

# DS-GA.3001 Embodied Learning and Vision

Mengye Ren

NYU

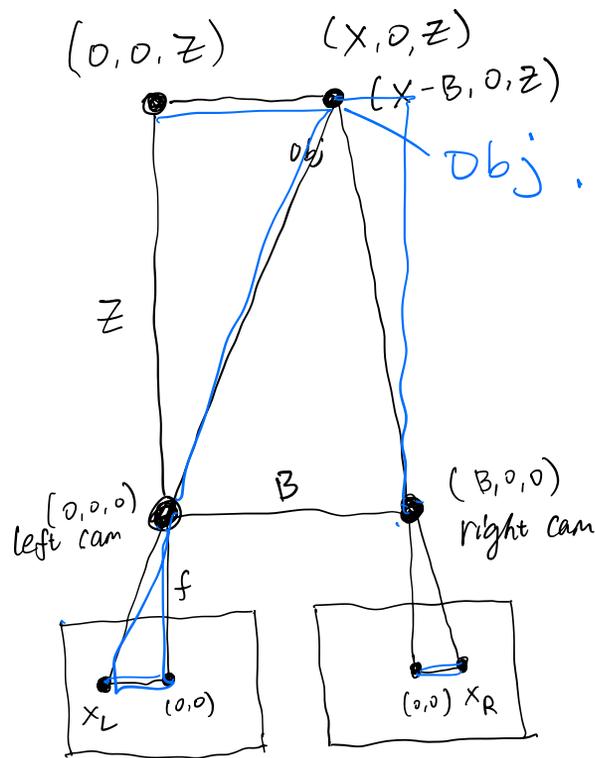
Spring 2026

[elvcourse.org](http://elvcourse.org)



# Classic Vision on Depth and Disparity

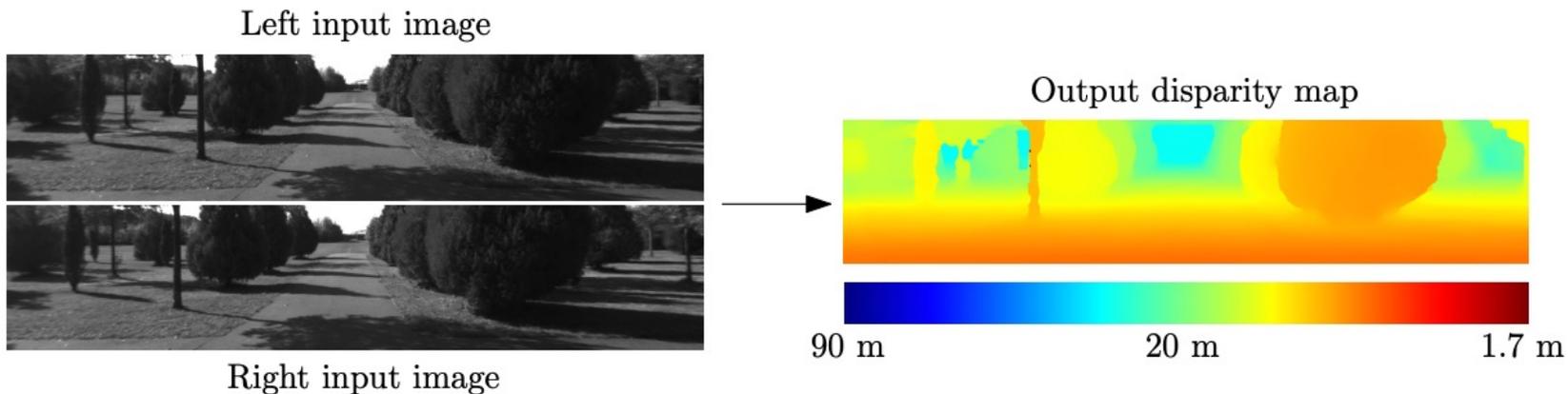
- One source of depth is from the displacement of pixels in a stereo setup.
- But we need to estimate disparity.



$$x_L = \frac{Xf}{Z}$$
$$x_R = \frac{(X-B)f}{Z}$$
$$\delta = x_L - x_R$$
$$= \frac{[X - (X-B)]f}{Z}$$
$$= \frac{Bf}{Z}$$
$$Z = \frac{Bf}{\delta}$$

# From 2D to 3D: Depth Network

- A network that can output disparity.
- Using LiDAR or depth camera as groundtruth supervision.



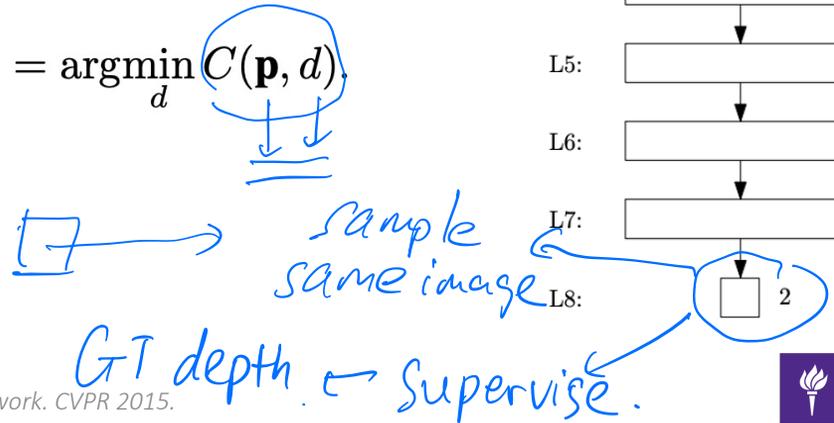
# The Energy-Based Approach

- The energy penalize matching with high cost (unary), and when neighboring pixels have disparity differences greater or equal to one (pairwise).
- Cost network: Train with binary classification

Energy  $E(D) = \sum_{\mathbf{p}} \left( C_{CBCA}^4(\mathbf{p}, D(\mathbf{p})) \right)$  *unary.*

*neighbors.*  $\left\{ \begin{aligned} &+ \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \times \mathbb{1}\{|D(\mathbf{p}) - D(\mathbf{q})| = 1\} \\ &+ \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \times \mathbb{1}\{|D(\mathbf{p}) - D(\mathbf{q})| > 1\} \end{aligned} \right\}$

$$D(\mathbf{p}) = \operatorname{argmin}_d C(\mathbf{p}, d)$$

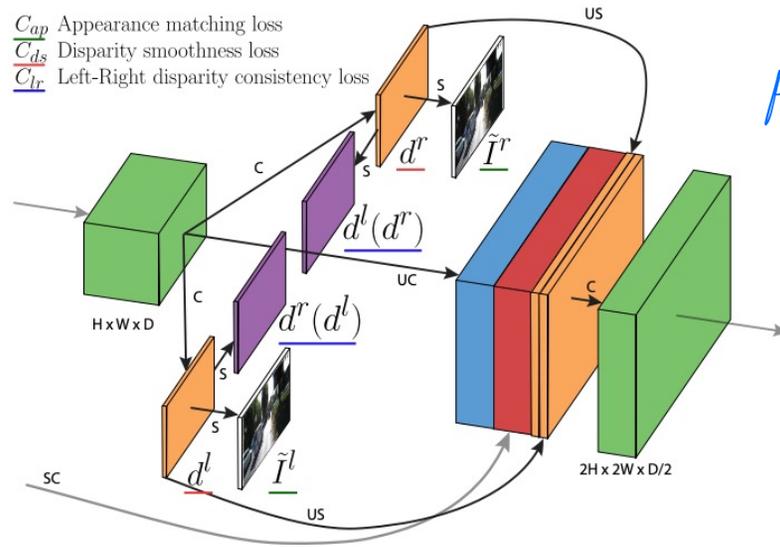


# Self-Supervised Depth

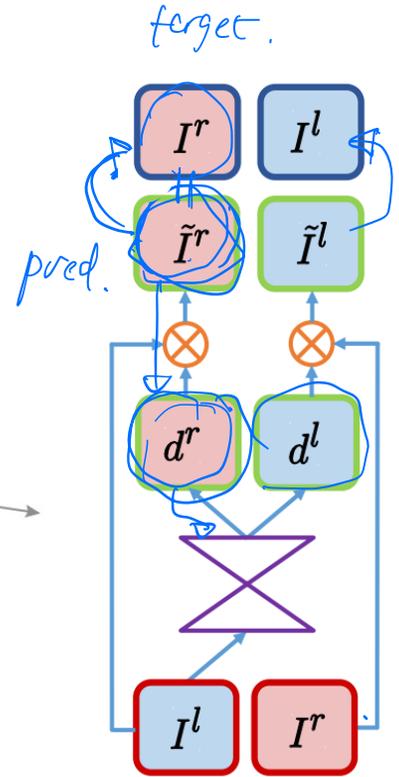
- Appearance matching loss

$$C_{ap}^l = \frac{1}{N} \sum_{i,j} \alpha \frac{1 - \text{SSIM}(I_{ij}^l, \tilde{I}_{ij}^l)}{2} + (1 - \alpha) \|I_{ij}^l - \tilde{I}_{ij}^l\|.$$

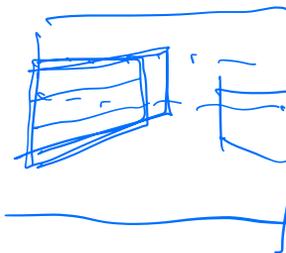
pixel.



$C_{ap}$  Appearance matching loss  
 $C_{ds}$  Disparity smoothness loss  
 $C_{lr}$  Left-Right disparity consistency loss



# Self-Supervised Depth



- Appearance matching loss

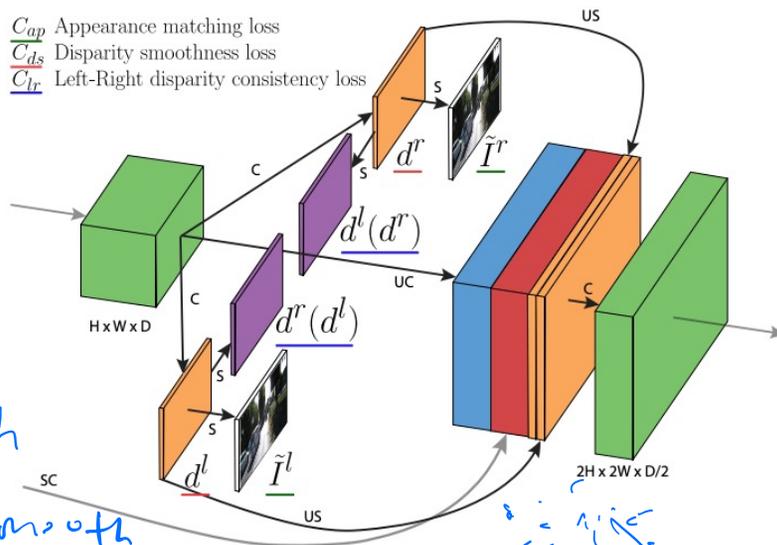
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$\rightarrow dx$   
 $\rightarrow dy$

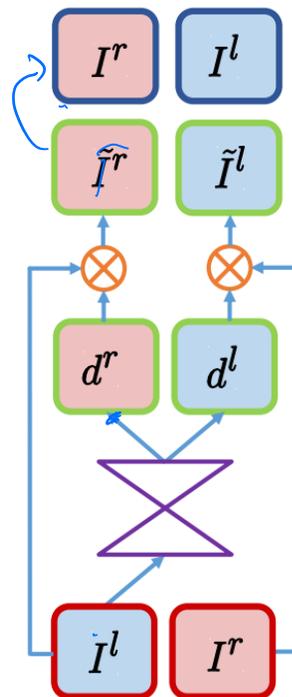
- Disparity smoothness loss

$$C_{ds}^l = \frac{1}{N} \sum_{i,j} \left[ |\partial_x d_{ij}^l| e^{-\|\partial_x I_{ij}^l\|} + |\partial_y d_{ij}^l| e^{-\|\partial_y I_{ij}^l\|} \right].$$

if image is smooth then depth should also be smooth.



