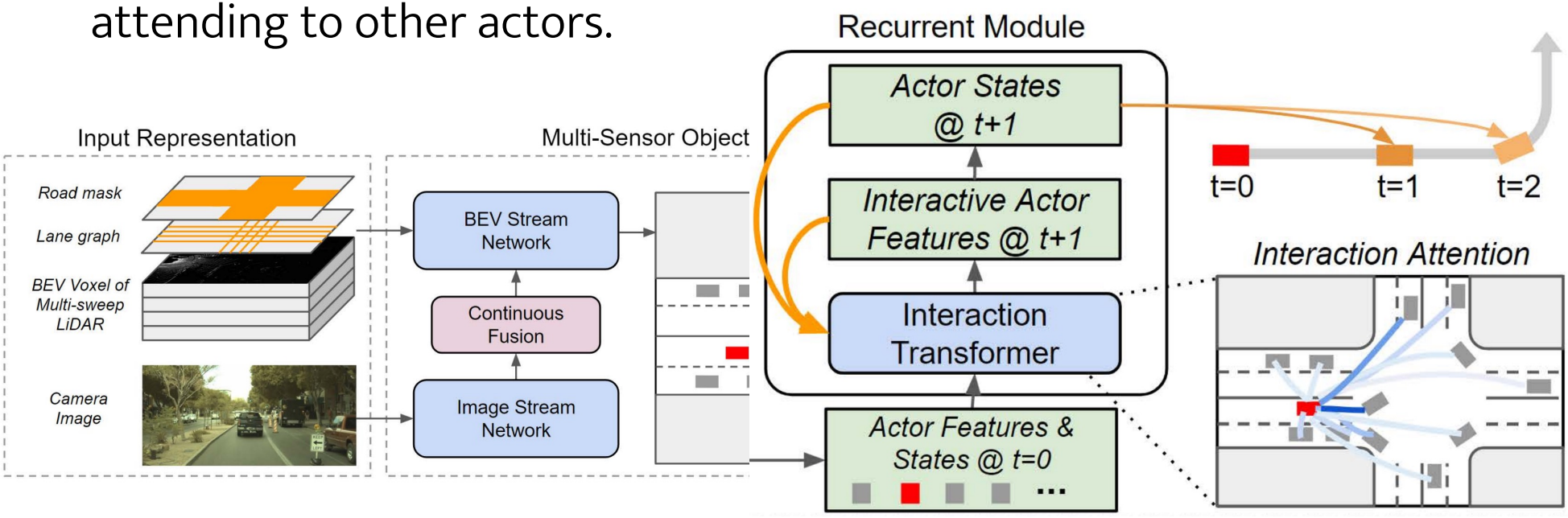

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Generating Raw Pixels

https://deepmind.google/blog/genie-3-a-new-frontier-for-world-models/

Genie 3: A new frontier for world models

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Genie 3: A new frontier for world models

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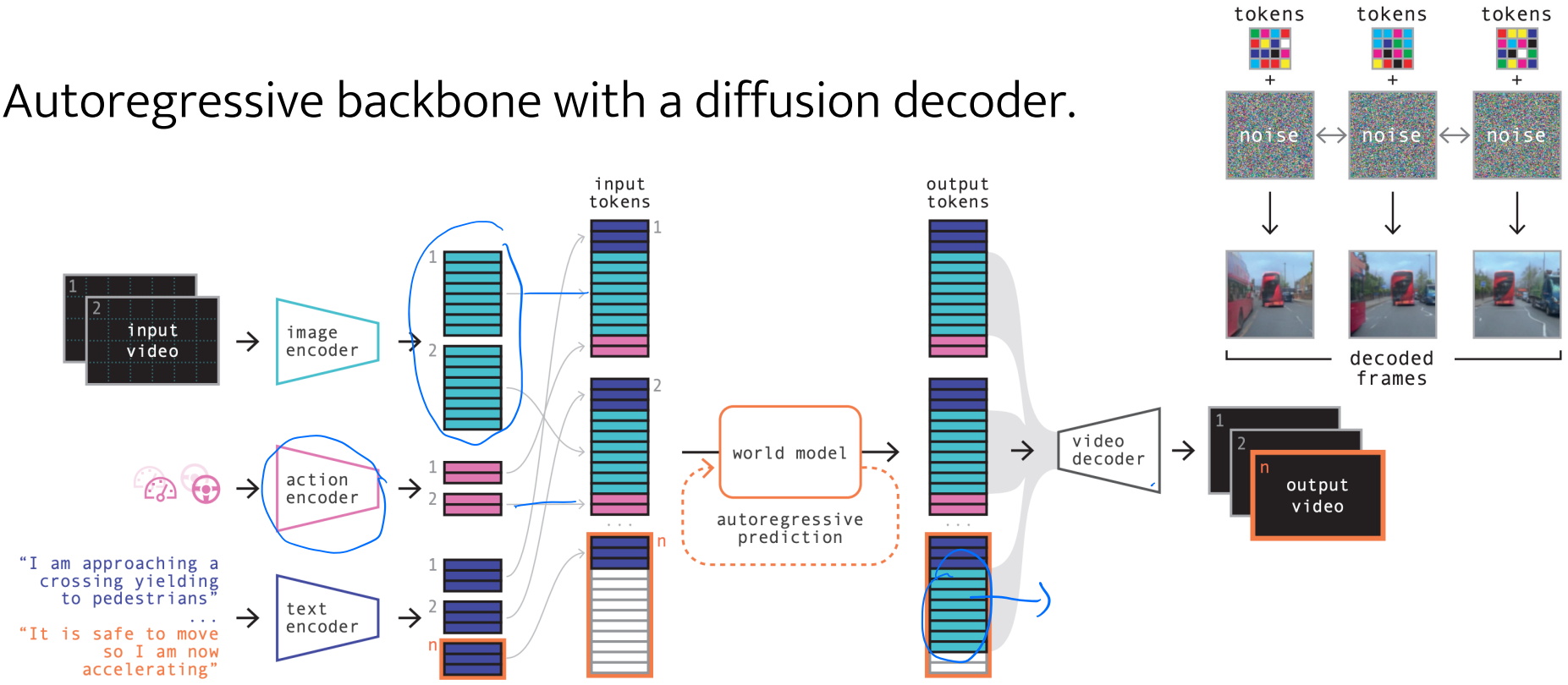
Action-Conditioned Video Generation

- Autoregressive backbone with a diffusion decoder.



Hu et al. GAIA-1: A Generative World Model for Autonomous Driving. arXiv 2023.

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https://www.youtube.com/watch?v=5Jx2QgEUZUI

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Hu et al. GAIA-1: A Generative World Model for Autonomous Driving. arXiv 2023.

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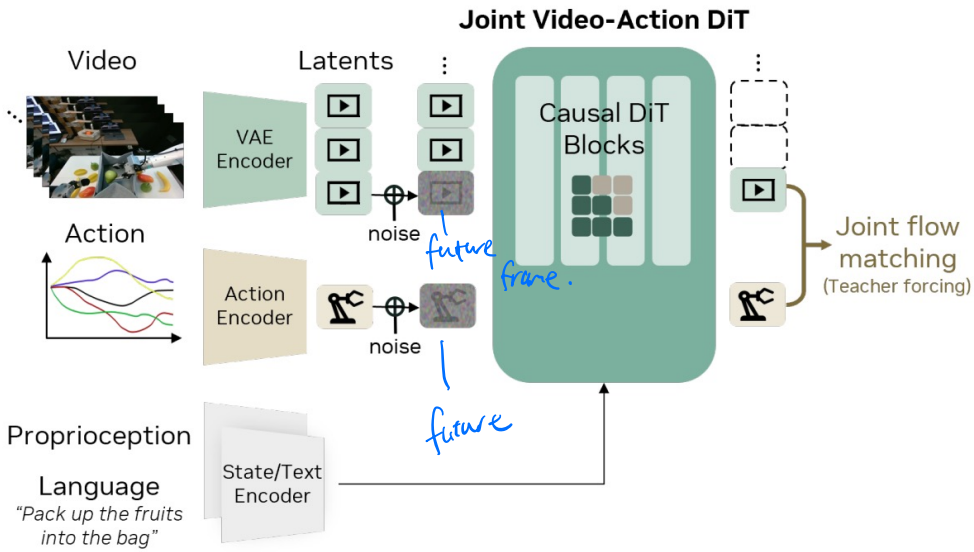
Action-Conditioned Video Generation

- DiT backbone with teacher forcing flow matching objective.
- Single actor robotic videos.



Ye et al. DreamZero: World Action Models are Zero-shot Policies. 2026

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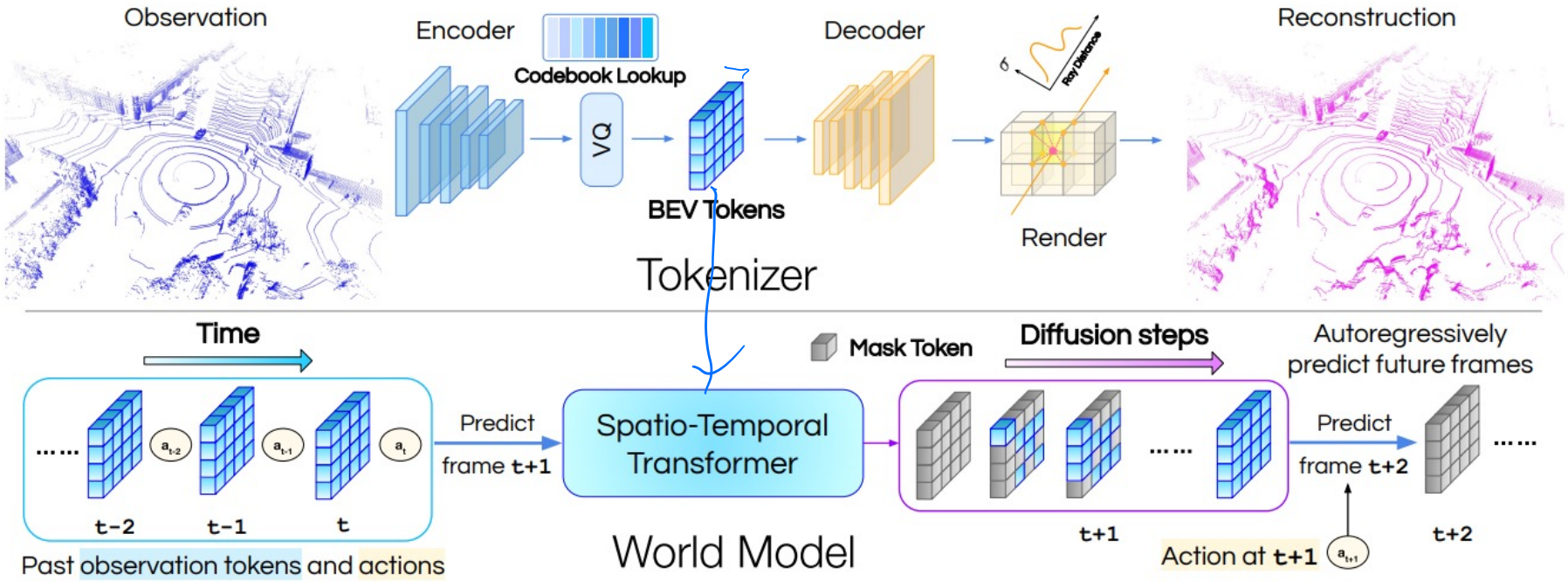
World Model in 3D volume prediction

- Autoregressively predict future 3D point clouds.



Zhang et al. Copilot4D: Learning Unsupervised World Models for Autonomous Driving via Discrete Diffusion. ICLR 2024.

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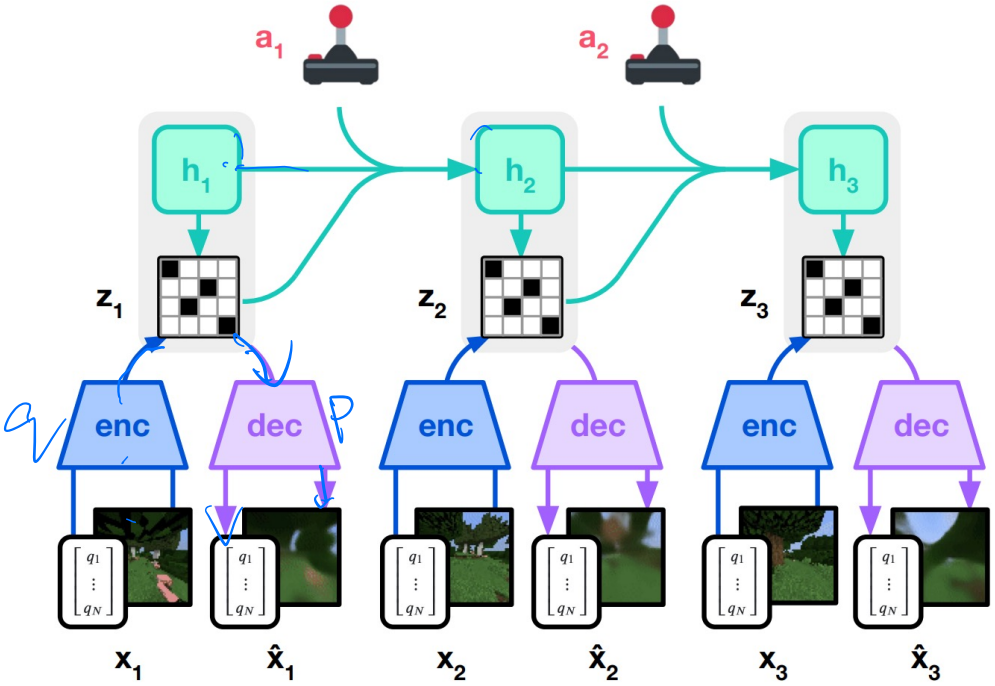
Latent Sequence World Model

- Autoencoder to ensure the latent representations are meaningful.

$$z_t \sim q_\phi(z_t \mid h_t, x_t)$$

$$\hat{x}_t \sim p_\phi(\hat{x}_t \mid h_t, z_t)$$



JEPA

Hafner et al. Mastering Diverse Domains through World Models. 2023.