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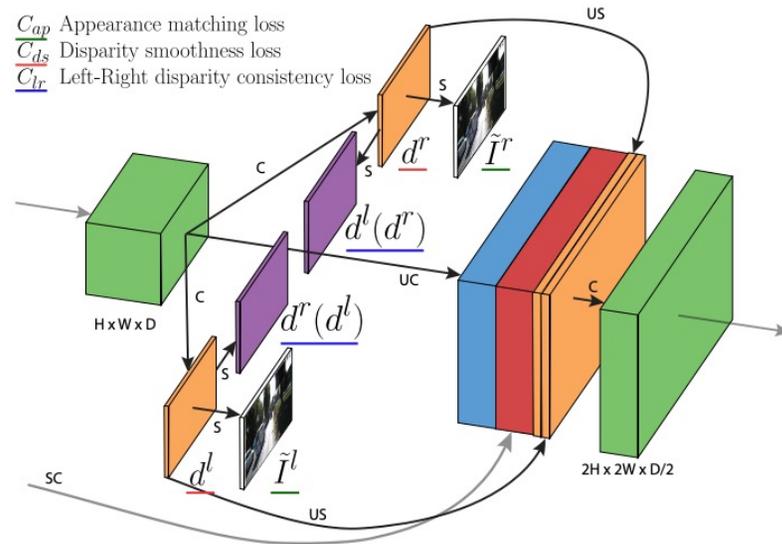
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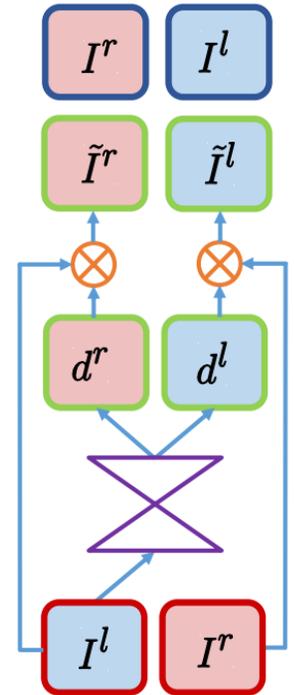
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- Left-right disparity consistency loss

$$C_{lr}^l = \frac{1}{N} \sum_{i,j} |d_{ij}^l - d_{ij+d_{ij}^l}^r|.$$



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# Motion, Optical Flow

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$$I(\underline{x}, \underline{y}, \underline{t}) = I(\underbrace{x + \Delta x}_{\text{unk}}, \underbrace{y + \Delta y}_{\text{unk}}, \underline{t + \Delta t}).$$

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$$I(x + \Delta x, y + \Delta y, t + \Delta t) = \underline{I(x, y, t)} + \underbrace{\left(\frac{\partial I}{\partial x}\right)}_{\underline{\quad}} \Delta x + \underbrace{\left(\frac{\partial I}{\partial y}\right)}_{\underline{\quad}} \Delta y + \underbrace{\left(\frac{\partial I}{\partial t}\right)}_{\underline{\quad}} \Delta t.$$

Taylor  
expansion.

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$$\frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0.$$

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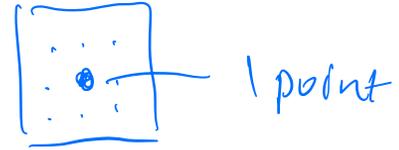
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$$\begin{aligned} I_x &= \frac{\partial I}{\partial x} \\ I_y &= \frac{\partial I}{\partial y} \end{aligned} \quad \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0.$$

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$$I_x = \frac{\partial I}{\partial x}$$

$$I_y = \frac{\partial I}{\partial y}$$

$$\frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0.$$

$$I_x u + I_y v + I_t = 0.$$

horizontal displacement

2 unk. equation

# Classical Approach

- Under-constrained system

$$I_x u + I_y v + I_t = 0.$$

# Classical Approach

- Under-constrained system  $I_x u + I_y v + I_t = 0.$
- Use a local patch and assume smooth motion

$$\mathbf{A}\mathbf{u} = \mathbf{b}$$
$$\begin{pmatrix} \underline{I_x(\mathbf{p}_1)} & \underline{I_y(\mathbf{p}_1)} \\ \vdots & \vdots \\ \underline{I_x(\mathbf{p}_{N^2})} & \underline{I_y(\mathbf{p}_{N^2})} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} I_t(\mathbf{p}_1) \\ \vdots \\ I_t(\mathbf{p}_{N^2}) \end{pmatrix}$$

# Classical Approach

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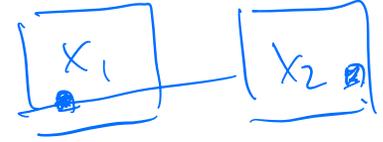
- Rigid, contains many assumptions

*path smoothness.* *brightness* *lighting* *occlusion.*

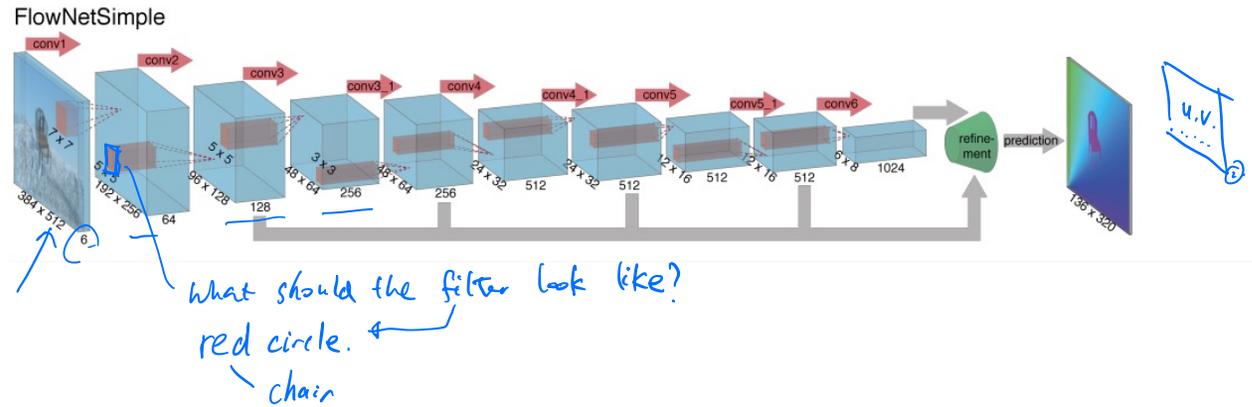
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# Correlation Volume Approach

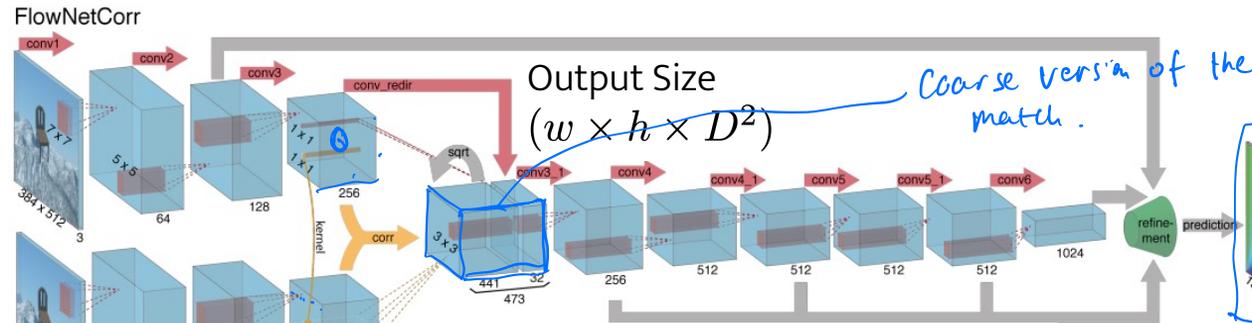
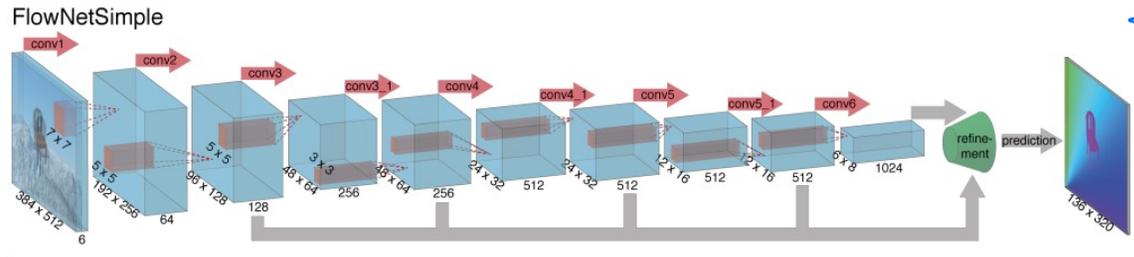


- Simple Approach: Concatenate the two images together.



# Correlation Volume Approach

- Simple Approach: Concatenate the two images together.
- Correlation: Extract some levels of features, and convolve one feature on top of another.



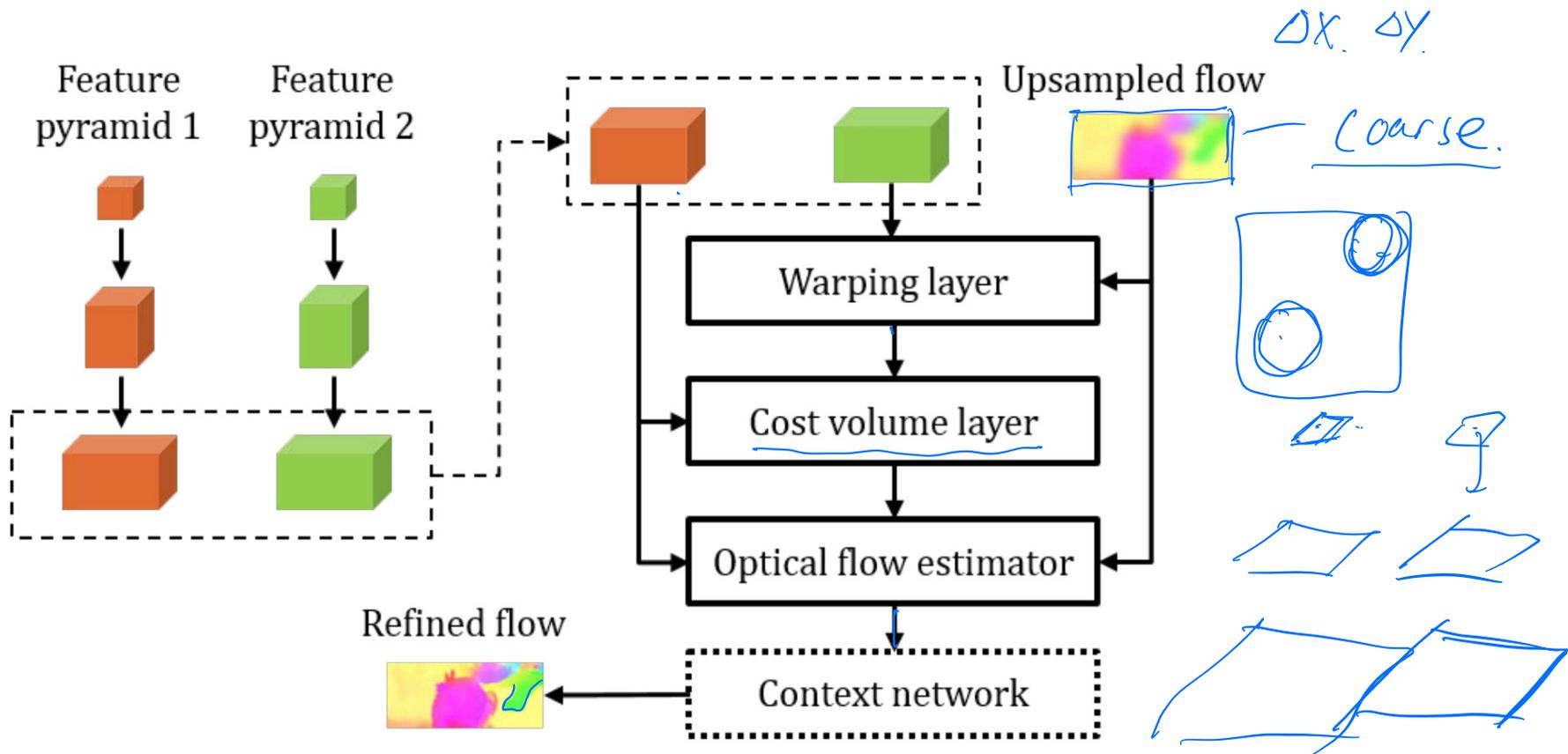
$$c(\mathbf{x}_1, \mathbf{x}_2) = \sum_{\mathbf{o} \in [-k, k] \times [-k, k]} \langle \mathbf{f}_1(\mathbf{x}_1 + \mathbf{o}), \mathbf{f}_2(\mathbf{x}_2 + \mathbf{o}) \rangle$$

features of one image.

$H \times W \times P \times P$ .

Simulation  
↓  
G.T.

# Iterative Refining through Feature Pyramid

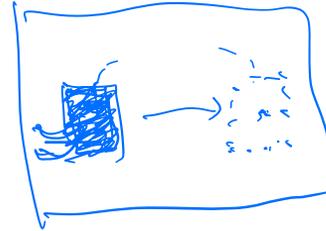


# Unsupervised Flow

- Photometric Consistency (Appearance)

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- Photometric Consistency (Appearance)
- Occlusion Estimation
  - Forward-backward consistency



$$\text{warp}(I_t \rightarrow I_{t+1})$$

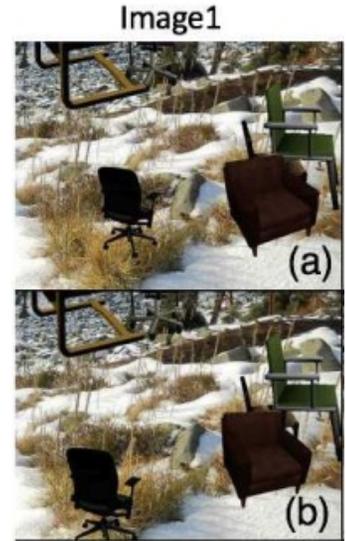
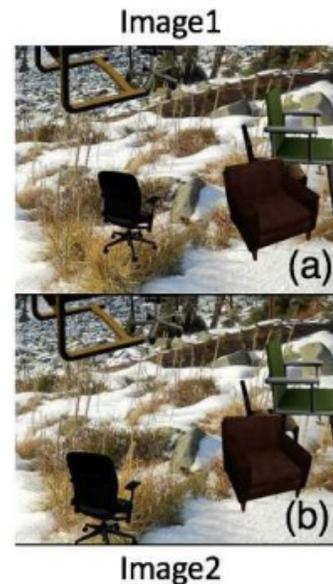


Image2  
Wang et al., 2018

# Unsupervised Flow

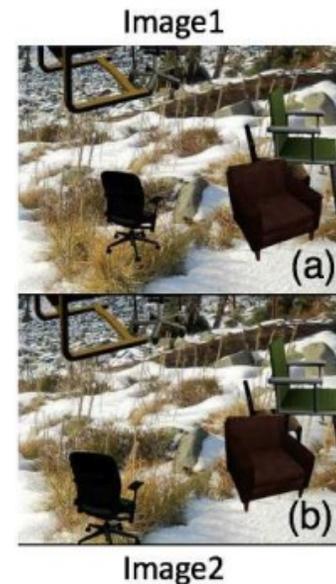
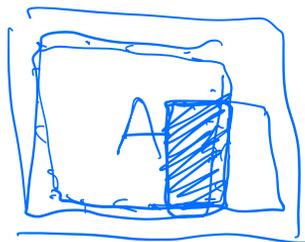
- Photometric Consistency (Appearance)
- Occlusion Estimation
  - Forward-backward consistency
- Smoothness



Wang et al., 2018

# Unsupervised Flow

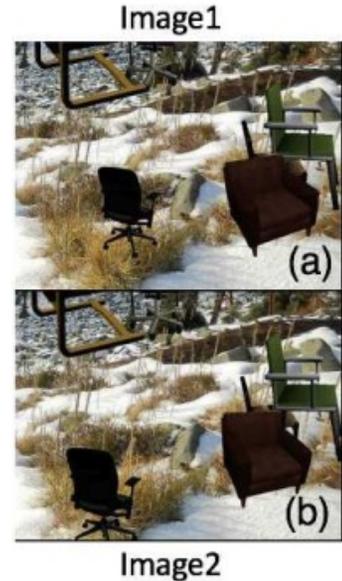
- Photometric Consistency (Appearance)
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- Self-supervision: Ensure consistent flow at different augmentation (e.g. crops)



Wang et al., 2018

# Unsupervised Flow

- Photometric Consistency (Appearance)
- Occlusion Estimation
  - Forward-backward consistency
- Smoothness
- Self-supervision: Ensure consistent flow at different augmentation (e.g. crops)
- Can 3D information help us reason about motion?

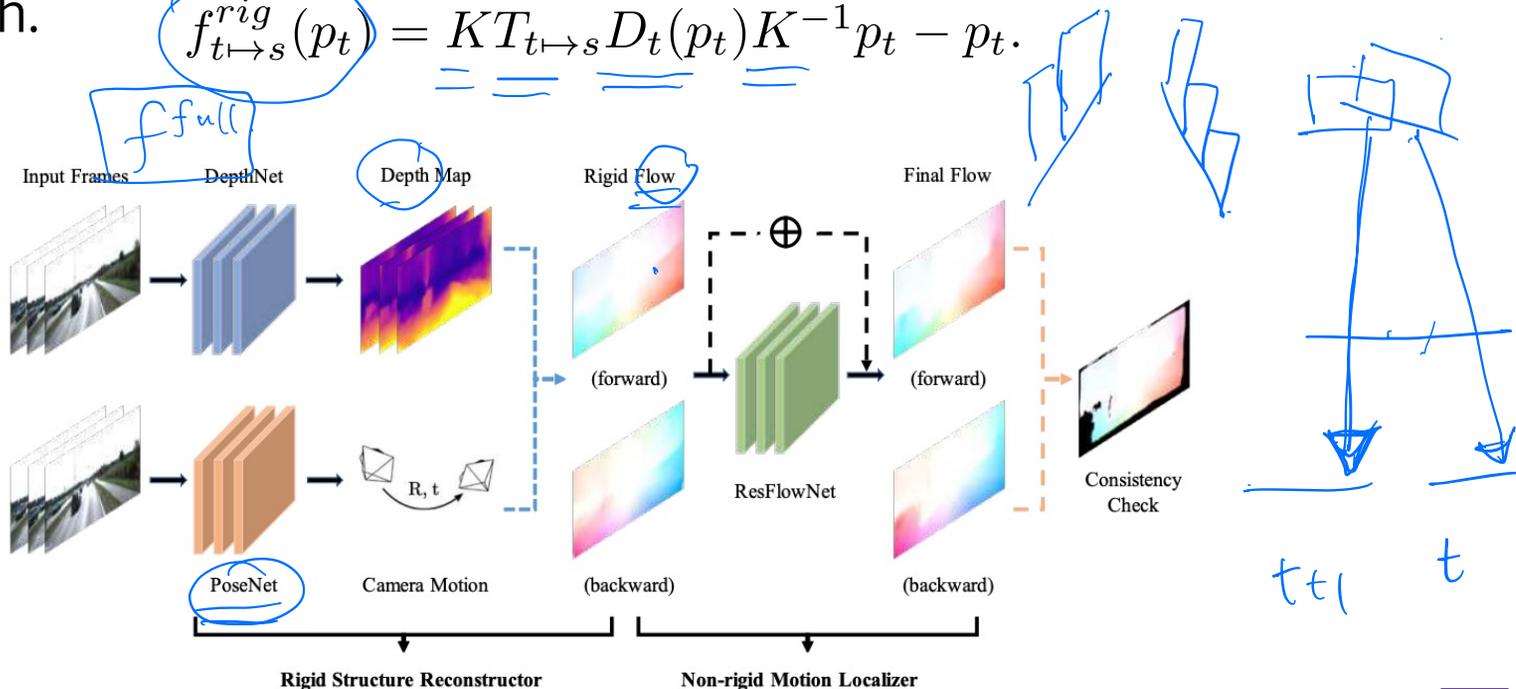


Wang et al., 2018

# Depth, Flow, and Pose Movement

- The static objects follow rigid flow: determined by camera motion and depth.

$$f_{t \rightarrow s}^{rig}(p_t) = \underline{\underline{K T_{t \rightarrow s} D_t(p_t) K^{-1} p_t - p_t}}$$



# Training Losses

- Appearance Loss (Warping):

$$\mathcal{L}_{rw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{rig})}{2} + (1 - \alpha) \|I_t - \tilde{I}_s^{rig}\|_1.$$

$$\mathcal{L}_{fw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{full})}{2} + (1 - \alpha) \|I_t - \tilde{I}_s^{full}\|_1.$$

*rigid.*

*full.*

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- Smoothness Loss:

$$\mathcal{L} = \sum_{p_t} \underbrace{|\nabla D(p_t)|}_{\text{smoothness}} \cdot \underbrace{(\exp(-|\nabla I(p(t))|))}_{\text{smoothness}}^T.$$

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- Forward-Backward Consistency:

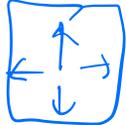
$$\mathcal{L} = \sum_{p_t} [\delta(p_t)] \cdot \|\Delta f_{t \rightarrow s}^{full}(p_t)\|_1.$$

if  $\Delta$  flow is too big, then ignore.