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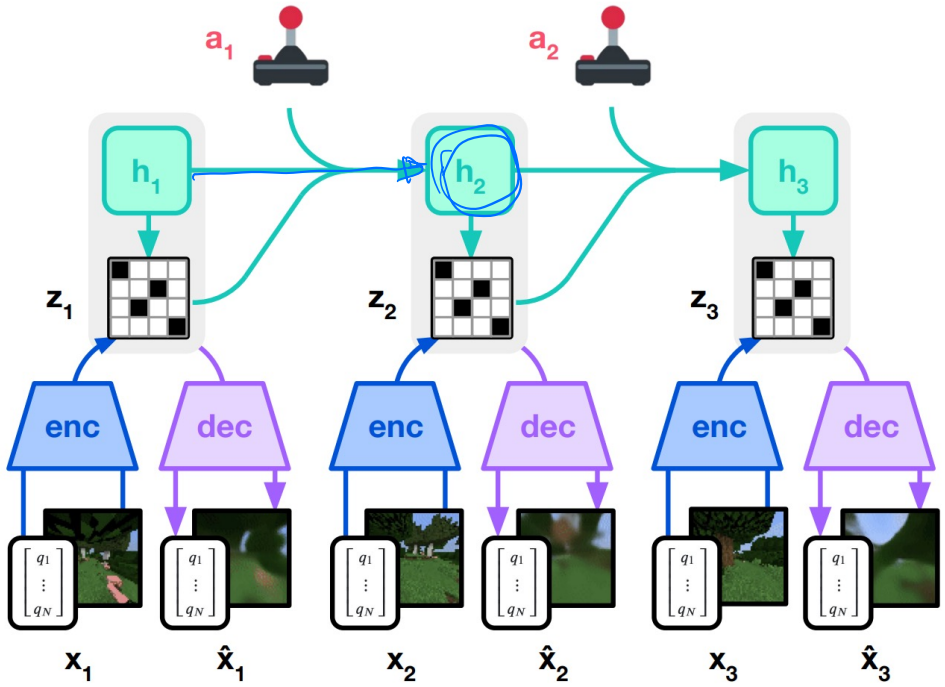
- Learn a sequence model to predict the latent conditioned on previous action.

$$h_t = f_\phi(h_{t-1}, z_{t-1}, a_{t-1})$$

$$\hat{z}_t \sim p_\phi(\hat{z}_t \mid h_t)$$



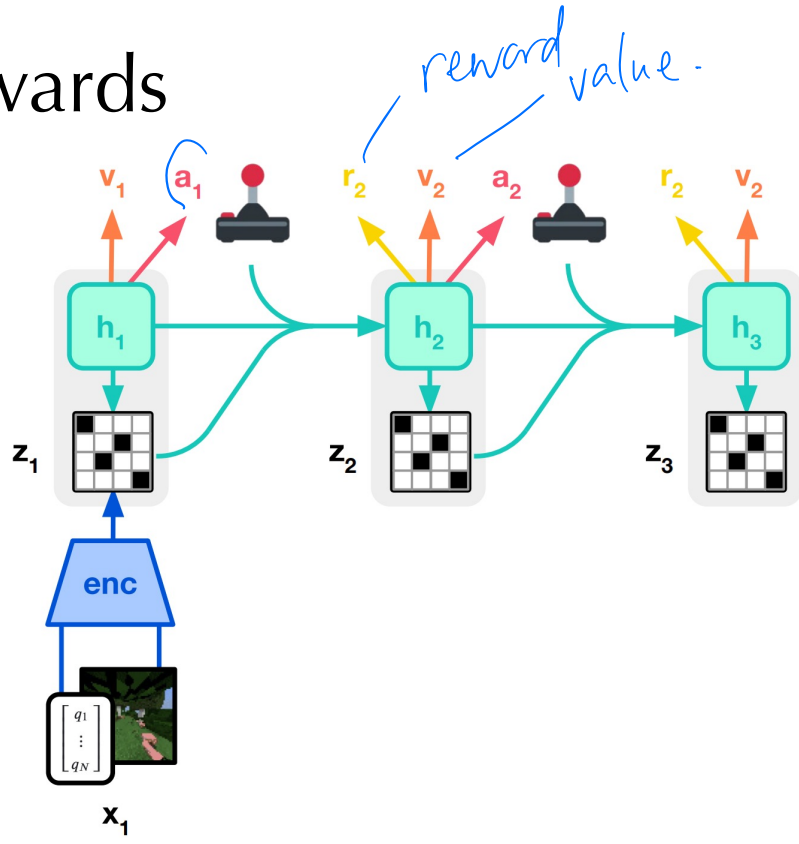
Hafner et al. Mastering Diverse Domains through World Models. 2023.

NYU

Incorporating Rewards

- Predicting reward

$$\hat{r}_t \sim p_\phi(\hat{r}_t \mid h_t, z_t)$$



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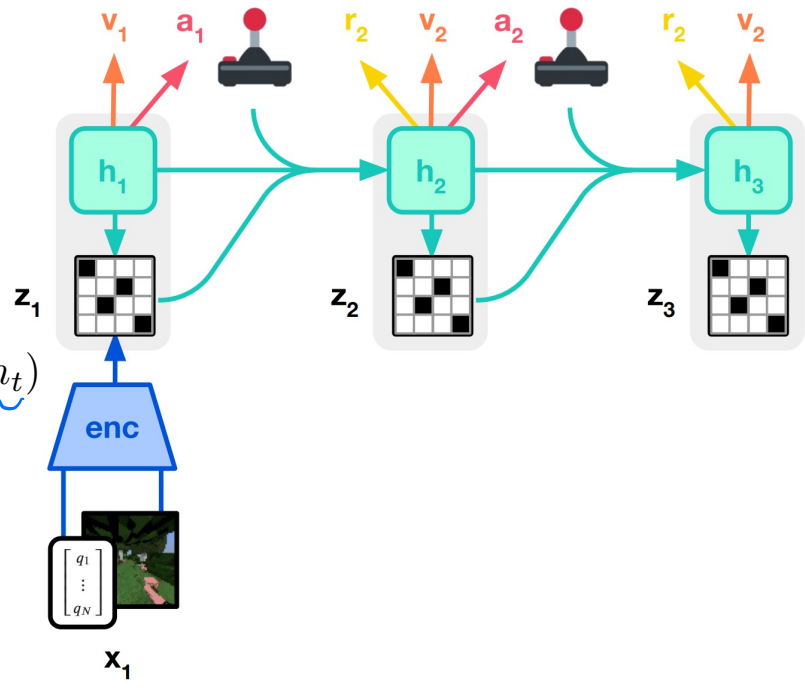
Incorporating Rewards

- Predicting reward

$$\hat{r}_t \sim p_\phi(\hat{r}_t \mid h_t, z_t)$$

- Reconstruction, reward, continue

$$\mathcal{L}_{\text{pred}}(\phi) = -\log p_\phi(x_t \mid z_t, h_t) - \log p_\phi(r_t \mid z_t, h_t) - \log p_\phi(c_t \mid z_t, h_t)$$



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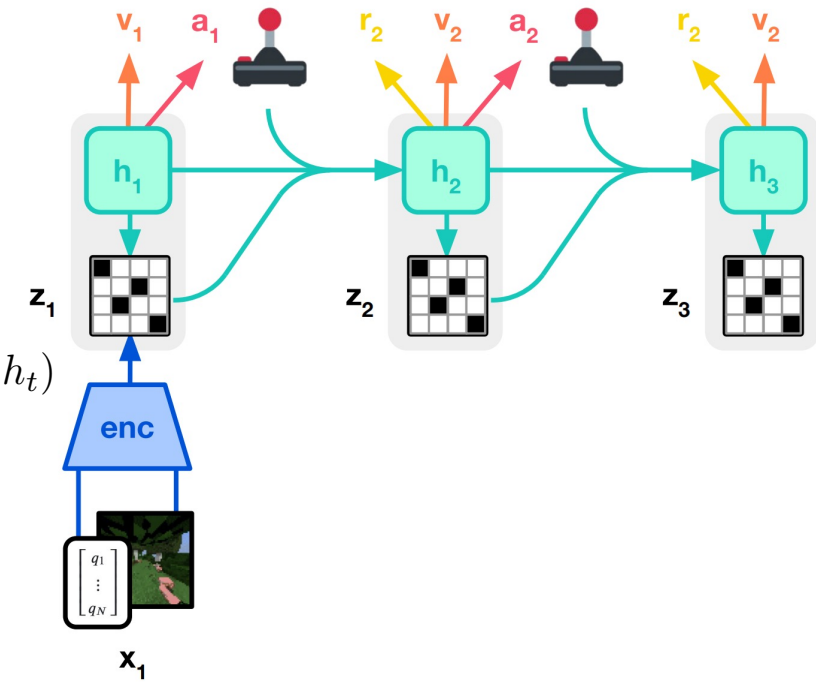
- Dynamics: Predicting future z

$$\mathcal{L}_{\text{dyn}}(\phi) = \max(1, \mathrm{KL}[\mathrm{sg}(q_\phi(z_t \mid h_t, x_t) \parallel p_\phi(z_t|h_t))]$$



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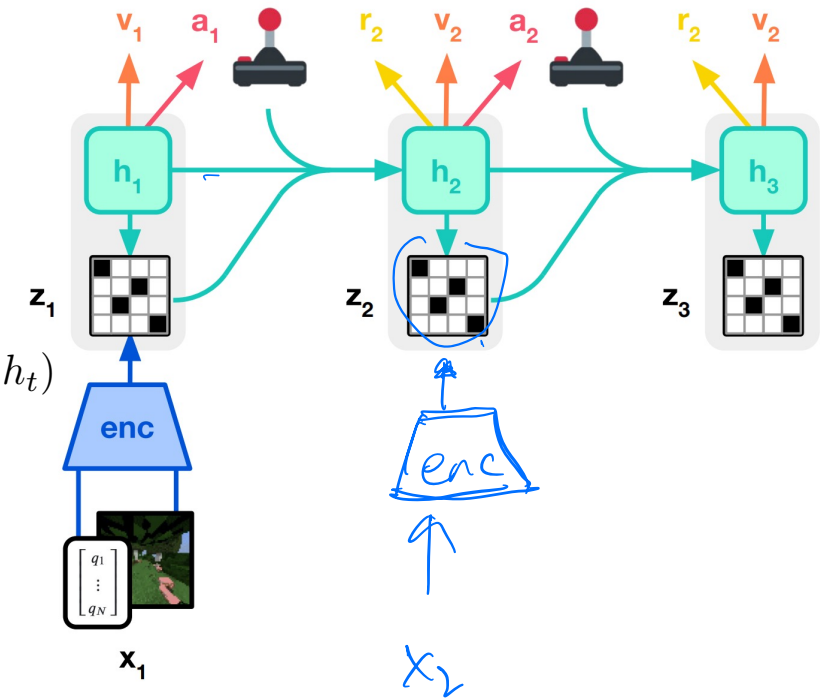
- Align representation

$$\mathcal{L}_{\text{rep}} = \max(1, \mathrm{KL}[q_\phi(z_t|h_t, x_t) \parallel \mathrm{sg}(p_\phi(z_t|h_t))])$$

JEPA



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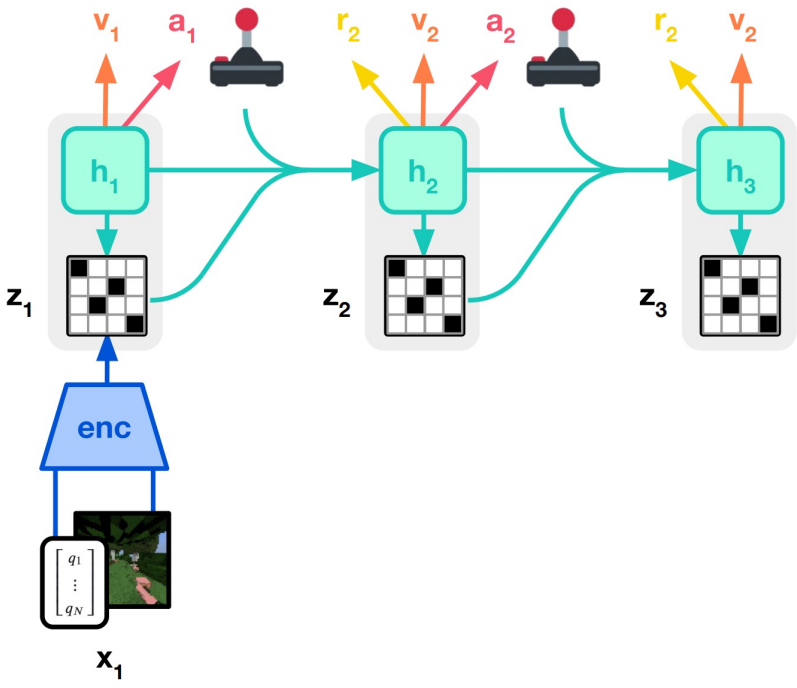
Incorporating Rewards

- How to use WM in planning? Predicting value by simulate a batch of trajectories.



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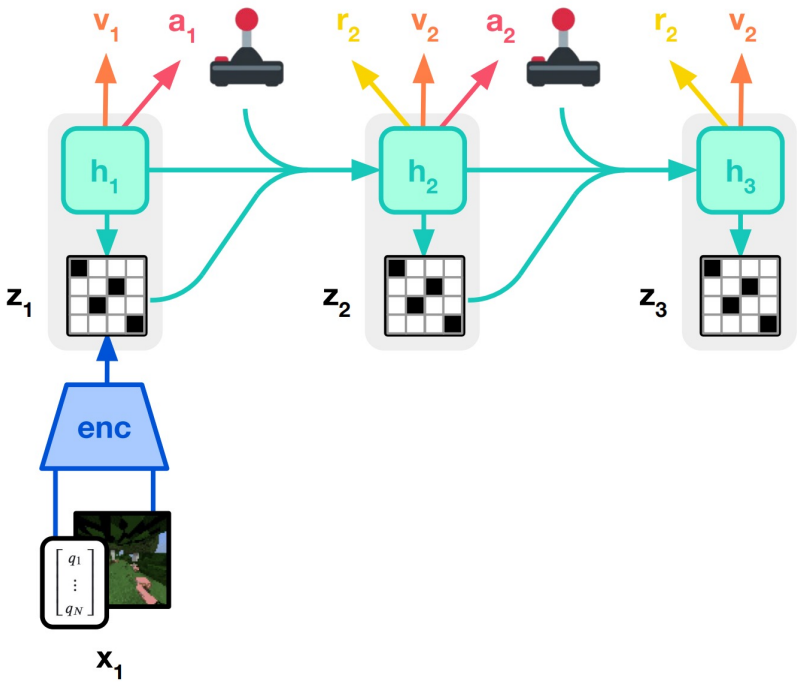

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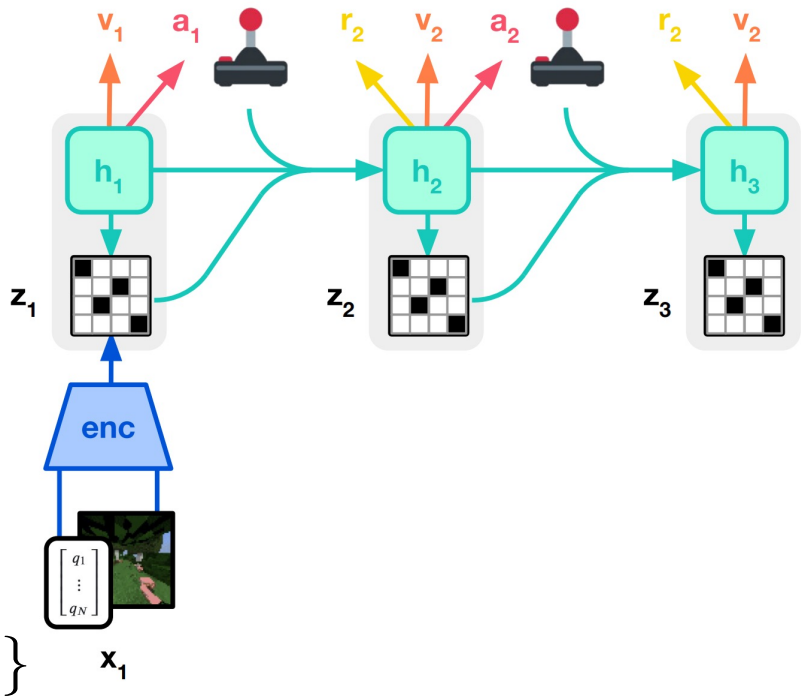
  $a_t \sim \pi_\theta(a_t \mid s_t)$ $\quad$ $v_\psi(R_t \mid s_t)$
- Learning a critic:

Categorical distribution

$$\mathcal{L}_\phi = -\sum_{t=1}^{T} \log p_\phi(R_t^\lambda \mid s_t) \quad s_t = \{h_t, z_t\}$$

Sum of discounted future rewards

$$R_t^\lambda = r_t + \gamma c_t[(1-\lambda)v_t + \lambda R_{t+1}^\lambda] \qquad R_T^\lambda = v_T$$



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Levine. Policy Gradients. Deep RL course. 2017.