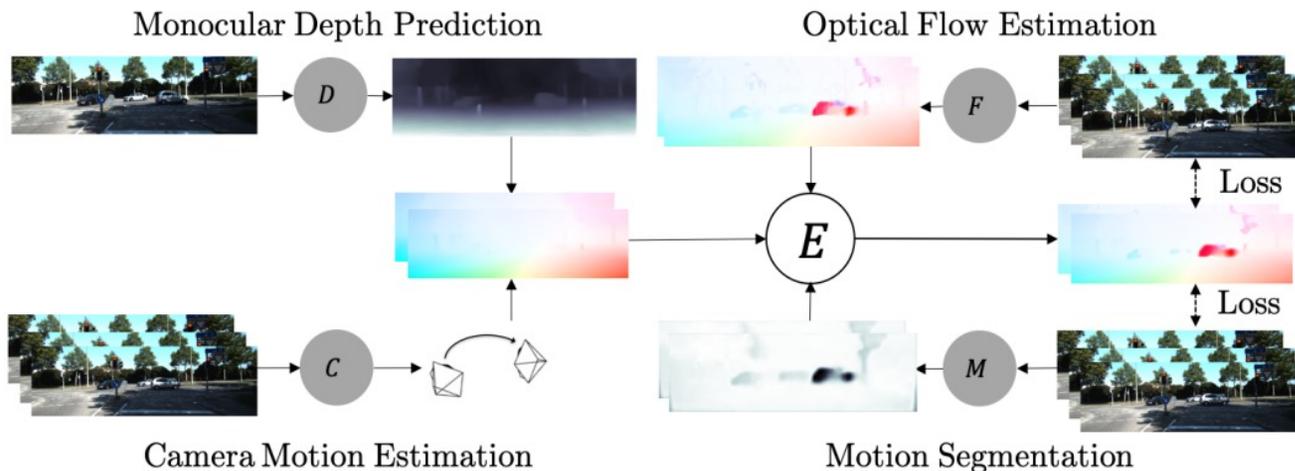
$$\delta(p_t) = \|f_{t \rightarrow s}^{full}(p_t)\|_2 \max\{\alpha, \beta \|f_{t \rightarrow s}^{full}(p_t)\|_2\}.$$

# Summary

→ horizontal + depth

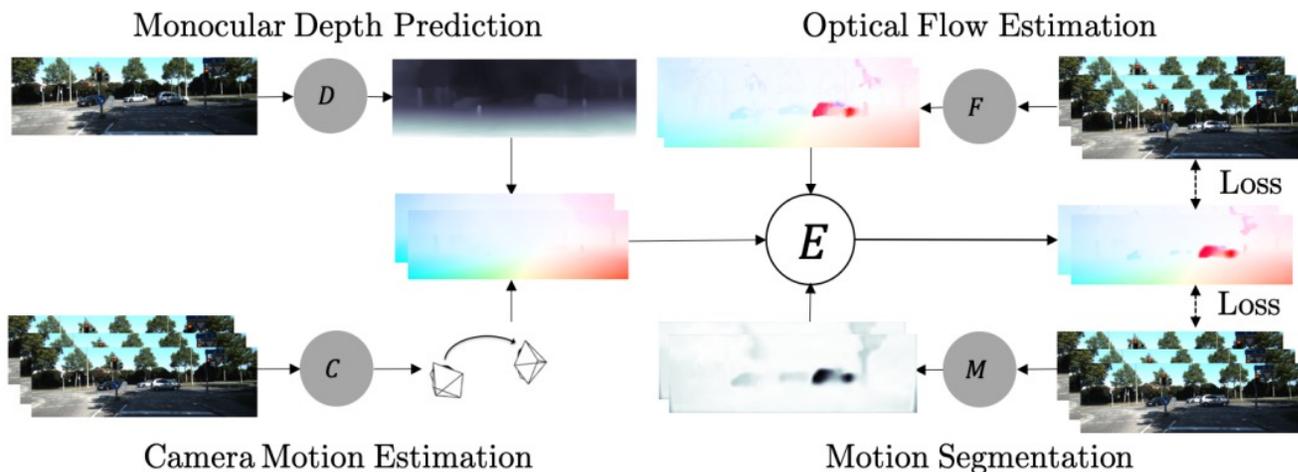
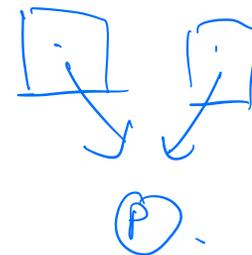
flow 

- Leverage cross correlation structure for spatial similarity matching.



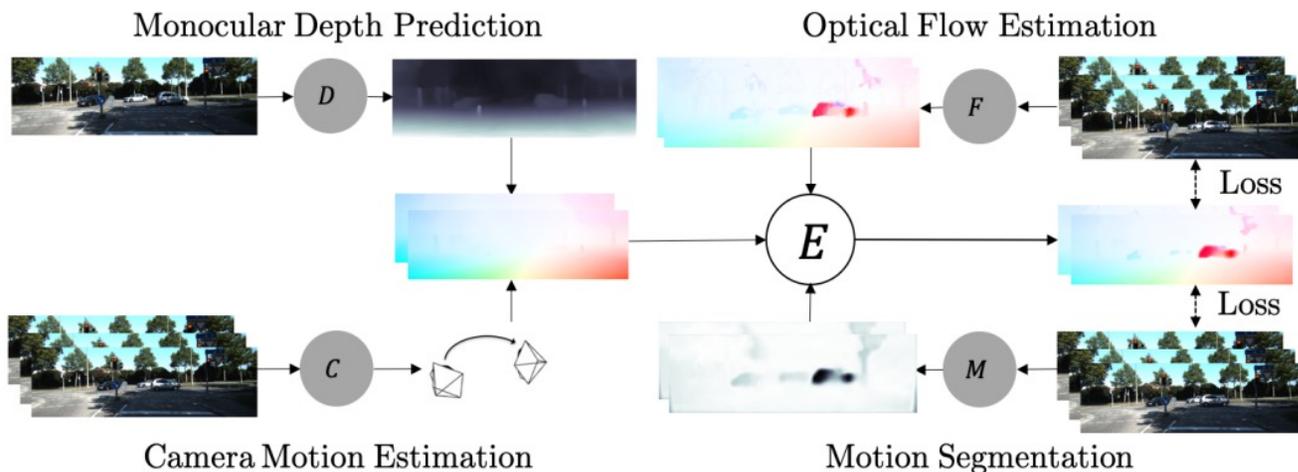
# Summary

- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.



# Summary

- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.
- Can form triangulation for self-supervision. *static object.*



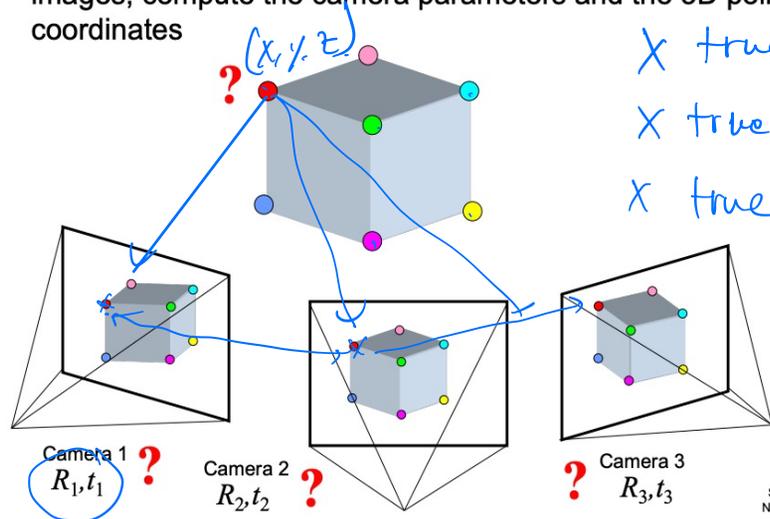
# Classical Mapping

Depth  
Motion

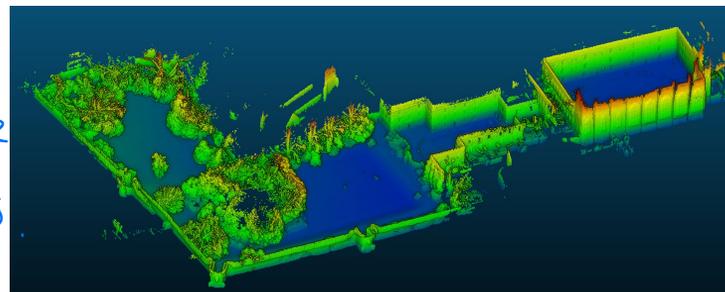
- Estimating 3D structure and location from 2D observations.

X GPS

- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



X true cam pose  
X true key points  
X true 3d



Slide credit:  
Noah Snavely

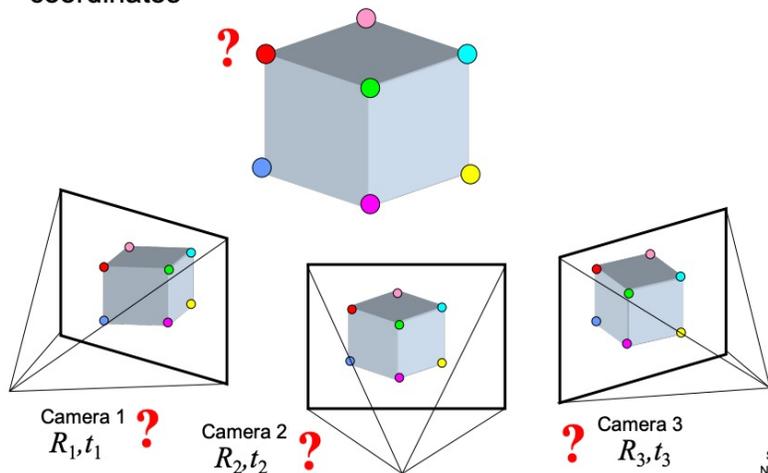
Garg & Jain

# Classical Mapping

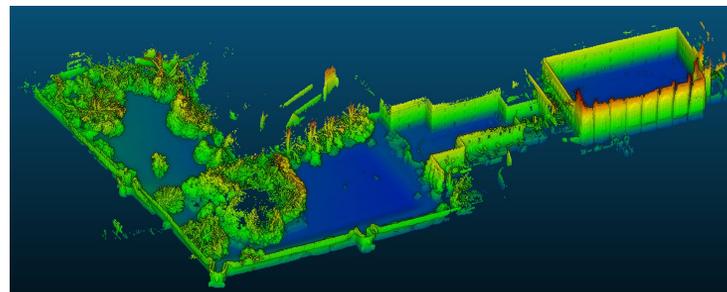
- Estimating 3D structure and location from 2D observations.
- Simultaneous Localization and Mapping. *3d structure.*

*pose.*

- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



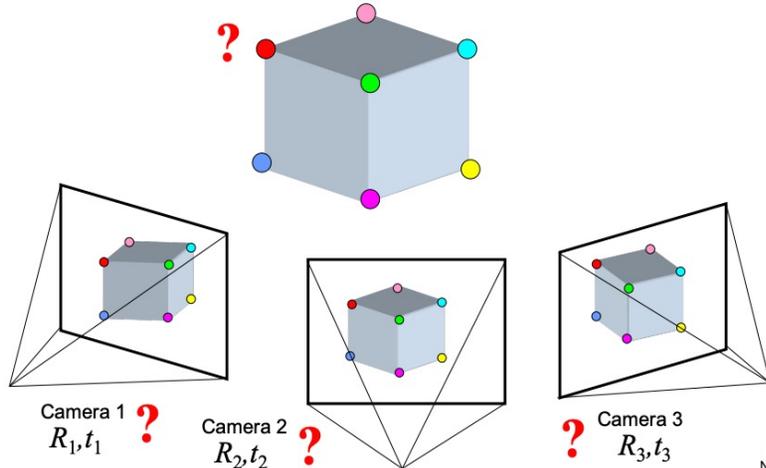
Slide credit:  
Noah Snavely



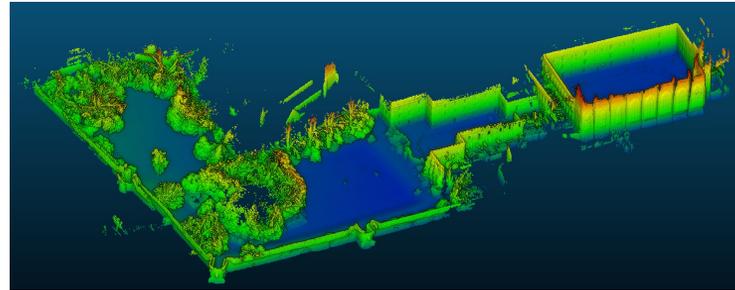
Garg & Jain

# Classical Mapping

- Estimating 3D structure and location from 2D observations.
- Simultaneous Localization and Mapping.
- Common Techniques: Extended Kalman Filter, GraphSLAM
- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



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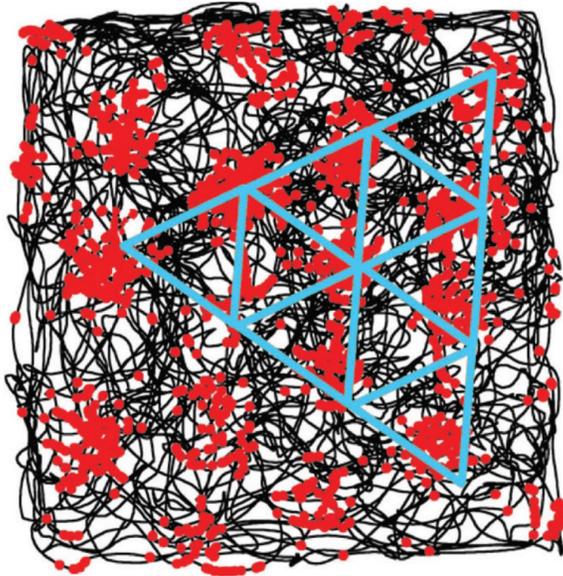
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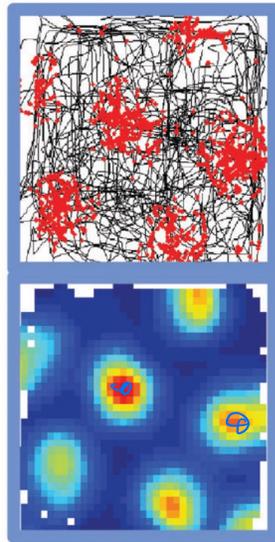
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- Great for 3D reconstruction but downstream tasks may not need a full precision explicit map.
- May not fully understand dynamic objects (averaging across multiple scans).
- Is there a more end-to-end version?

# Mapping in the Brain: Grid and Place Cells

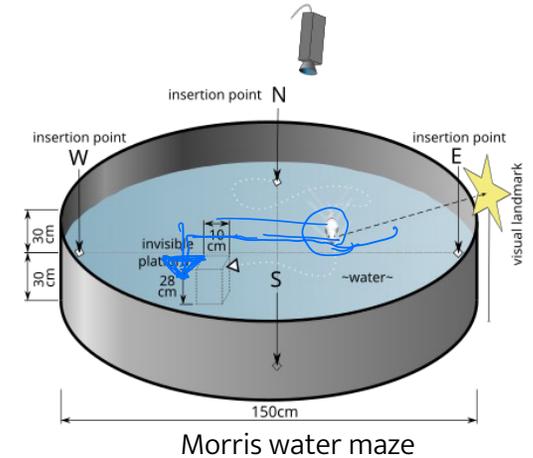
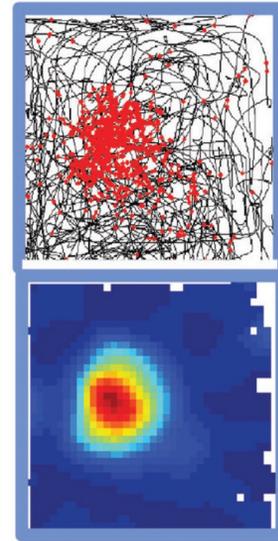
Metric.



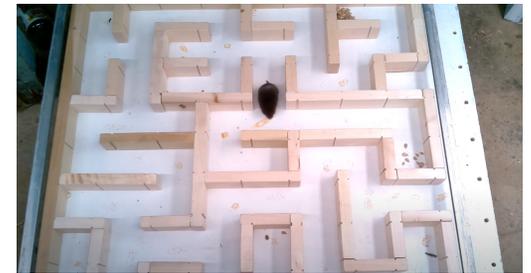
Grid



Place

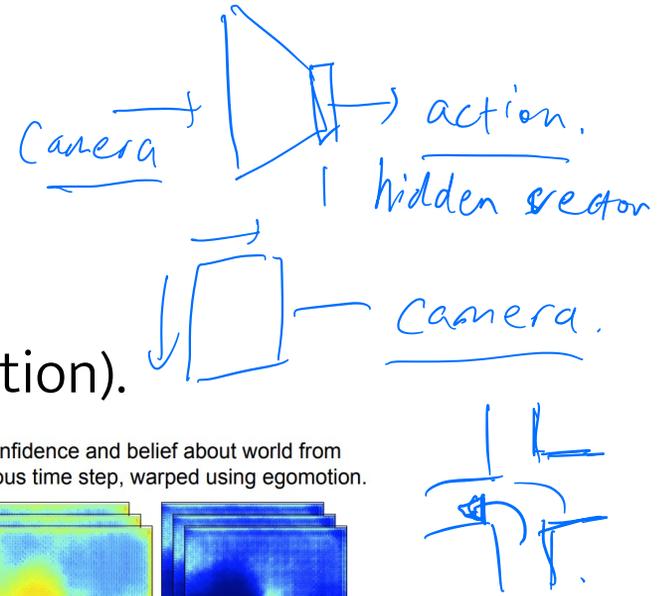


Morris water maze

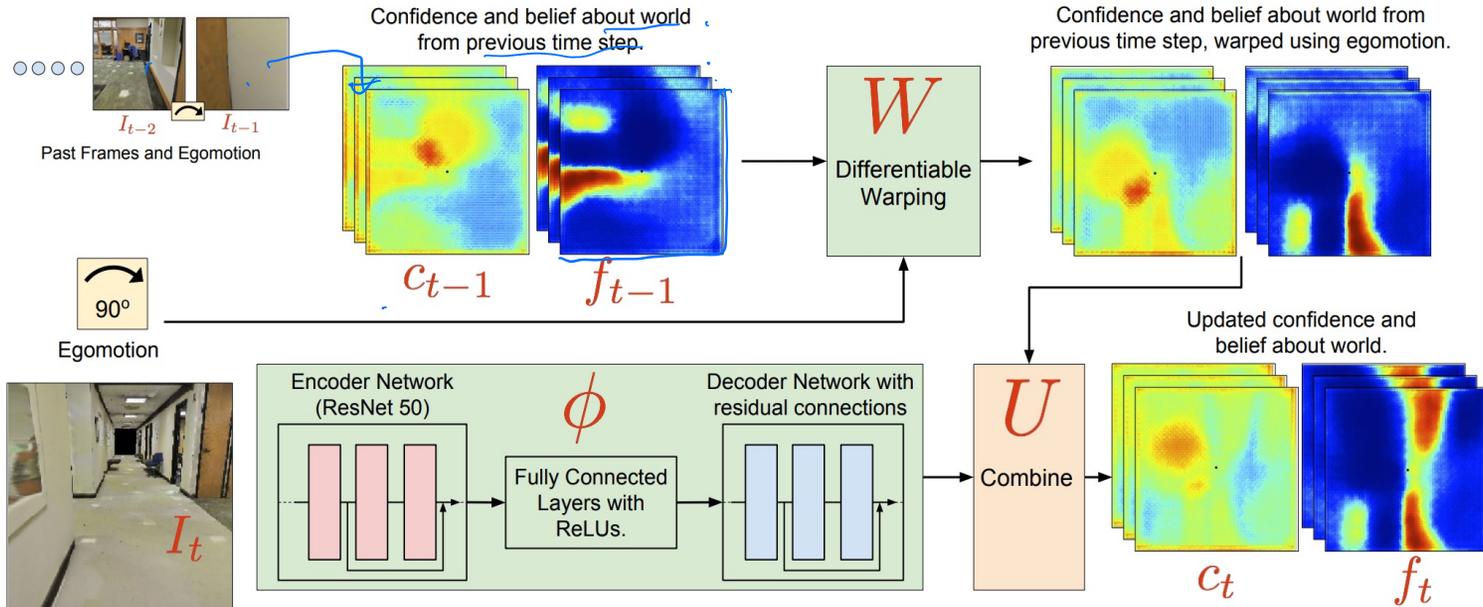


Matthias Wandel, 2018

# Neural Mapping



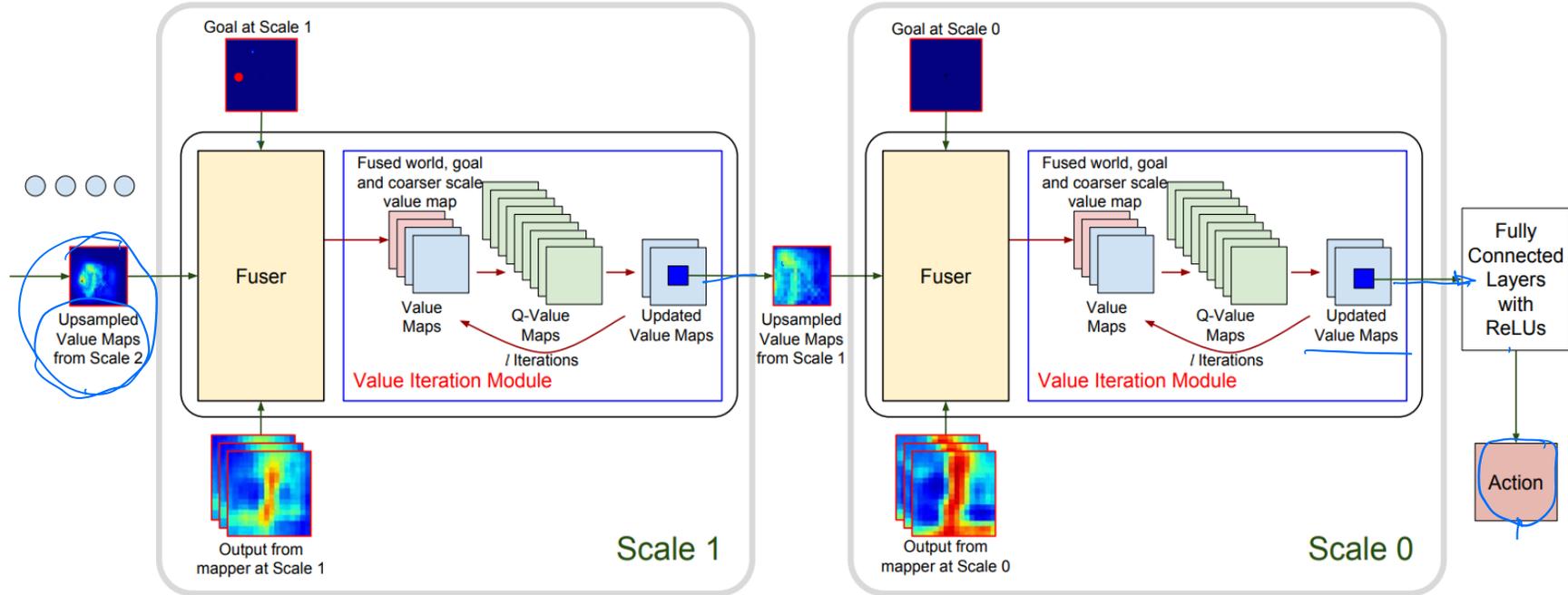
- Can we learn a mapping representation?
- Metric space, top-down warping (known egomotion).