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- REINFORCE algorithm [Williams 1992]

Normalization

Entropy Regularizer

$$\mathcal{L}(\theta) = -\sum_{t=1}^{T} \mathrm{sg}((R_t^\lambda - v_\phi(s_t))/\max(1, S)) \log \pi_\theta(a_t \mid s_t) + \eta H[\pi_\theta(a_t \mid s_t)]$$

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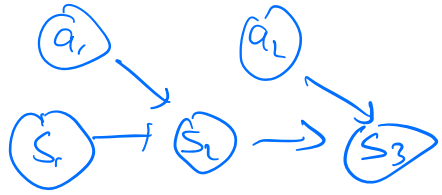

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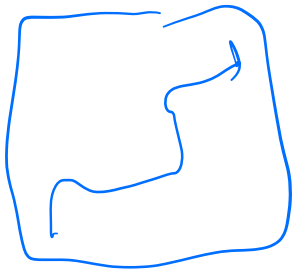
When Do We Need A Learned Actor?

- For low dimensional or discrete problems, we can directly take the argmax of the value function.



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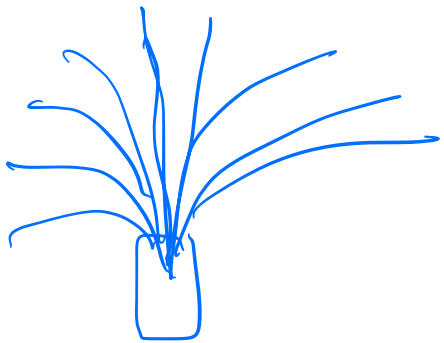

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- It compresses planning into a reactive policy.

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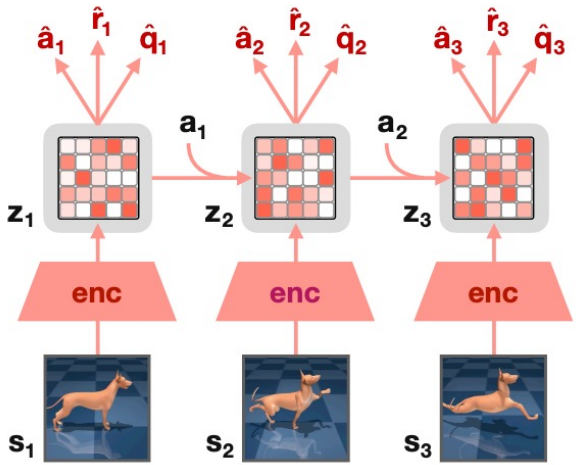
When Do We Need A Learned Actor?

- For low dimensional or discrete problems, we can directly take the argmax of the value function.
- For problems with a good model, we can roll out and sample many future trajectories. Evaluation can be done in real time with GPU.
- It compresses planning into a reactive policy.
- For general control problems, learning a separate actor can be a general solution without invoking domain knowledge.

Temporal Difference MPC

- Instead of learning an actor, we can directly update the actor network at test-time by optimizing the learned Q function from TD.



No reconstruction. JEPA.

Hansen et al. Temporal Difference Learning for Model Predictive Control. ICML 2022.

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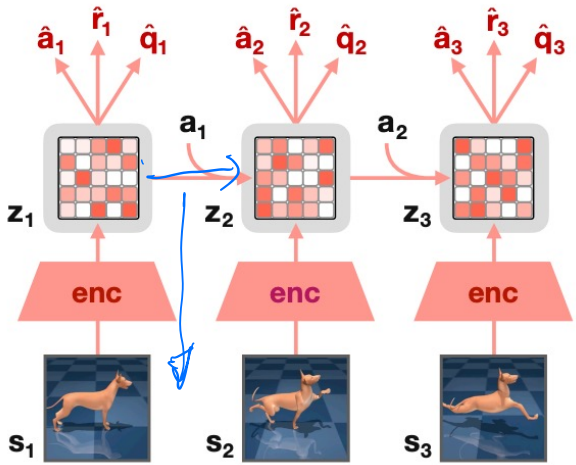

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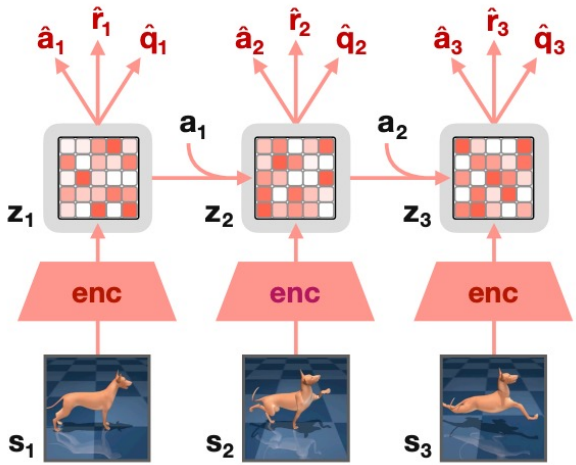

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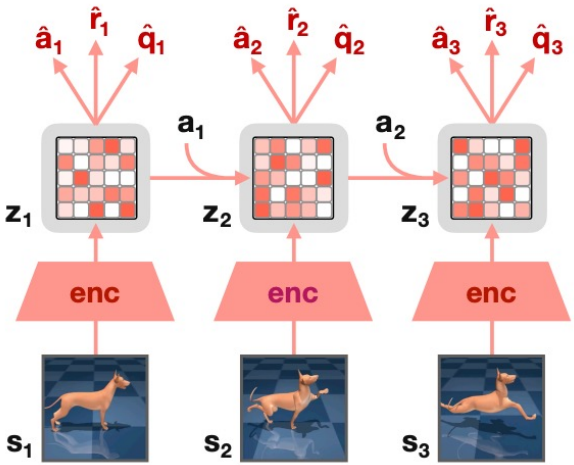
- TD: $\hat{Q}(z_t, a_t) \approx r_t + \gamma \bar{Q}(z_{t+1}, a_{t+1})$
- Planning:

$$J(A) = \sum_{k=0}^{H-1} \gamma^k \hat{r}(z_{t+k}, a_{t+k}) + \gamma^H \hat{Q}(z_{t+H}, a_{t+H})$$

estimate reward within horizon.

$$\mu^* = \arg\min_{\mu} \left(\mathbb{E}_{A \sim \mathcal{N}(\mu, \Sigma)}[-J(A)] + \lambda D_{\mathrm{KL}}(\mathcal{N}(\mu, \Sigma) \parallel \mathbb{P}_{\mathrm{prior}}) \right)$$



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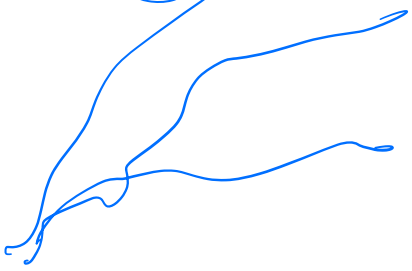
Sample Diverse Trajectories From Learned Prior

- Initialize and sample from policy prior

$$a_{t.i} \sim \mathcal{N}(\mu_\theta(z_t), \sigma_\theta(z_t)^2)$$



Max-Entropy

$$\mathcal{L}_{\text{prior}} = -\mathbb{E}_{a \sim \pi_\theta(\cdot|z)} \left[\hat{Q}_\theta(z, a) - \alpha \log \pi_\theta(a|z) \right]$$

Hansen et al. TD-MPC2: Scalable, Robust World Models for Continuous Control. ICLR 2024.

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$$R(\tau_i) = \sum_{k=0}^{H-1} \gamma^k \hat{r}_{t+k,i} + \gamma^H \hat{Q}(\hat{z}_{t+H,i}, a_{t+H,i})$$

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gradient-free.

- Weighted average smoothing and adaptation

$$w_i = \frac{\exp\left(\frac{1}{\lambda} R(\tau_i)\right)}{\sum_{j=1}^{N} \exp\left(\frac{1}{\lambda} R(\tau_j)\right)} \qquad a_t^{new} = \sum_{i=1}^{N} w_i a_{t,i}$$

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Latent MPC Using Gradient Descent

- MPC is a differentiable computation graph.
- We can directly optimize action variables.

Zhou et al. DINO-WM: World Models on Pre-trained Visual Features enable Zero-shot Planning. ICML 2025.

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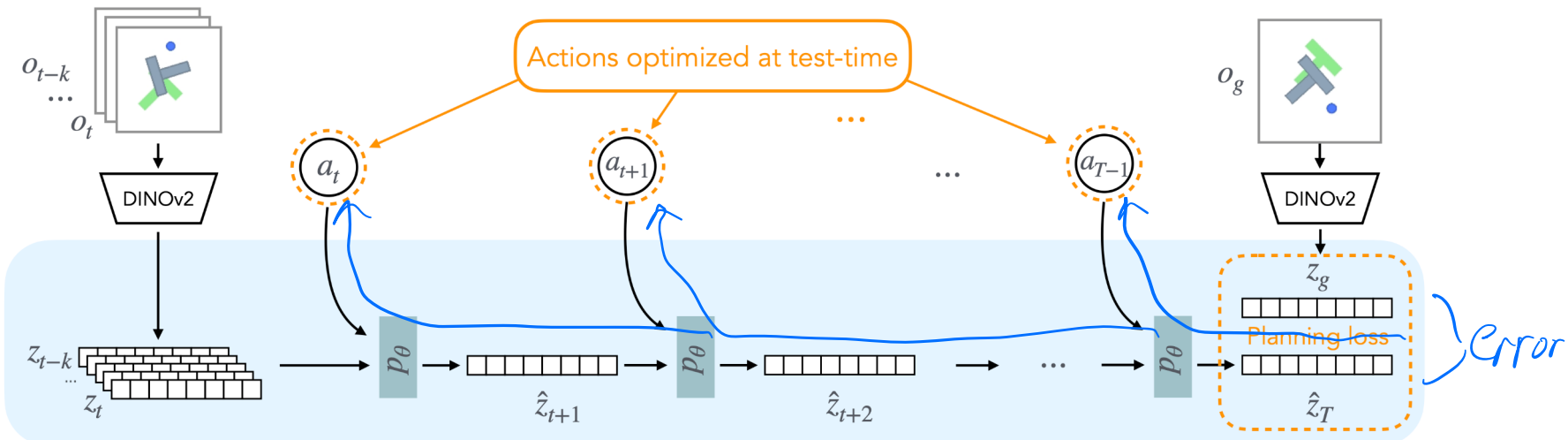

Zhou et al. DINO-WM: World Models on Pre-trained Visual Features enable Zero-shot Planning. ICML 2025.

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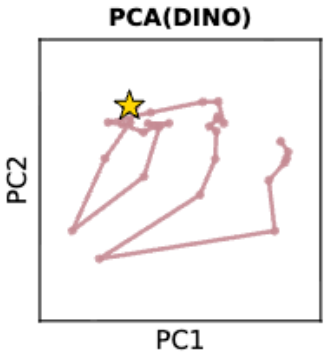
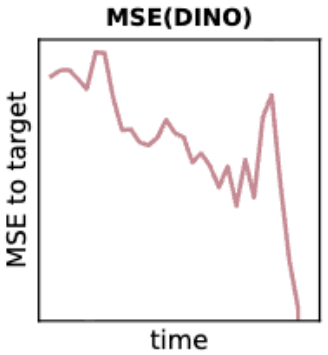
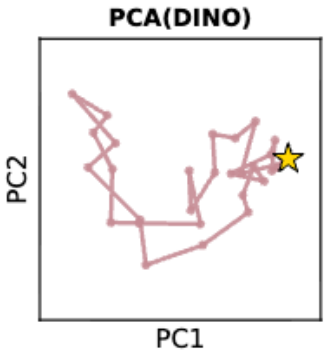
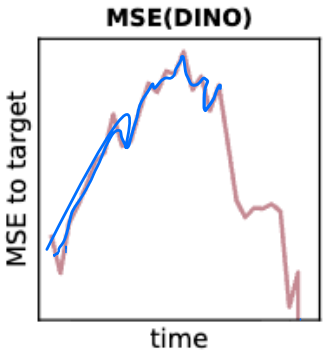
Learn Better Latent WM Representations

- The representation space is often jagged.
- Hard to optimize action sequences at test-time with gradient descent.









Wang et al. Temporal Straightening for Latent Planning. WMW 2026.