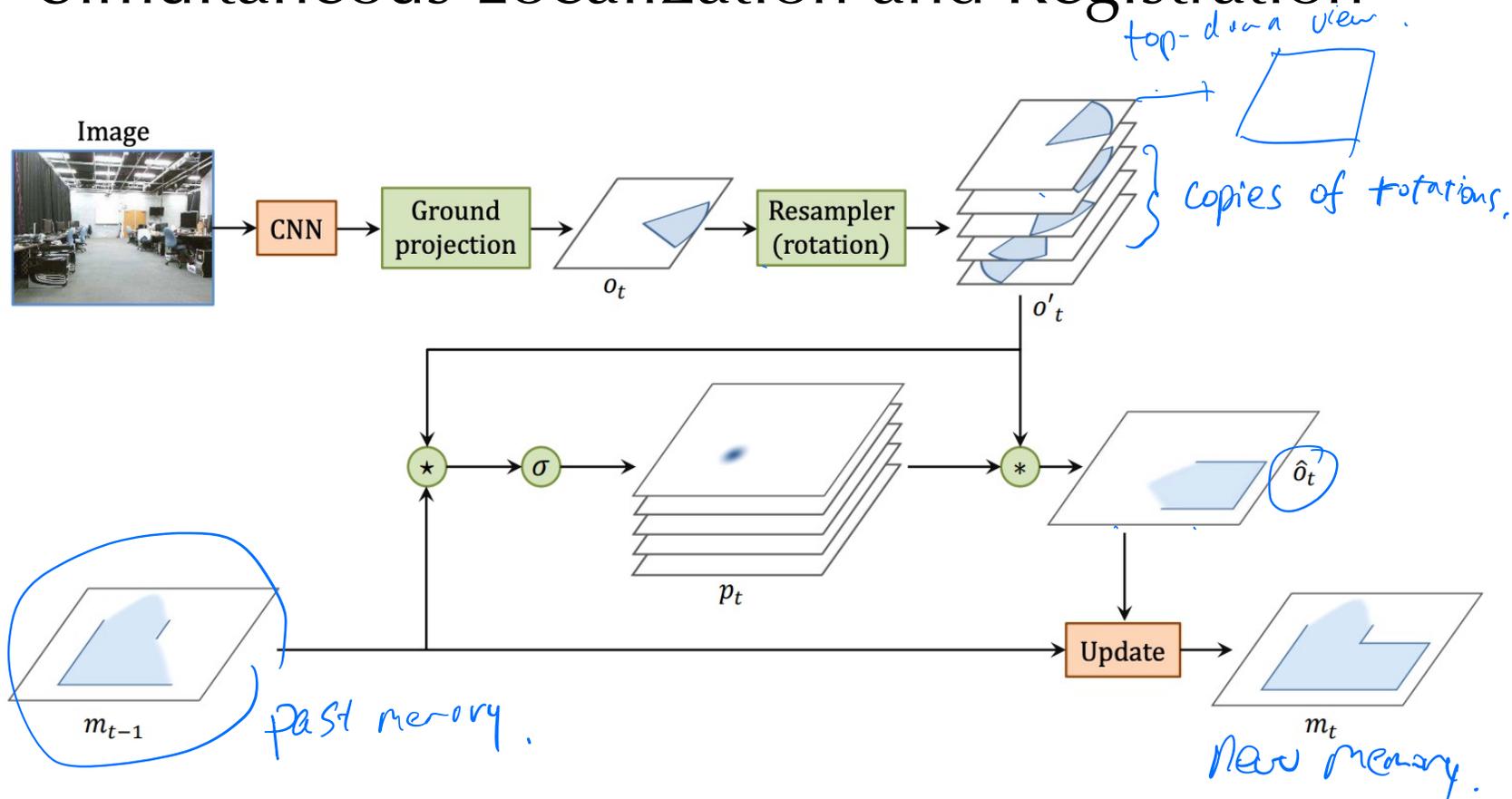
# Hierarchical Planning

↗ *egocentric*

- How do we use the learned map (allocentric) feature of the world?



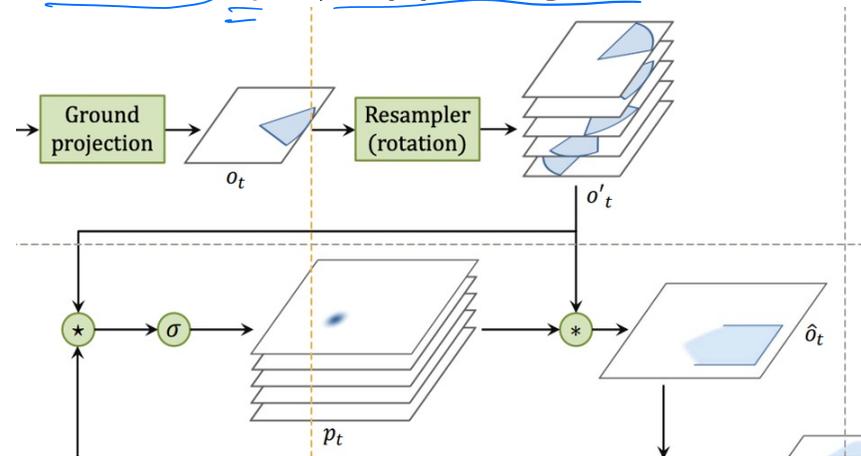
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$$o'_{ijkl} = [R(o, 2\pi l/r)]_{ijk}.$$



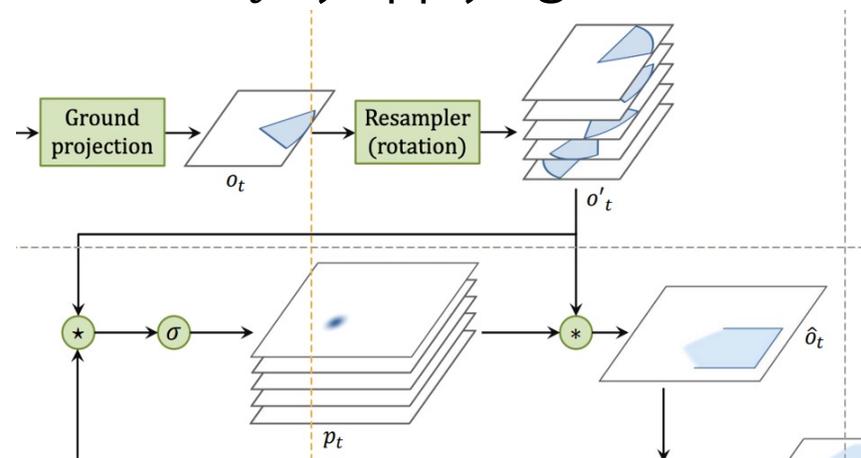
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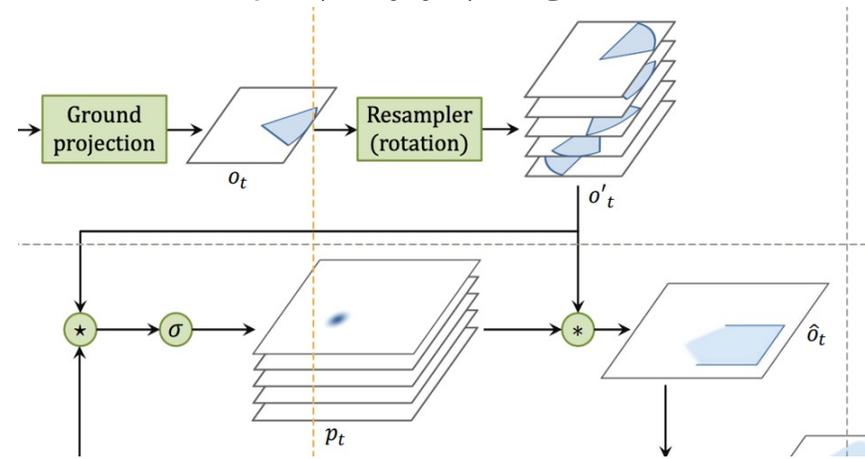
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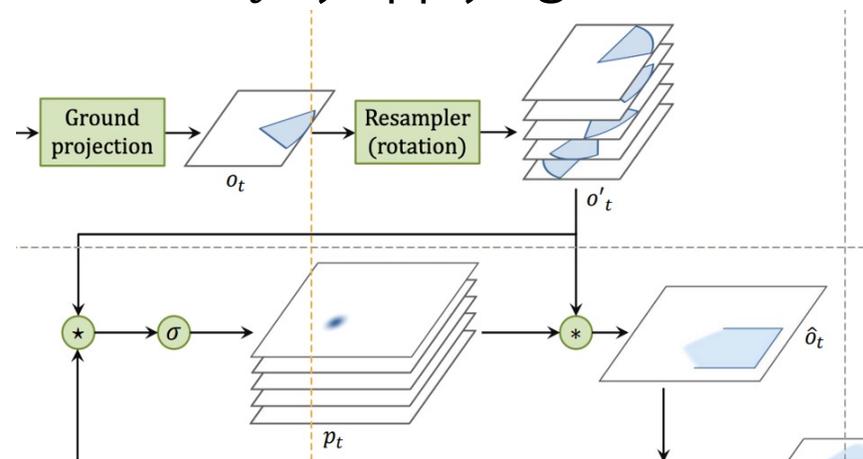
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planner.