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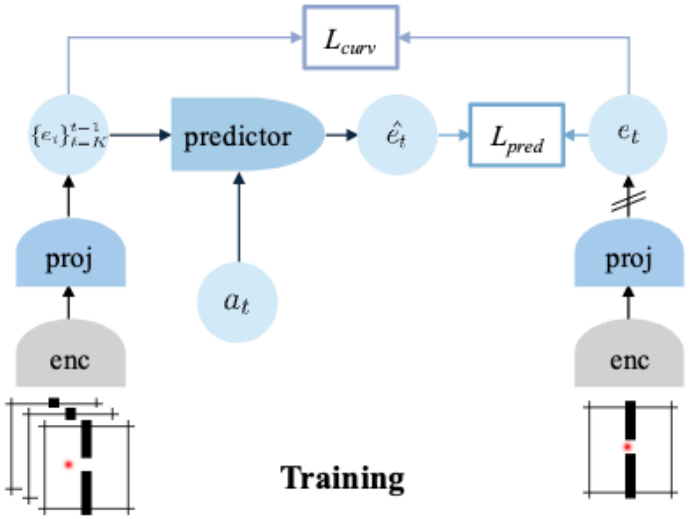
Straightening Objective

- In addition to predictors, maximizing cosine similarity across adjacent velocities (minimizing curvature).



$$\mathcal{L}_{pred} = ||\hat{z}_{t+1} - \text{sg}(z_{t+1})||_2^2.$$

JEPA

$$v_t = z_{t+1} - z_t, \quad v_{t+1} = z_{t+2} - z_{t+1}.$$

$$\mathcal{L}_{curve} = 1 - \frac{v_t \cdot v_{t+1}}{||v_t||_2 \cdot ||v_{t+1}||_2}.$$

Wang et al. Temporal Straightening for Latent Planning. WMW 2026.

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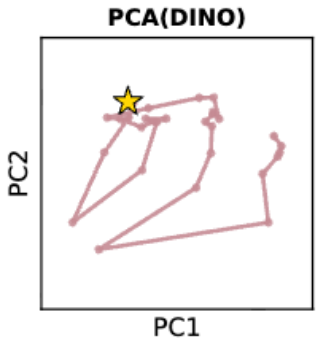
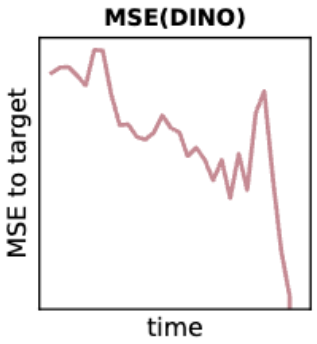
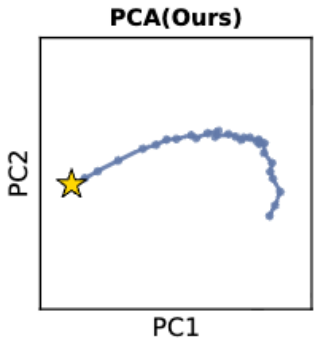
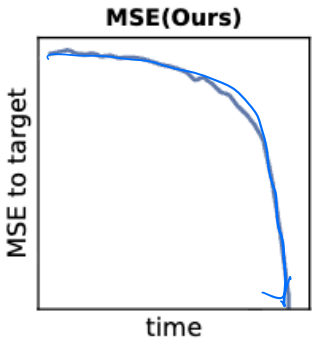
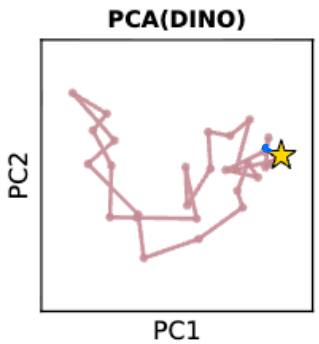
Straightened Representations

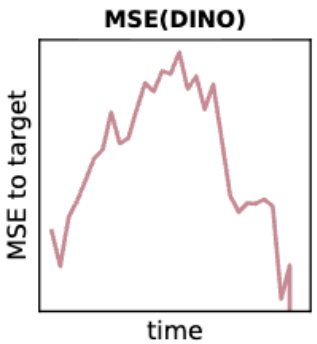


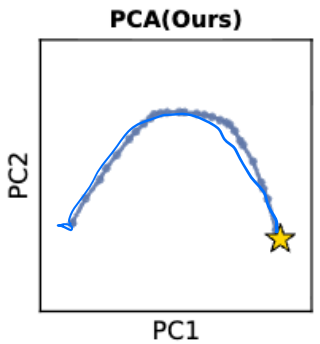


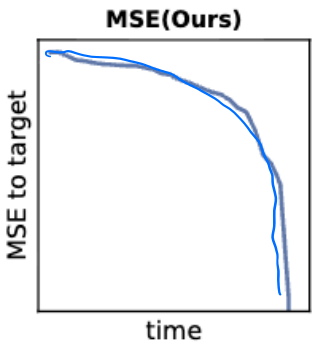












Wang et al. Temporal Straightening for Latent Planning. 2026.

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Why Straightening Helps?

- Encourages the predictor to be linear – more generalizable.

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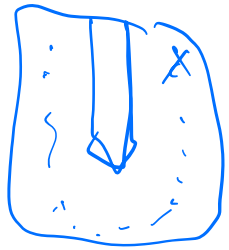

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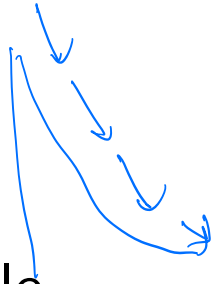

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$$\nabla_{\mathbf{a}} C(z_T(\mathbf{a})) = \left(\frac{\partial z_T}{\partial \mathbf{a}}\right)^T \nabla_z C(z_T)$$

$$H = \underbrace{\left(\frac{\partial z_T}{\partial \mathbf{a}}\right)^T \nabla_z^2 C(z_T) \left(\frac{\partial z_T}{\partial \mathbf{a}}\right)}_{\mathcal{T}_1\text{:Cost Convexity}} + \underbrace{\nabla_z C(z_T) \cdot \frac{\partial^2 z_T}{\partial \mathbf{a}^2}}_{\mathcal{T}_2\text{:Dynamics Curvature}}$$

Closer Zero.

Wang et al. Temporal Straightening for Latent Planning. 2026.

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O(1) wrt time.

- Conditioning effect: Convergence of convex gradient optimization depends on κ (condition number). For straight trajectories, condition number is constant.

$$\|\mathbf{a}^{(k)} - \mathbf{a}^*\| \leq \left(\frac{\kappa - 1}{\kappa + 1}\right)^k \|\mathbf{a}^{(0)} - \mathbf{a}^*\|$$

Wang et al. Temporal Straightening for Latent Planning. 2026.

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Summary: World Models

- Explicit object representation
  - Traditional, light weight, instance-specific, hard to learn jointly

WM for external actors.
Low-dim spaces

WM for simple, single actor environment, visualization

WM for general planning and test-time MPC

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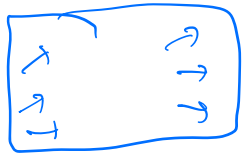
Summary: World Models



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- Global latent, RNNs
  - General-purpose, flexible
  - Planning-aware representations

Collect TD reward and train value function / critic.

Model predictive control.

WM for general planning and test-time MPC

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Learnable Planning and Cost

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Imitation Learning

- The explicit policy model, supervised learning (behavior cloning)

$$\hat{a} = f_\theta(x) \qquad \mathcal{L} = \min_i \|a_i - \hat{a}\|_2^2 \qquad \mathcal{L} = -\log \hat{a}_j$$

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- Energy-based (cost-based) approach

$$\tau^\star = \operatorname{argmin}_\tau E(x, \tau)$$

$$p(\tau \mid x) = \frac{\exp(E(x,\tau))}{\int_\tau \exp(E(x,\tau))}$$

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- Dataset Aggregation (DAgger)
  - Learned policy may deviate from experts
  - Need to collect more groundtruths

Initialize $\mathcal{D} \leftarrow \emptyset$.
Initialize $\hat{\pi}_1$ to any policy in Π.
for $i = 1$ **to** N **do**
  Let $\pi_i = \beta_i \pi^* + (1 - \beta_i)\hat{\pi}_i$.
  Sample T-step trajectories using π_i.
  Get dataset $\mathcal{D}_i = \{(s, \pi^*(s))\}$ of visited states by π_i and actions given by expert.
  Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \bigcup \mathcal{D}_i$.
  Train classifier $\hat{\pi}_{i+1}$ on $\mathcal{D}$.
end for
Return best $\hat{\pi}_i$ on validation.

Algorithm 3.1: DAGGER Algorithm.

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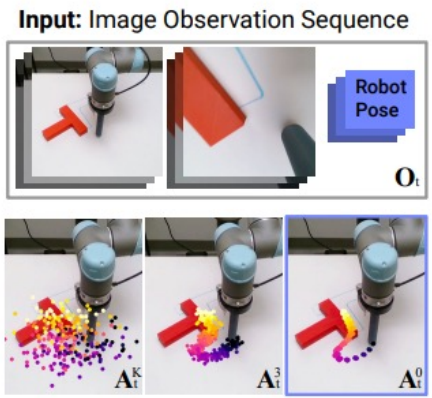
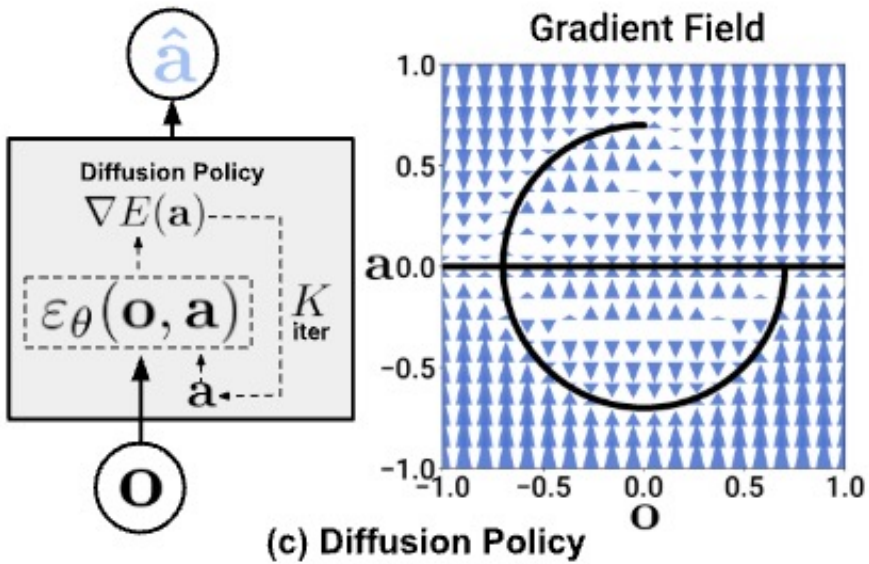
Direct Policy Learning from Diffusion

- Error prediction network is conditioned on observation features.

$$A_t^{k-1} = \alpha(A_t^k - \gamma\epsilon_\theta(O_t, A_t^k, k) + \mathcal{N}(0, \sigma^2 I)).$$

$$\mathcal{L} = MSE(\epsilon^k, \epsilon_\theta(O_t, A_t + \epsilon^k, k)).$$





Chi et al. Diffusion Policy: Visuomotor Policy Learning via Action Diffusion. RSS 2023

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Learning Cost

- Typically, we have some rough ideas on what the cost should look like
  - E.g. Avoid obstacle with squared distance barrier.

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- Typically, we have some rough ideas on what the cost should look like
  - E.g. Avoid obstacle with squared distance barrier.
- But there are costs that are implicitly defined.
  - E.g. Human comfort
- Can be learned from human demonstrations (also called IRL)

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Max-Margin Planning with Explicit Cost Volume

- If we have an explicit cost volume, the cost of a trajectory can be directly queried.

Zeng et al. End-to-end Interpretable Neural Motion Planner. CVPR 2019.

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- We can use the max-margin objective to make the groundtruth trajectory have lower costs.

$$\underset{\theta}{\operatorname{argmin}} \sum_{\{(\hat{x}_i^t, \hat{y}_i^t)\}_{i=1\ldots N}} \max_i \sum_{t=1}^{T} C_\theta^t[x_t, y_t] - C_\theta^t[\hat{x}_i^t, \hat{y}_i^t] + d_i^t$$

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- Find the lowest cost trajectory among a batch of samples.

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- Low-dimensional/known dynamics problems: External samplers

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- Low-dimensional/known dynamics problems: External samplers
- In general, needs to perform optimization (e.g. DP)

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EBM Planning

- Energy-based framework also needs negative samples.

Florence et al. Implicit Behavioral Cloning. CoRL 2021.

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- “Pick” the groundtruth sample among others.

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EBM Planning

- Energy-based framework also needs negative samples.
- “Pick” the groundtruth sample among others.
- If there isn’t an external sampler, we can either use autoregressive energy of sampling one dimension at a time, or gradient-based Langevin MCMC.

Florence et al. Implicit Behavioral Cloning. CoRL 2021.

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EBM Planning

- Energy-based framework also needs negative samples.
- “Pick” the groundtruth sample among others.
- If there isn’t an external sampler, we can either use autoregressive energy of sampling one dimension at a time, or gradient-based Langevin MCMC.

Langevin MCMC: $\tilde{\mathbf{y}}_i^k = \tilde{\mathbf{y}}_i^{k-1} - \lambda \left(\frac{1}{2} \nabla_{\mathbf{y}} E_\theta(\mathbf{x}_i, \mathbf{y}_i^{k-1}) + \omega^k \right), \omega^k \sim \mathcal{N}(0, \sigma).$

Florence et al. Implicit Behavioral Cloning. CoRL 2021.

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Loss: $\mathcal{L} = \sum_i -\log(p_\theta(\mathbf{y}_i \mid \mathbf{x}, \{\tilde{\mathbf{y}}_i\}_j) \qquad p_\theta(\mathbf{y}_i \mid \mathbf{x}, \{\tilde{\mathbf{y}}_i\}_j = \frac{e^{-E_\theta(\mathbf{x}_i, \mathbf{y}_i)}}{e^{-E_\theta(\mathbf{x}_i, \mathbf{y}_i)} + \sum_j e^{-E_\theta(\mathbf{x}_i + \tilde{\mathbf{y}}_{i,j})}}$

Florence et al. Implicit Behavioral Cloning. CoRL 2021.

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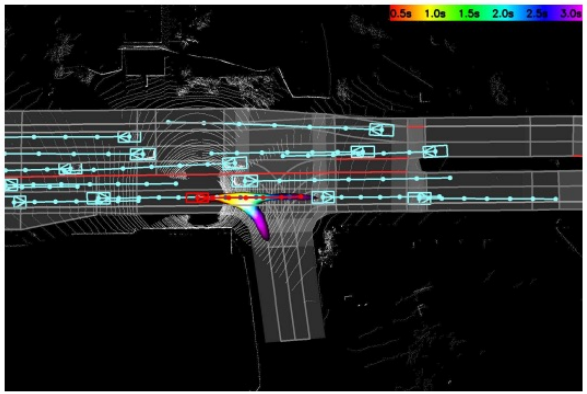
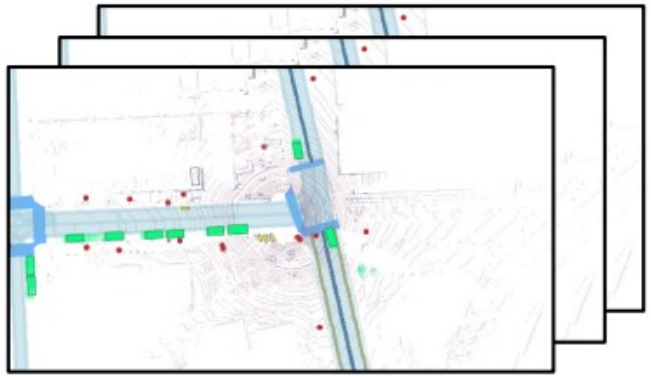
Non-Parametric Cost Volume for 2D Planning

- Interpretability (both costs and planner inputs)

Rasterization



Semantic Occupancy



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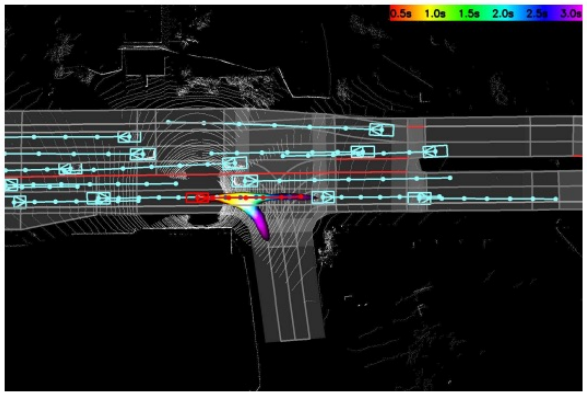
Non-Parametric Cost Volume for 2D Planning

- Interpretability (both costs and planner inputs)
- Use spatial geometry to form cost from explicit objects

Rasterization



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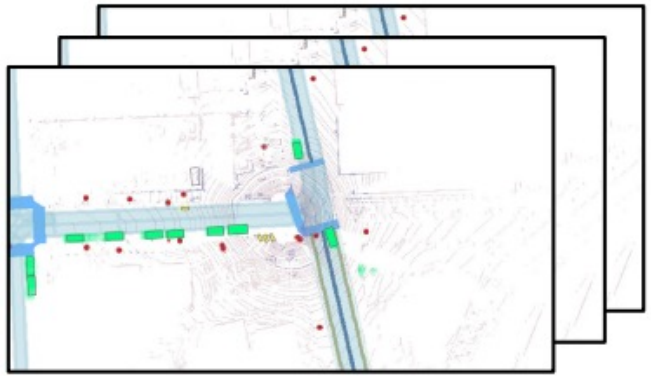

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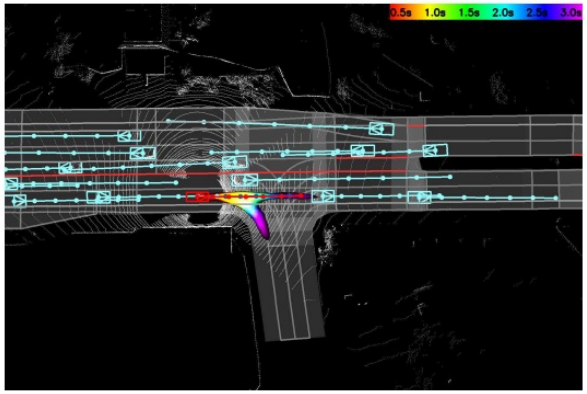
Non-Parametric Cost Volume for 2D Planning

- Interpretability (both costs and planner inputs)
- Use spatial geometry to form cost from explicit objects
- Predict spatial cost volume
  - Rasterize the scene for spatial inputs
  - Predict soft occupancy volumes (present and future)

Rasterization



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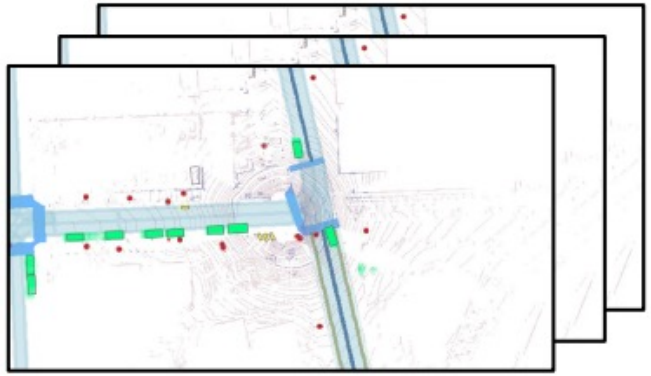

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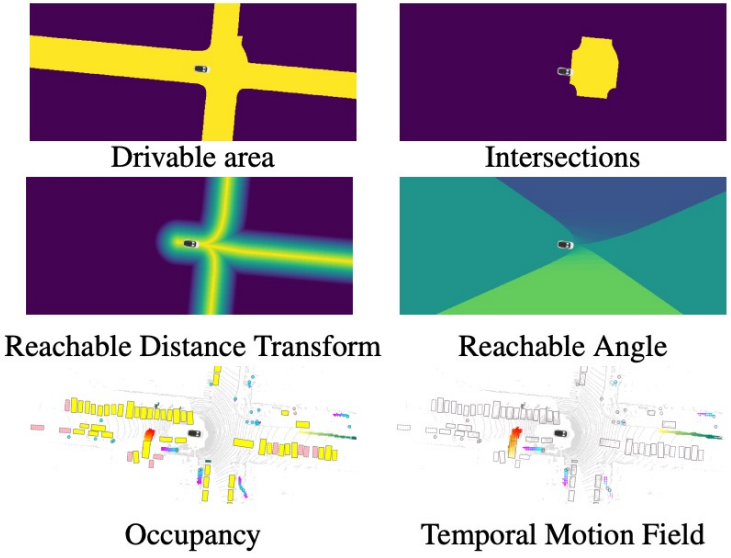
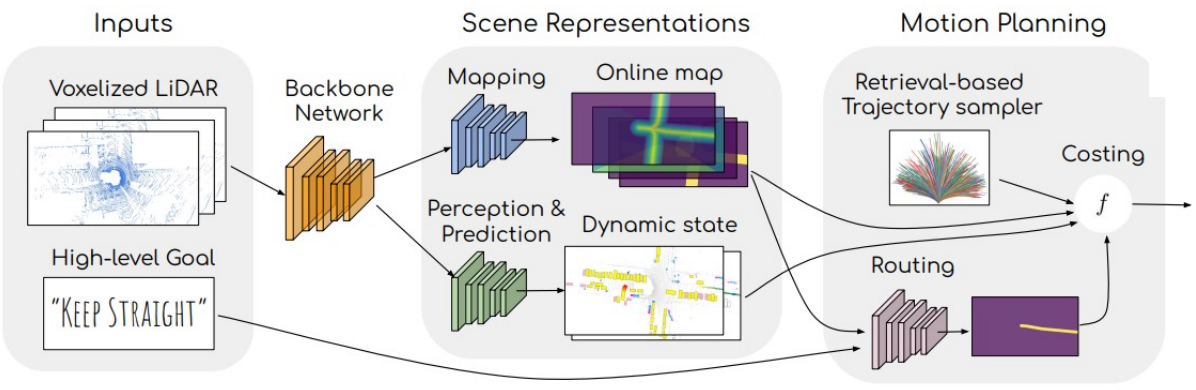
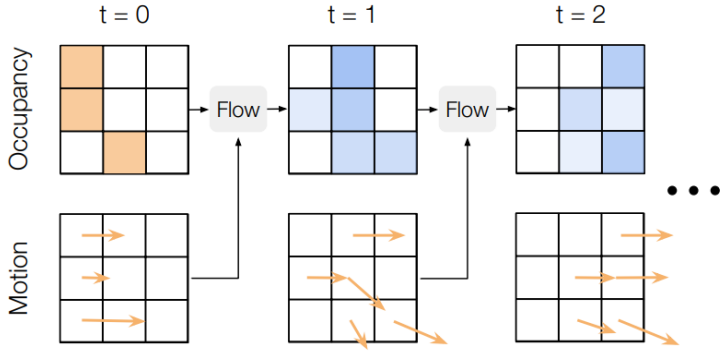
Learning Through Interpretable Predictions

- Semantic occupancy, motion field, mapping, etc. as intermediate predictions.



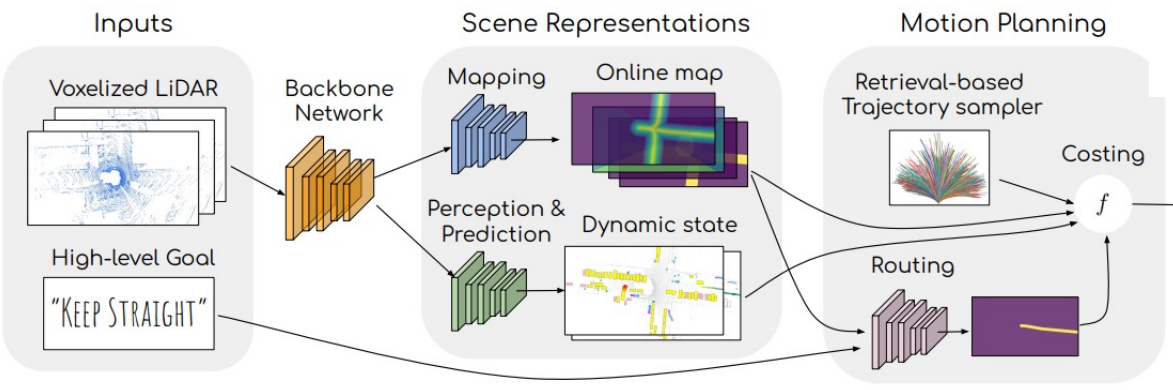


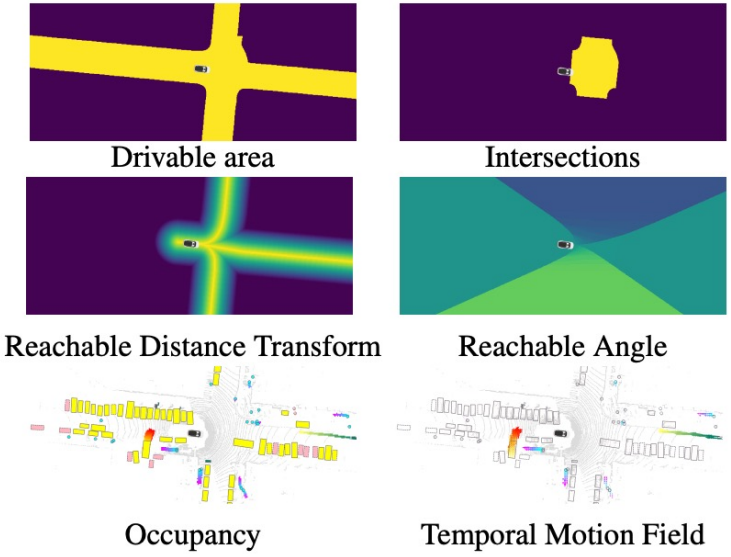


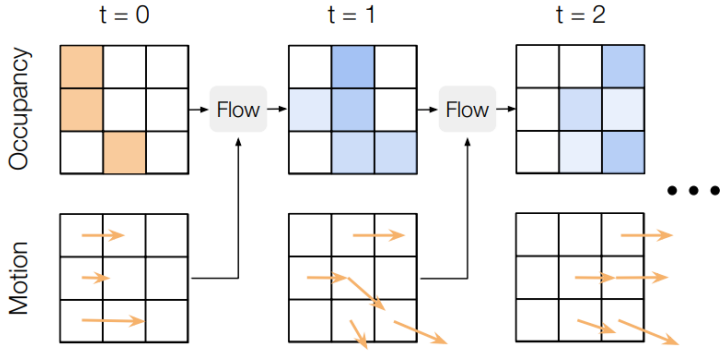
Casas et al. MP3: A Unified Model to Map, Perceive, Predict and Plan. CVPR 2021.

Learning Through Interpretable Predictions

- Semantic occupancy, motion field, mapping, etc. as intermediate predictions.
- Differentiable, supports end-to-end interpretable learning from perception to planning.







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Value Iteration Networks

- A network design for predicting cost volumes that are grounded from the classic value iteration algorithm.

Tamar et al. Value Iteration Networks. NeurIPS 2016.

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$$\pi^*(s) = \operatorname{argmax}_a Q_\infty(s,a)$$

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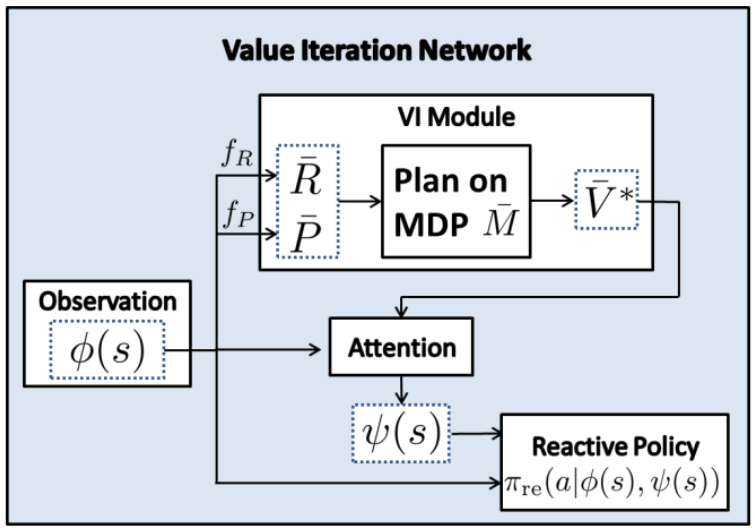
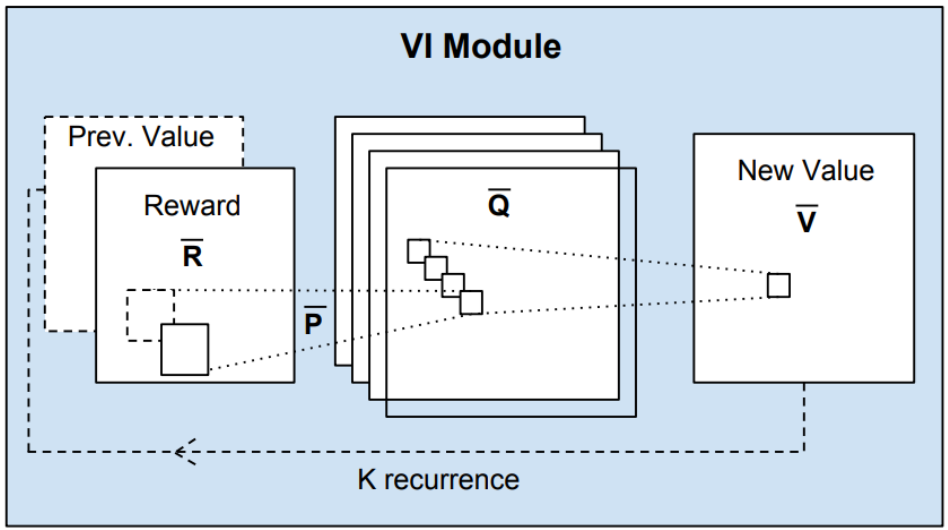
Value Iteration Networks