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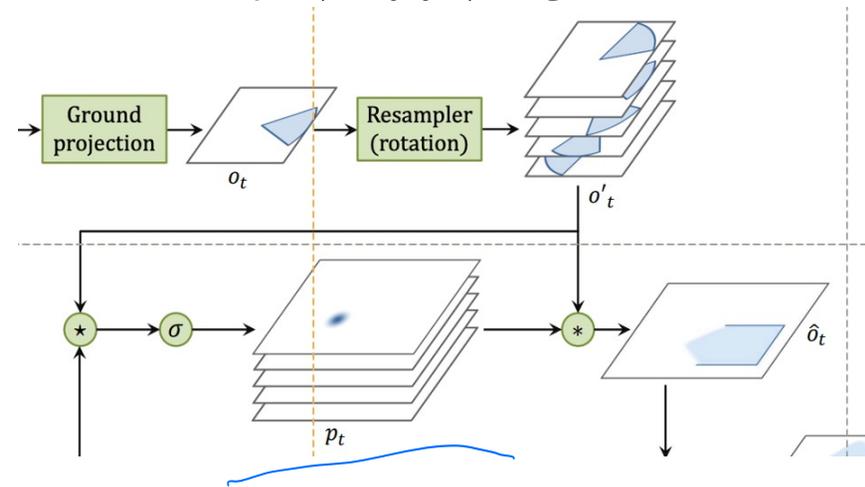
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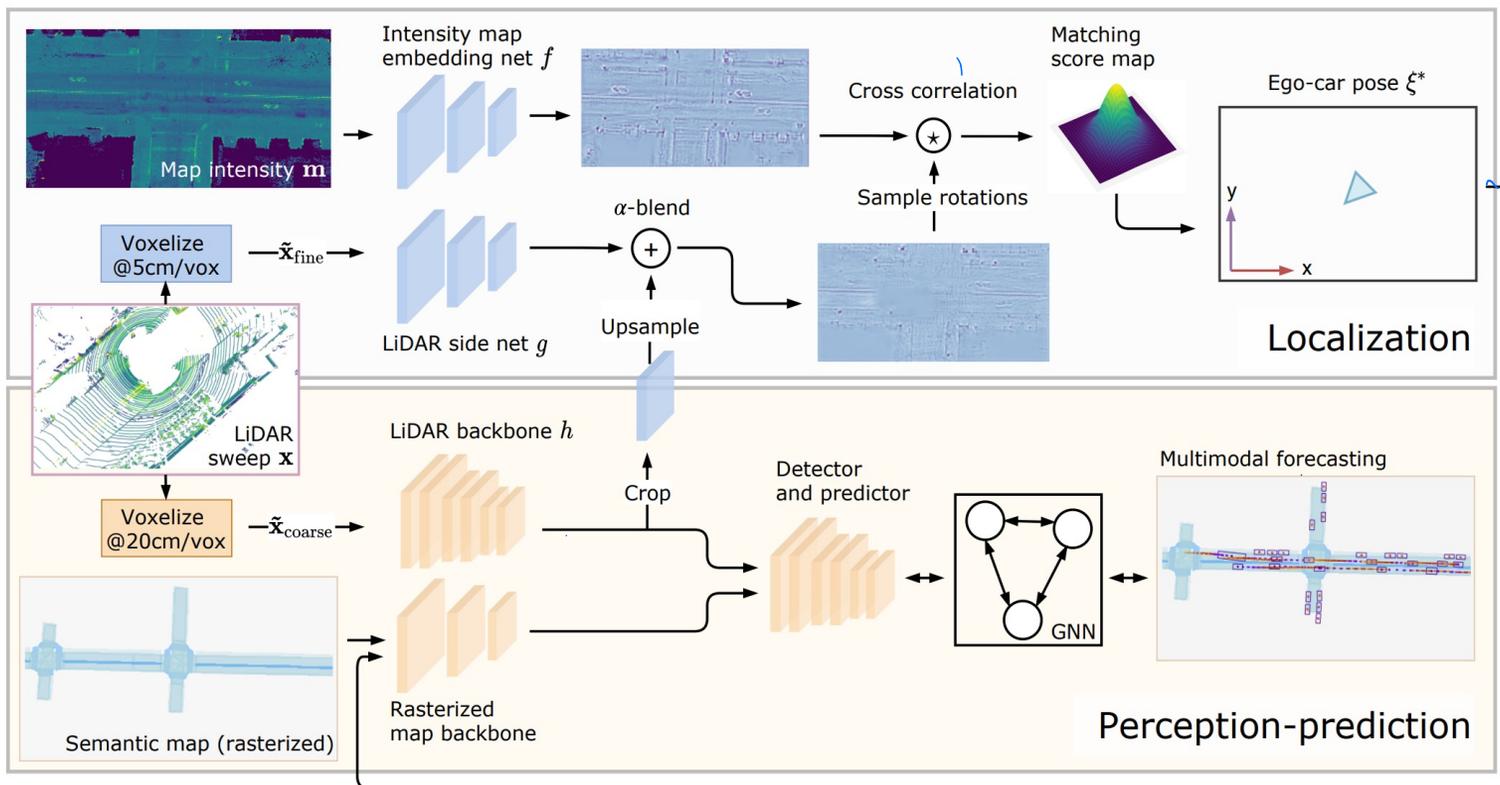
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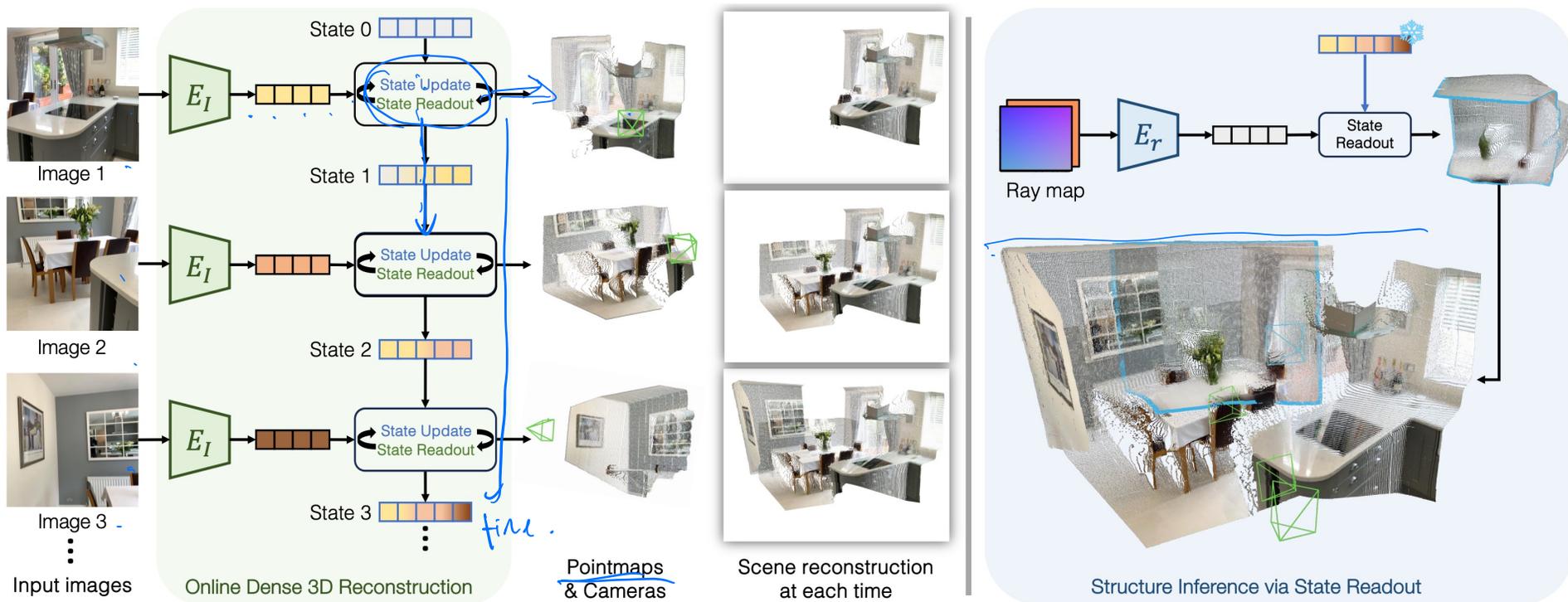
Loss:

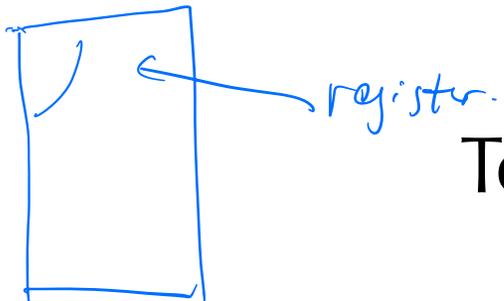
$$\mathcal{L}(p) = -\log \sum_t p_{H_t W_t R_{tt}}.$$

# Joint Localization, Perception and Prediction



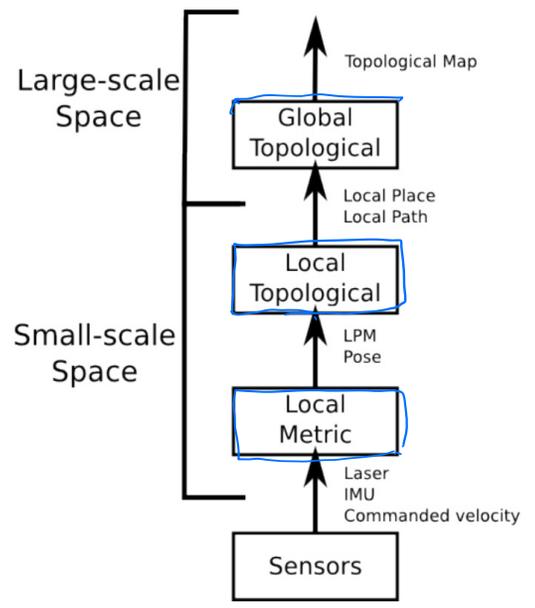
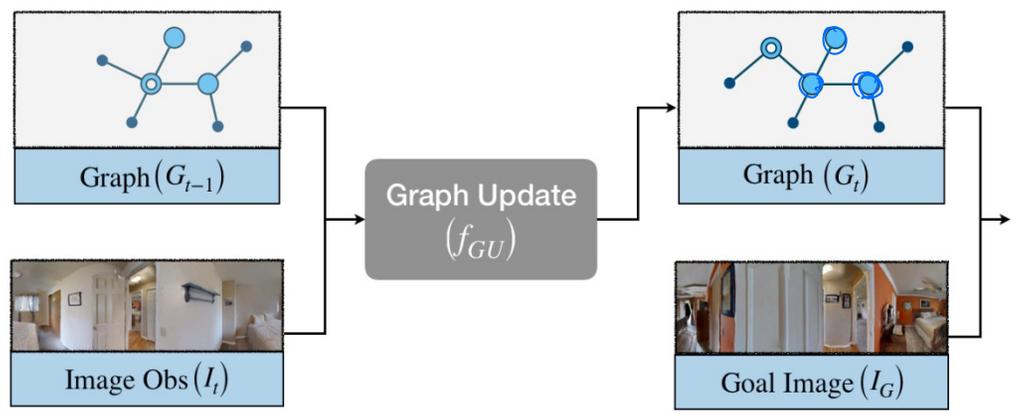
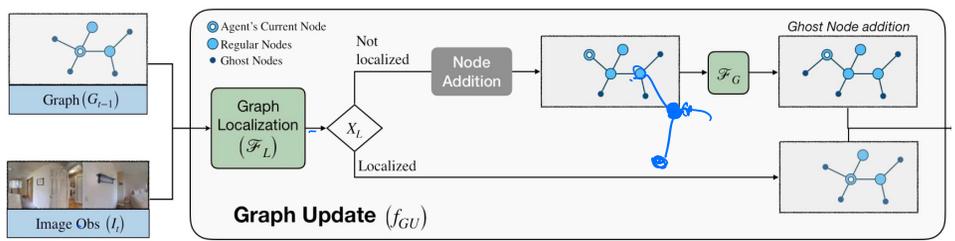
# Continuous 3D Perception and Mapping





# Topological Mapping

- High-level graph representation
- Each node contains more summarized information
- Enables global planning



Johnson. Topological Mapping and Navigation in Real-World Environments. 2018.  
 Chaplot et al. Neural Topological SLAM for Visual Navigation. CVPR 2020.

# Summary

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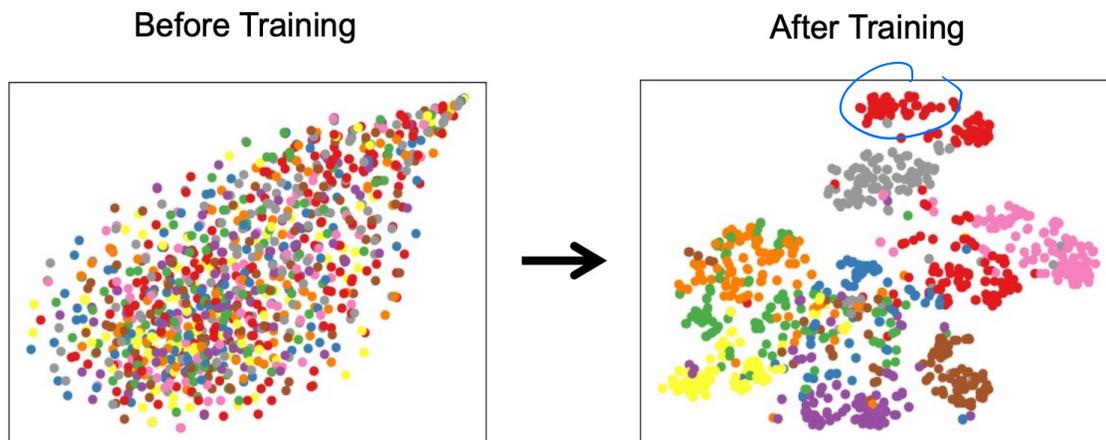
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- Design joint learning frameworks.
- Using geometric transformation to ground representations.
- Useful for planning (a few weeks from now).