- Reward and previous value are fed into a network to generate Q of A channels. Transition matrix is convolutional kernel. Then Max-Pooling.

$$Q_n(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a) V_n(s') \quad V_{n+1}(s) = \max_a Q_n(s,a)$$





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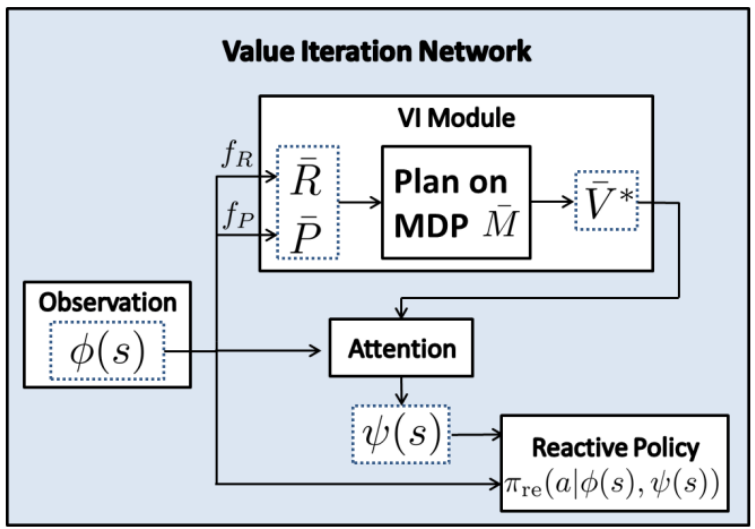
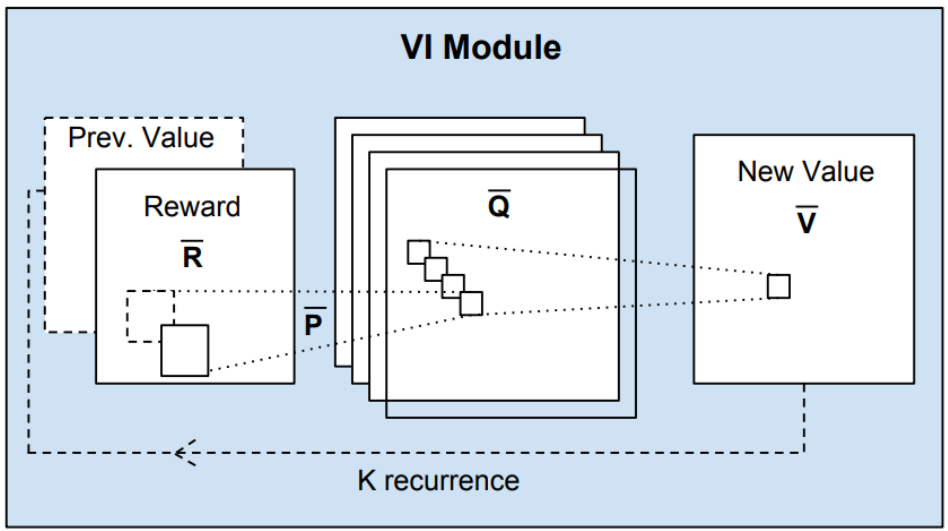
Value Iteration Networks

- Select the current state and choose an action from softmax.

$$\hat{a} \sim \mathrm{softmax}_a(Q(s,a))$$





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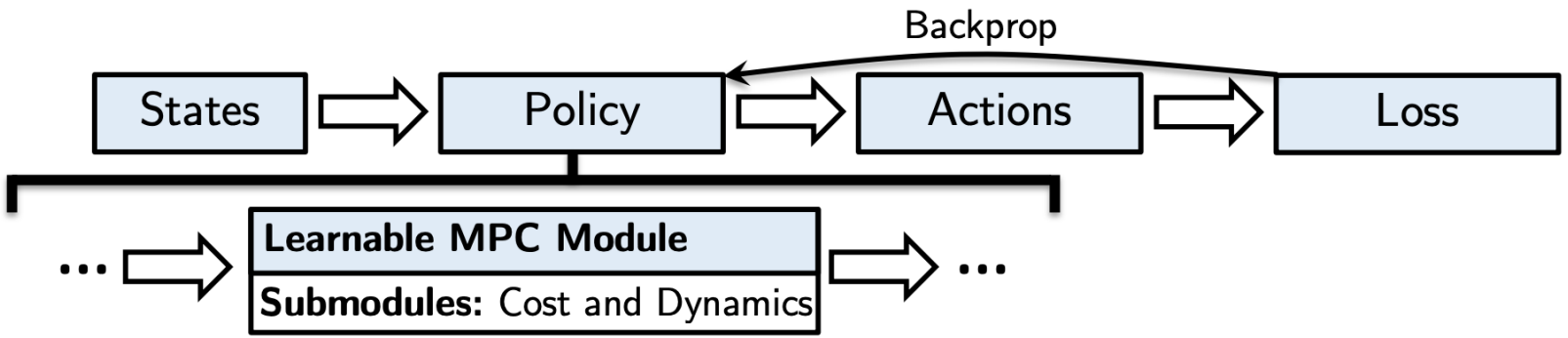
Backprop through Planning

- Treat planning as an end-to-end layer. Can be used for RL/Imitation.



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Backprop through Planning

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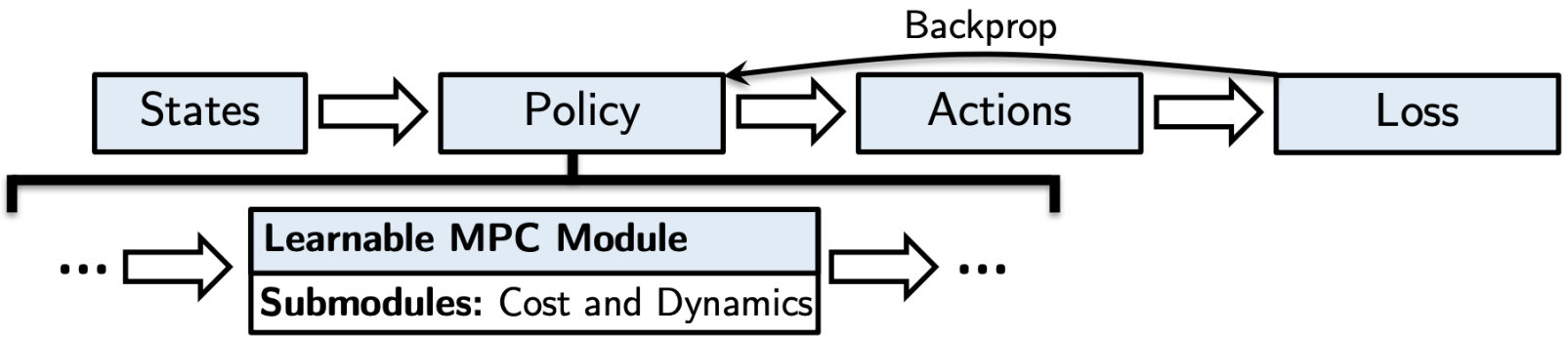
- Option 1: Unrolling a finite number of steps



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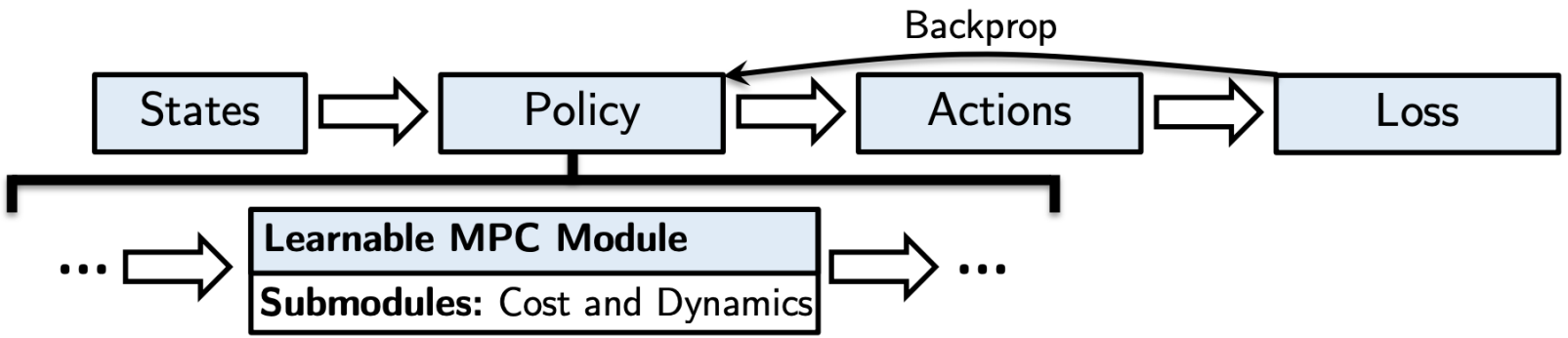
- Option 2: Solve till convergence, backprop for a finite step



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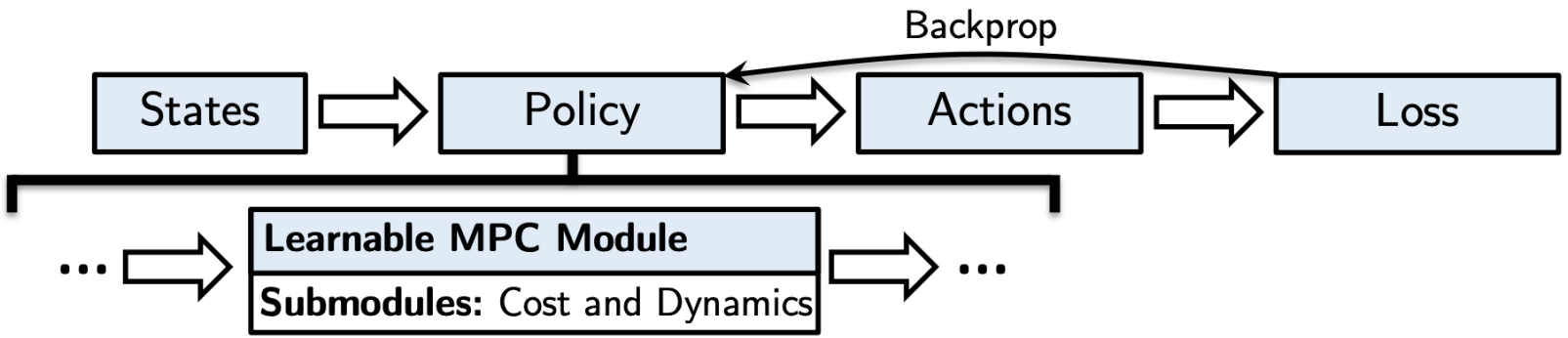
- Option 1: Unrolling a finite number of steps
- Option 2: Solve till convergence, backprop for a finite step
- Option 3: Converged at fixed point: Implicit differentiation



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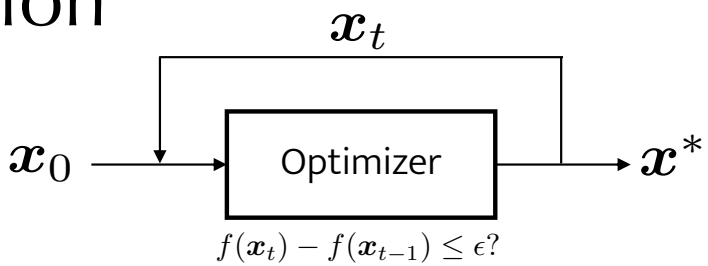
Implicit Differentiation

- Unconstrained case

$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\operatorname{argmin}} f(\boldsymbol{x}; \boldsymbol{\theta}).$$



Liao et al. Reviving and Improving Recurrent Back-Propagation. ICML 2018.

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Implicit Differentiation

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$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\operatorname{argmin}} f(\boldsymbol{x}; \boldsymbol{\theta}).$$

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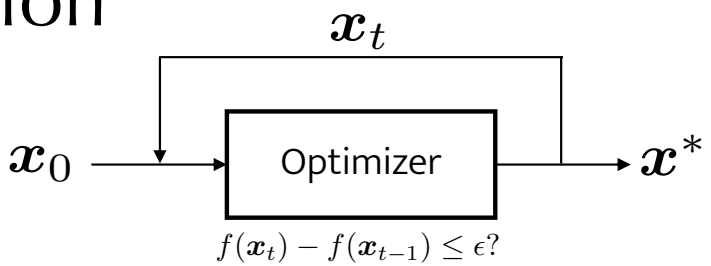

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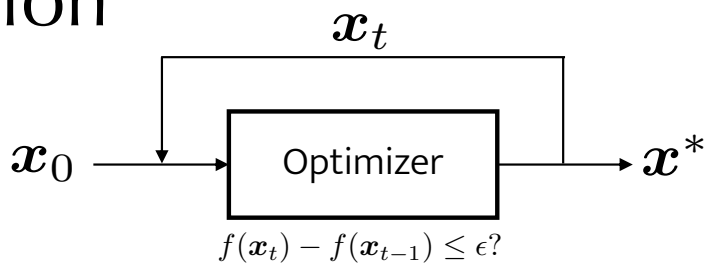

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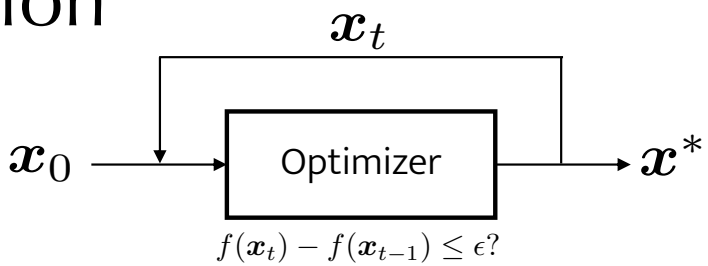

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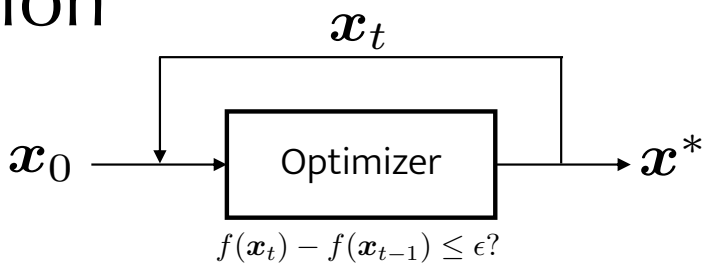

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Implicit Differentiation

- How to compute Hessian inverse vector product?

Liao et al. Reviving and Improving Recurrent Back-Propagation. ICML 2018.

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Implicit Differentiation

- How to compute Hessian inverse vector product?
- Conjugate gradient, solve $A\boldsymbol{x} = \boldsymbol{b}$

Conjugate Gradient Method

$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$
if $\mathbf{r}_0$ is sufficiently small, then return $\mathbf{x}_0$ as the result
$\mathbf{p}_0 := \mathbf{r}_0$
$k := 0$
repeat
$\alpha_k := \frac{\mathbf{r}_k^\mathsf{T}\mathbf{r}_k}{\mathbf{p}_k^\mathsf{T}\mathbf{A}\mathbf{p}_k}$
$\mathbf{x}_{k+1} := \mathbf{x}_k + \alpha_k\mathbf{p}_k$
$\mathbf{r}_{k+1} := \mathbf{r}_k - \alpha_k\mathbf{A}\mathbf{p}_k$
if $\mathbf{r}_{k+1}$ is sufficiently small, then exit loop
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$k := k + 1$
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return $\mathbf{x}_{k+1}$ as the result

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Implicit Differentiation

- How to compute Hessian inverse vector product?
- Conjugate gradient, solve $A\boldsymbol{x} = \boldsymbol{b}$
- Neumann series (finite truncation)

$$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k.$$

  - Same as backprop the last K steps (Option 2).
  - Memory savings.

Conjugate Gradient Method

$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$
if $\mathbf{r}_0$ is sufficiently small, then return $\mathbf{x}_0$ as the result
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Differentiable LQR

- Now add linear equality constraints on the dynamics and initialization.

$$\tau_{1:T} = \{x_t, u_t\}_{1:T}$$

$$\underset{\tau_{1:T}}{\text{argmin}} \sum_{t=1}^{T} \frac{1}{2}\tau_t^\top C_t \tau_t + c_t^\top \tau_t$$

$$\text{subject to } x_{t+1} = F_t \tau_t + f_t, x_1 = x_{\text{init}}.$$

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General QP:

$$\boldsymbol{x}^* = \text{argmin} \frac{1}{2} \boldsymbol{x}^\top Q \boldsymbol{x} + \boldsymbol{c}^\top \boldsymbol{x}$$

$$\text{subject to } \; A\boldsymbol{x} = \boldsymbol{b}.$$

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- KKT:

$$\underset{\tau_{1:T}}{\text{argmin}} \sum_{t=1}^{T} \frac{1}{2} \tau_t^\top C_t \tau_t + c_t^\top \tau_t$$

$$\text{subject to } x_{t+1} = F_t \tau_t + f_t, x_1 = x_{\text{init}}.$$

General QP:

$$\boldsymbol{x}^* = \text{argmin} \frac{1}{2} \boldsymbol{x}^\top Q \boldsymbol{x} + \boldsymbol{c}^\top \boldsymbol{x}$$

$$\text{subject to } \; A\boldsymbol{x} = \boldsymbol{b}.$$

Amos et al. Differentiable MPC for End-to-end Planning and Control. NeurIPS 2018.

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Differentiable LQR

- Now add linear equality constraints on the dynamics and initialization.

$$\tau_{1:T} = \{x_t, u_t\}_{1:T}$$

- Chain rule: $\frac{\partial \ell}{\partial \theta} = \frac{\partial \ell}{\partial \tau^\star_{1:T}} \frac{\partial \tau^\star_{1:T}}{\partial \theta}$.
- KKT:

$$\begin{bmatrix} Q & A^\top \\ A & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^* \\ \boldsymbol{\lambda}^* \end{bmatrix} = \begin{bmatrix} -\boldsymbol{c} \\ \boldsymbol{b} \end{bmatrix}$$

$$K \begin{bmatrix} \boldsymbol{x}^* \\ \boldsymbol{\lambda}^* \end{bmatrix} = v.$$

$$\underset{\tau_{1:T}}{\text{argmin}} \sum_{t=1}^{T} \frac{1}{2} \tau_t^\top C_t \tau_t + c_t^\top \tau_t$$

$$\text{subject to } x_{t+1} = F_t \tau_t + f_t, x_1 = x_{\text{init}}.$$

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$$\text{subject to } \; A\boldsymbol{x} = \boldsymbol{b}.$$

In classic LQR solver, the Riccati recursion solves this linear system.

Amos et al. Differentiable MPC for End-to-end Planning and Control. NeurIPS 2018.

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Differentiable LQR

- Apply differentiation $\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}}\left(K\begin{bmatrix}\boldsymbol{x}^*\\ \boldsymbol{\lambda}^*\end{bmatrix}\right)=\frac{\mathrm{d}v}{\mathrm{d}\boldsymbol{\theta}}.$ $\frac{\mathrm{d}K}{\mathrm{d}\boldsymbol{\theta}}\begin{bmatrix}\boldsymbol{x}^*\\ \boldsymbol{\lambda}^*\end{bmatrix}+K\begin{bmatrix}\frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{\theta}}\\ \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{\theta}}\end{bmatrix}=\frac{\mathrm{d}v}{\mathrm{d}\boldsymbol{\theta}}$

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$$K\begin{bmatrix}\frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{\theta}}\\ \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{\theta}}\end{bmatrix}=K\begin{bmatrix}\frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{c}} & \frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{b}} & \frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}Q} & \frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}A}\\ \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{c}} & \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{b}} & \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}Q} & \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}A}\end{bmatrix}=\begin{bmatrix}-I & 0 & -x^* & -\lambda^*\\ 0 & I & 0 & -x^*\end{bmatrix}$$

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$$K\frac{\partial\ell}{\partial\boldsymbol{z}^*}\begin{bmatrix}\frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{c}} & \frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{b}}\\ \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{c}} & \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{b}}\end{bmatrix}=\begin{bmatrix}-\frac{\partial\ell}{\partial\boldsymbol{x}^*}\\ \mathbf{0}\end{bmatrix}\qquad K\boldsymbol{d}^*=\begin{bmatrix}-\frac{\partial\ell}{\partial\boldsymbol{x}^*}\\ \mathbf{0}\end{bmatrix}$$

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- Equivalent QP:

$$\boldsymbol{d}^*=\underset{\boldsymbol{d}}{\operatorname{argmin}}\ \frac{1}{2}\boldsymbol{d}^\top Q\boldsymbol{d}+\frac{\partial\ell}{\partial\boldsymbol{x}^*}^\top\boldsymbol{d},$$
$$\text{subject to}\quad A\boldsymbol{d}=\mathbf{0}.$$

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Differentiable LQR

- Apply differentiation $\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}}\left(K\begin{bmatrix}\boldsymbol{x}^*\\ \boldsymbol{\lambda}^*\end{bmatrix}\right) = \frac{\mathrm{d}v}{\mathrm{d}\boldsymbol{\theta}}. \qquad \frac{\mathrm{d}K}{\mathrm{d}\boldsymbol{\theta}}\begin{bmatrix}\boldsymbol{x}^*\\ \boldsymbol{\lambda}^*\end{bmatrix} + K\begin{bmatrix}\frac{\mathrm{d}\boldsymbol{x}^*}{\mathrm{d}\boldsymbol{\theta}}\\ \frac{\mathrm{d}\boldsymbol{\lambda}^*}{\mathrm{d}\boldsymbol{\theta}}\end{bmatrix} = \frac{\mathrm{d}v}{\mathrm{d}\boldsymbol{\theta}}$

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$$\boldsymbol{d}^* = \underset{\boldsymbol{d}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{d}^\top Q\boldsymbol{d} + \frac{\partial \ell}{\partial \boldsymbol{x}^*}^\top \boldsymbol{d}, \quad \text{subject to } A\boldsymbol{d} = \mathbf{0}.$$

$$\frac{\partial \ell}{\partial Q} = \frac{1}{2}(\boldsymbol{d}_{\boldsymbol{x}}^* \otimes \boldsymbol{x}^* + \boldsymbol{x}^* \otimes \boldsymbol{d}_{\boldsymbol{x}}^*)$$

$$\frac{\partial \ell}{\partial A} = \boldsymbol{d}_{\boldsymbol{\lambda}}^* \otimes \boldsymbol{x}^* + \boldsymbol{\lambda}^* \otimes \boldsymbol{d}_{\boldsymbol{x}}^*.$$

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Differentiable LQR

- The backward pass can also be formulated as a LQR problem.
- Swap c to $\nabla_{\tau^\star}\ell$ and f to 0.

Module 1 Differentiable LQR *(The LQR algorithm is defined in appendix A)*

Input: Initial state x_{init}
Parameters: $\theta = \{C, c, F, f\}$

Forward Pass:

1: $\tau^\star_{1:T} = \text{LQR}_T(x_{\text{init}}; C, c, F, f)$ ▷ Solve (2)
2: Compute $\lambda^\star_{1:T}$ with (7)

Backward Pass:

1: $d^\star_{\tau_{1:T}} = \text{LQR}_T(0; C, \nabla_{\tau^\star}\ell, F, 0)$ ▷ Solve (9), ideally reusing the factorizations from the forward pass
2: Compute $d^\star_{\lambda_{1:T}}$ with (7)
3: Compute the derivatives of ℓ with respect to C, c, F, f, and x_{init} with (8)

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Differentiable MPC

- What about general MPC?

$$\underset{x_{1:T} \in \mathcal{X}, u_{1:T} \in \mathcal{U}}{\operatorname{argmin}} \sum_{t=1}^{T} C_t(x_t, u_t)$$

$$\text{subject to } x_{t+1} = f(x_t, u_t), x_1 = x_{\text{init}}.$$

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- Use Taylor expansion to approximate.

$$\tilde{C}^i_{\theta,t} = C_{\theta,t}(\tau^i_t) + {p^i_t}^\top (\tau_t - \tau^i_t) + \tfrac{1}{2}(\tau_t - \tau^i_t)^\top H^i_t (\tau_t - \tau^i_t).$$

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- Fixed point iteration

$$\tau^{i+1}=\operatorname{argmin}_\tau \textstyle\sum_t^T \tilde{C}^i_t(\tau^i_t).$$

Amos et al. Differentiable MPC for End-to-end Planning and Control. NeurIPS 2018.

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Differentiable MPC

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- Fixed point iteration

$$\tau^{i+1} = \operatorname{argmin}_\tau \textstyle\sum_t^T \tilde{C}^i_t(\tau^i_t).$$

- Backward only depends on final quadratic approximation.

Amos et al. Differentiable MPC for End-to-end Planning and Control. NeurIPS 2018.

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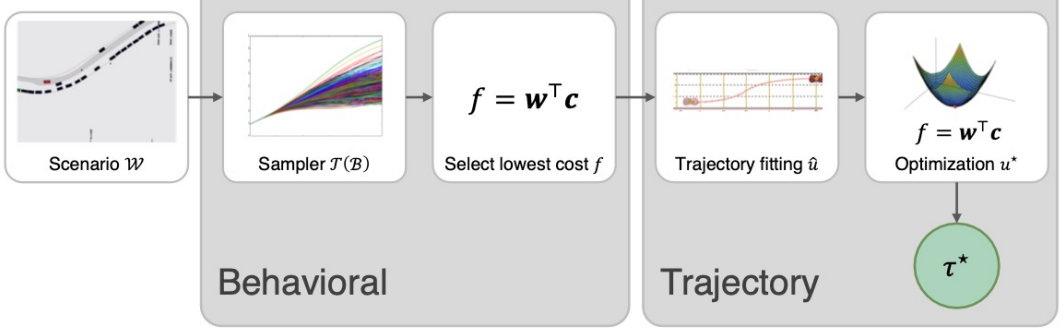
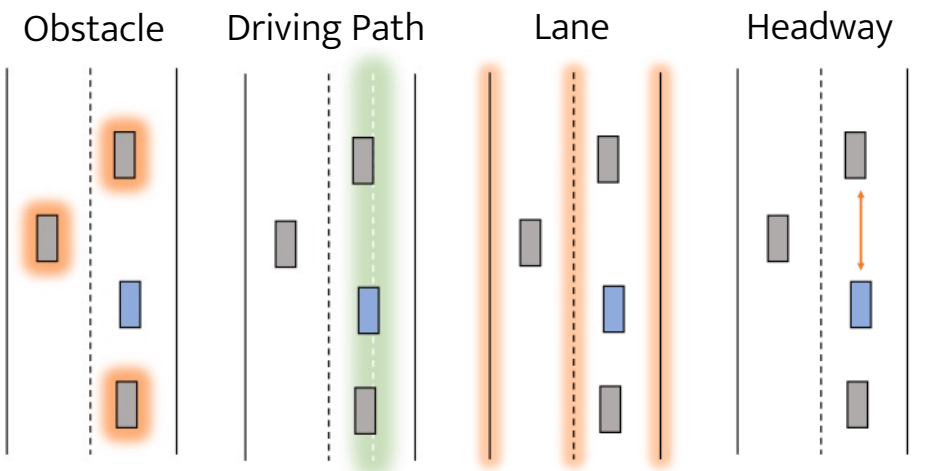
Behavioral vs. Trajectory Planning

- Gradient-based optimization provides a locally optimized trajectory.





Sadat et al. Jointly Learnable Behavior and Trajectory Planning for Self-Driving Vehicles. IROS 2019.

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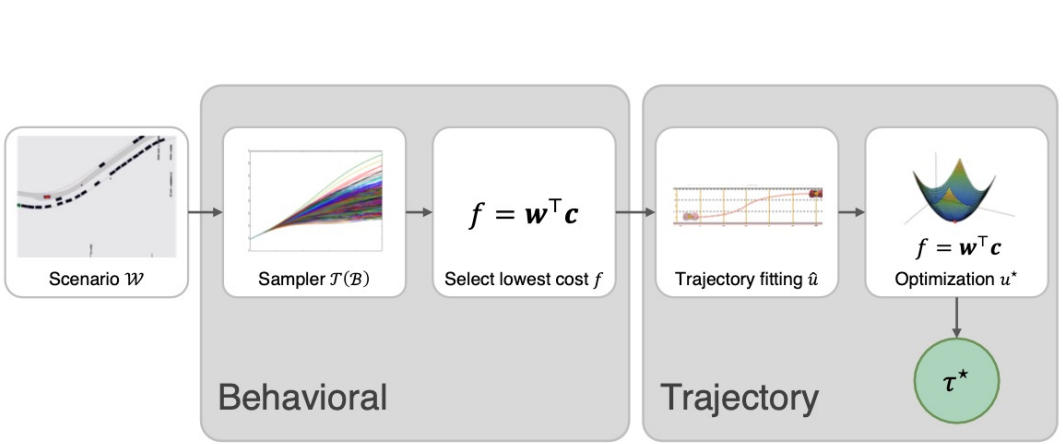
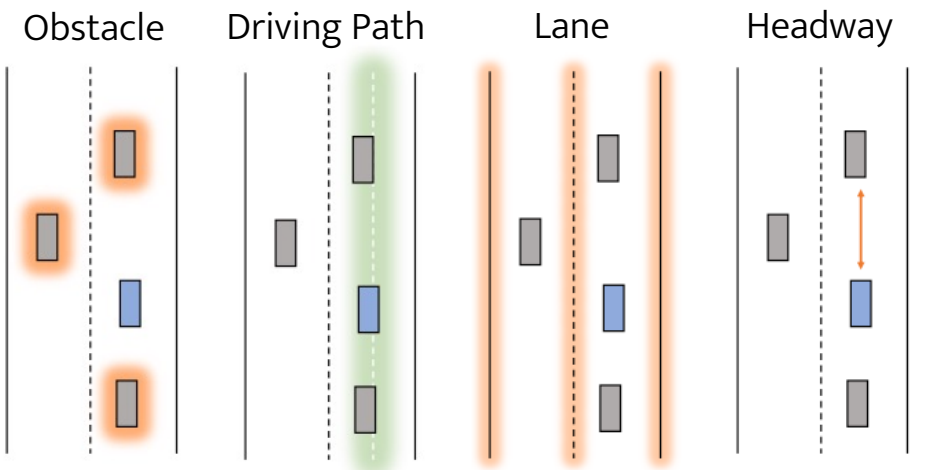
Behavioral vs. Trajectory Planning

- Gradient-based optimization provides a locally optimized trajectory.
- Samples may be needed for reasoning global structure.





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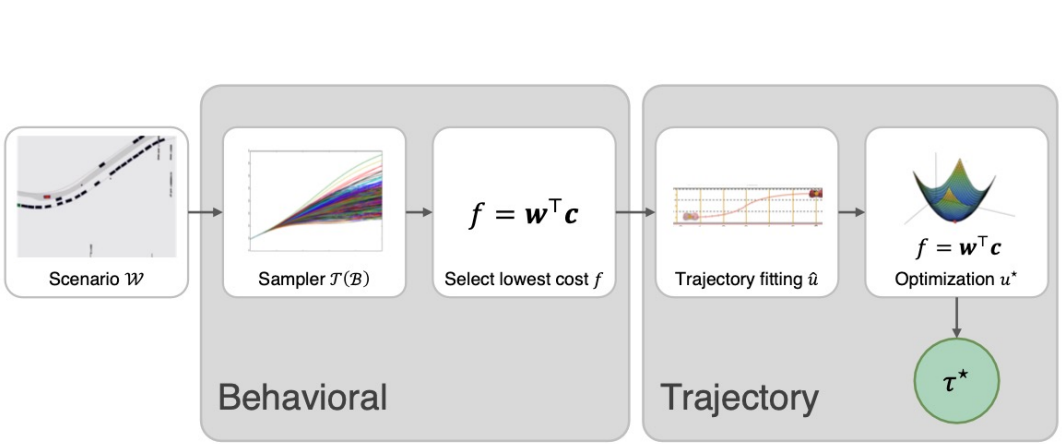
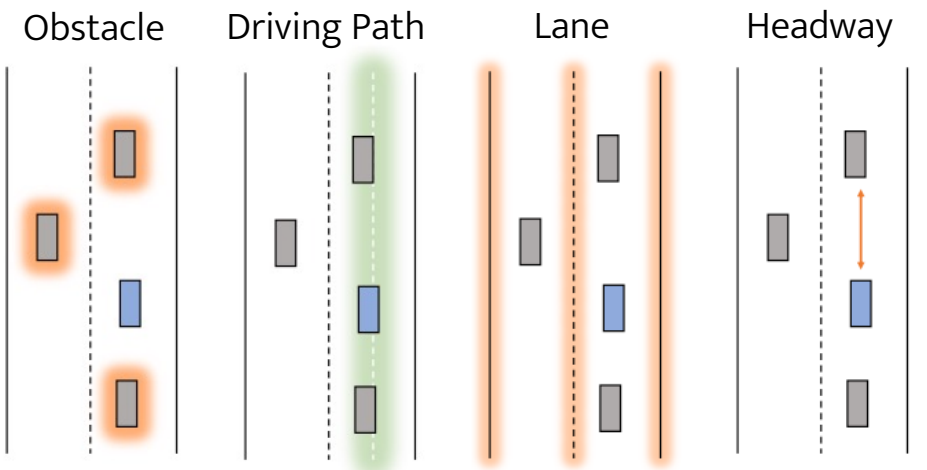
Behavioral vs. Trajectory Planning

- Gradient-based optimization provides a locally optimized trajectory.
- Samples may be needed for reasoning global structure.
- Can learn together using the same learned costs.





Sadat et al. Jointly Learnable Behavior and Trajectory Planning for Self-Driving Vehicles. IROS 2019.

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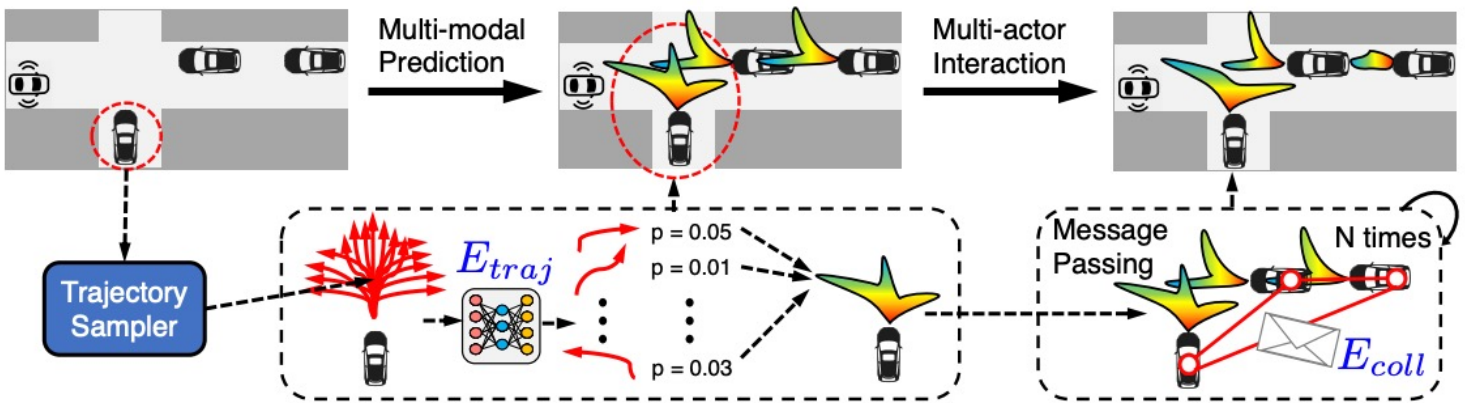
Planning with Social Reasoning

- Jointly reason the future trajectories of multiple agents as an energy-based graphical model.

$$p(\mathbf{s}_1, \ldots, \mathbf{s}_N \mid \mathbf{X}) = \frac{1}{Z} \exp(-E_\theta(\mathbf{s}_1, \ldots, \mathbf{s}_N) \mid \mathbf{X})$$



Zeng et al. DSDNet: Deep Structured Self-Driving Network. ECCV 2020.

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Planning with Social Reasoning

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- Trajectory Goodness + Collision.

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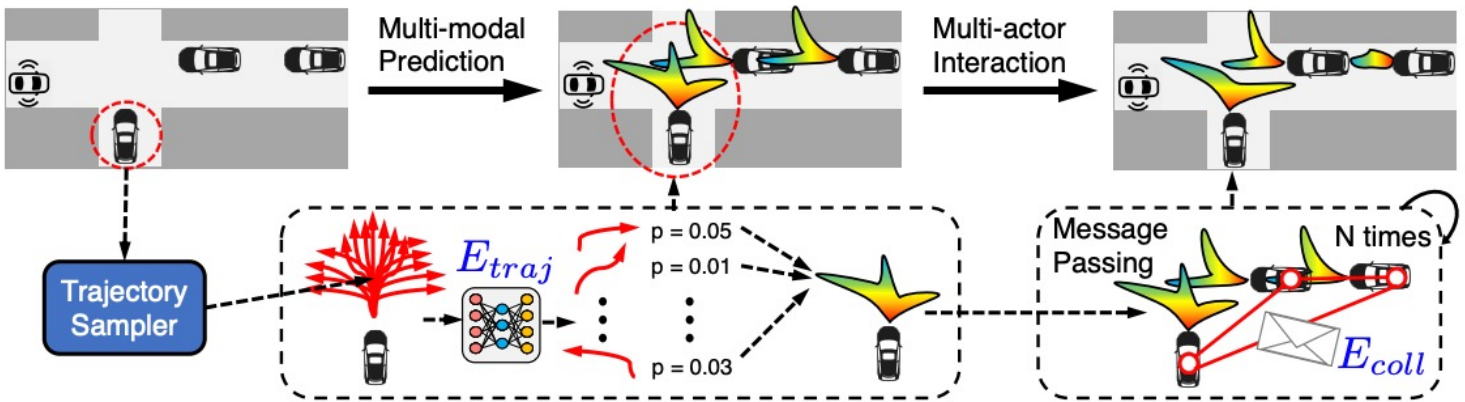
$$\sum_i E_\theta(s_i \mid X) + \sum_{i \neq j} E(\mathbf{s}_i, \mathbf{s}_j)$$

$$E(\mathbf{s}_i, \mathbf{s}_j) = \gamma \quad \text{if } \mathbf{s}_i \text{ collides } \mathbf{s}_j$$



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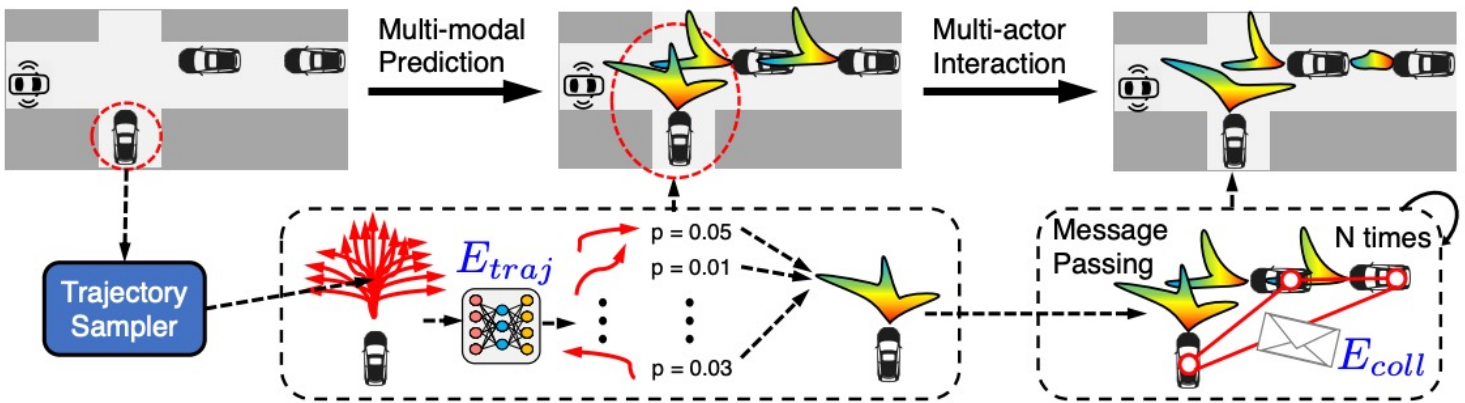

Zeng et al. DSDNet: Deep Structured Self-Driving Network. ECCV 2020.

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Planning with Social Reasoning

- Jointly reason the future trajectories of multiple agents as an energy-based graphical model.
- Trajectory Goodness + Collision.
- Batch of trajectory samples.
- Classification of groundtruth trajectory.

$$p(\mathbf{s}_1, \ldots, \mathbf{s}_N \mid \mathbf{X}) = \frac{1}{Z} \exp(-E_\theta(\mathbf{s}_1, \ldots, \mathbf{s}_N) \mid \mathbf{X})$$

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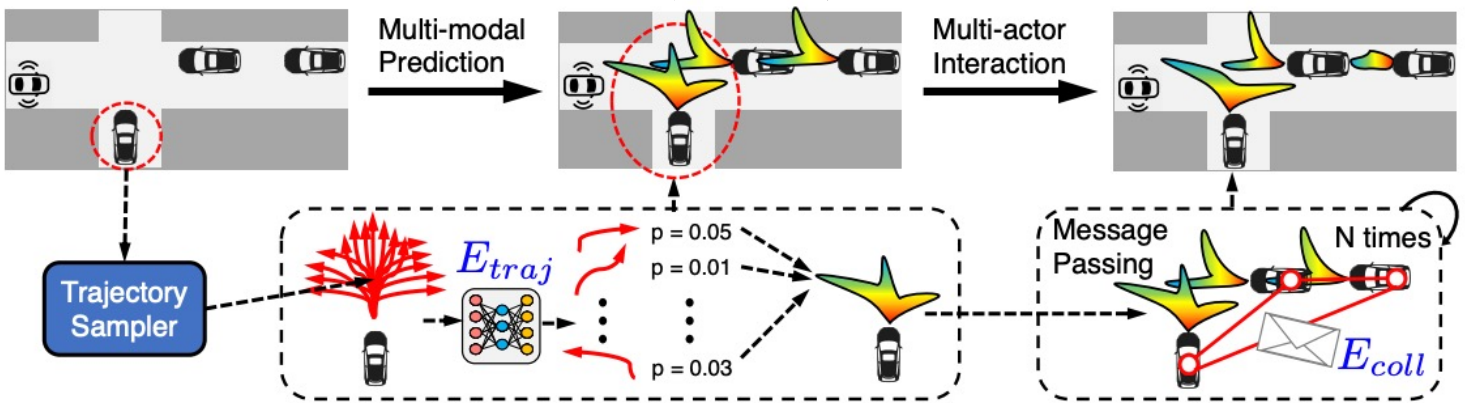

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Summary: End-to-End Planning and Control

- Direct Policy Prediction
  - Condition perception features into the model
  - Use of diffusion models

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- Direct Policy Prediction
  - Condition perception features into the model
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- Cost Learning (IRL) from Experts
  - Max-margin, max-entropy/EBM
  - Need negative samples
  - Can be combined with efficient external samplers
  - Cost volume prediction: parametric + non-parametric

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Summary: End-to-End Planning and Control

- Direct Policy Prediction
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  - Max-margin, max-entropy/EBM
  - Need negative samples
  - Can be combined with efficient external samplers
  - Cost volume prediction: parametric + non-parametric
- Differentiable Planner
  - Backprop through local optimization, learnable cost
  - Can be memory efficient, implicit differentiation

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