*Latent* Module 3:  
Self-Supervised Representation Learning and  
Object Discovery

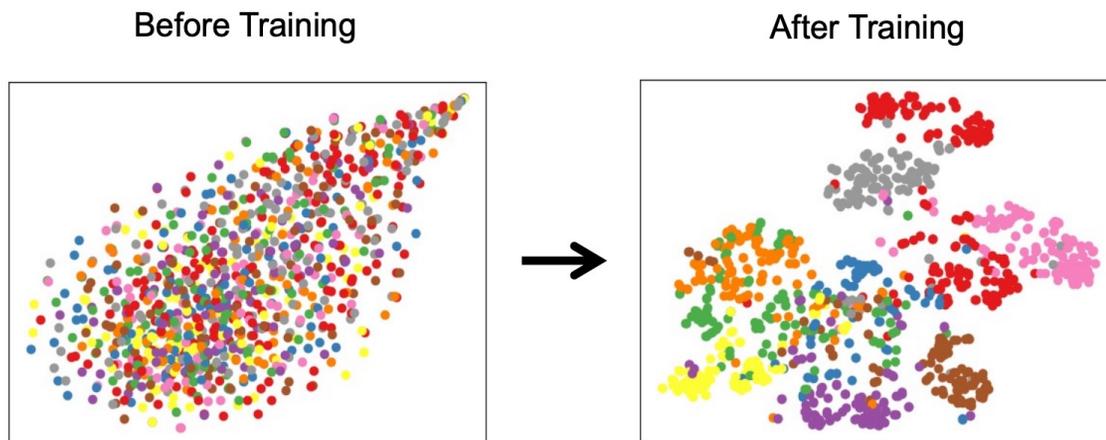
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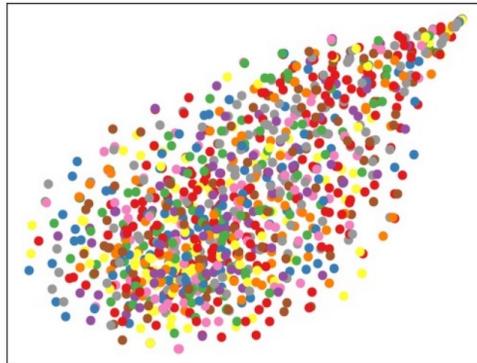




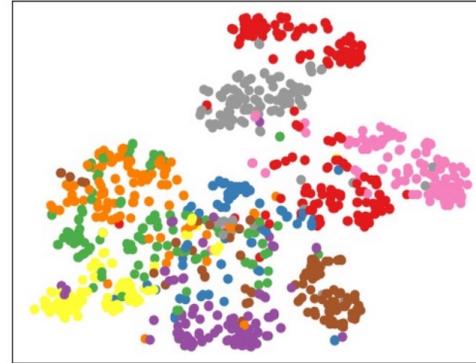
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Before Training



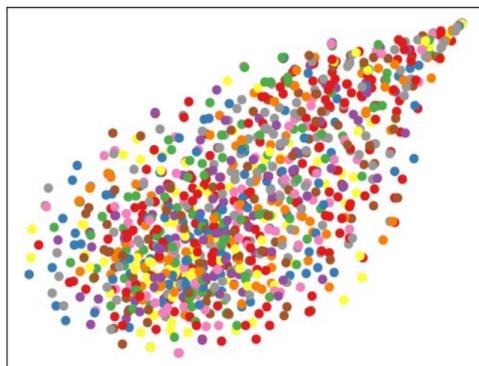
After Training



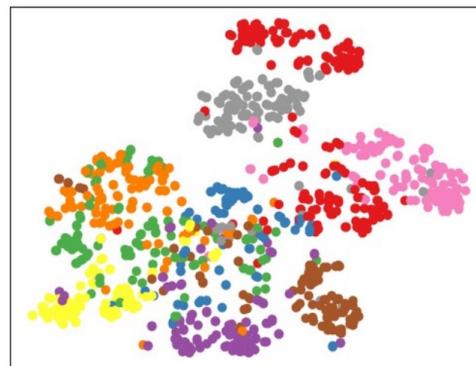
# Representation Learning

- Efficient encoding of the world that can help us recognize semantic concepts (high-level cognition).
- Efficient learning of visual data without extra supervision.
- Recognition of motion also requires global matching.
- Historically, largely driven by supervised classification.

Before Training

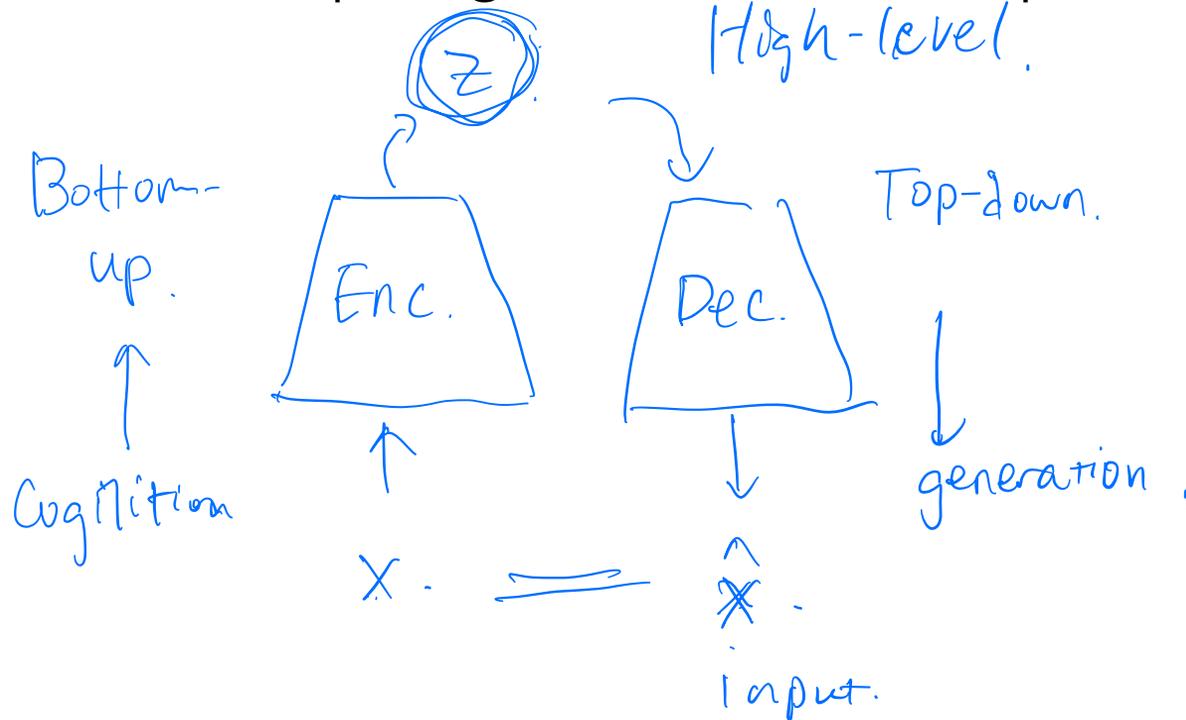


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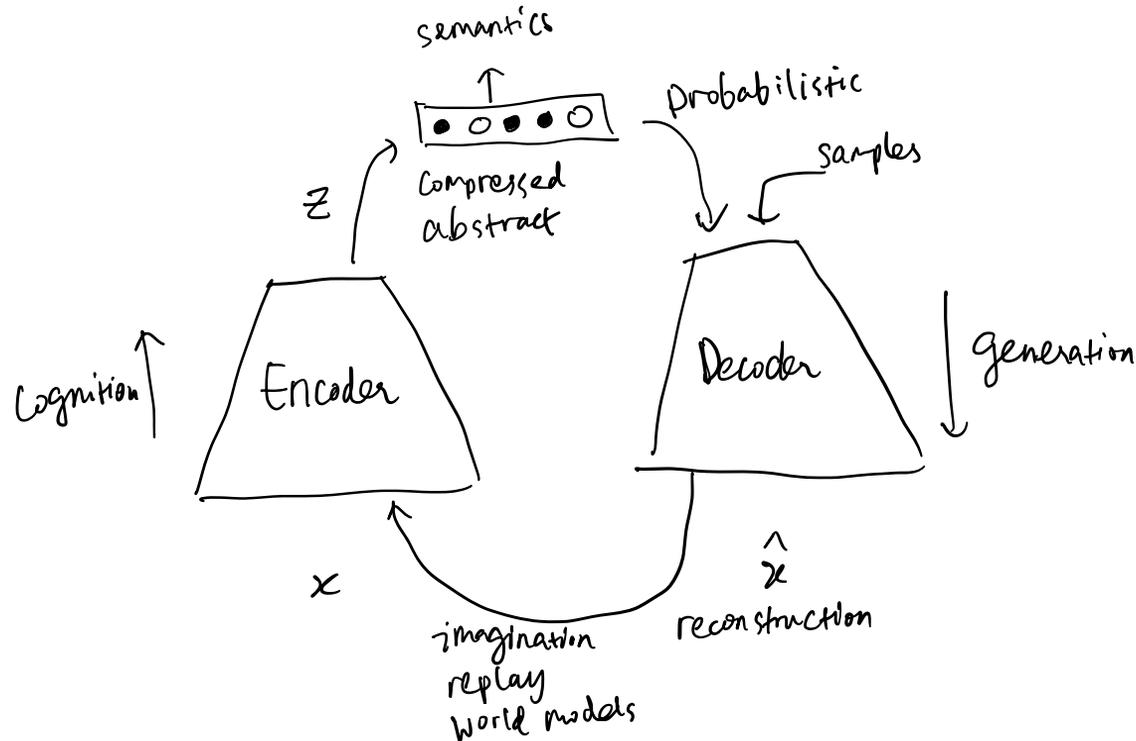
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- Encoder / bottom-up / cognition & decoder / top-down / generation



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# Denoising Autoencoder (DAE)

- Making representations robust to partial corruption

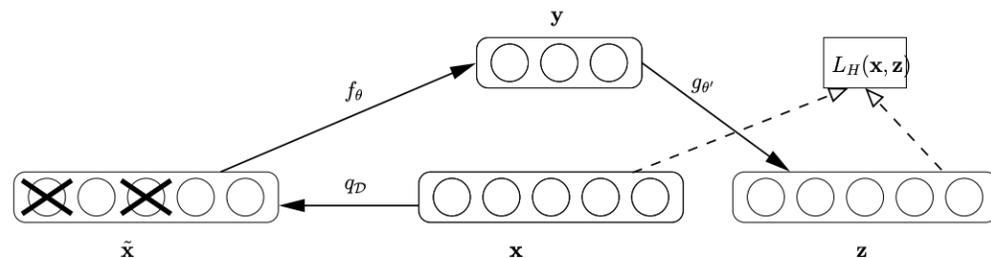
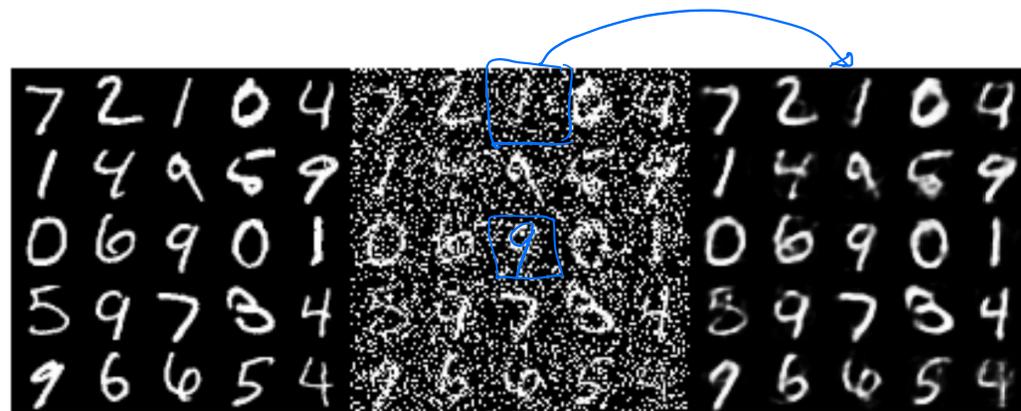


Figure 1. An example  $x$  is corrupted to  $\tilde{x}$ . The autoencoder then maps it to  $y$  and attempts to reconstruct  $x$ .



# Denoising Autoencoder (DAE)

- Making representations robust to partial corruption
- Low-dimensional manifold near which the data concentrate:  
 $p(x|\tilde{x}) = B_{g_{\theta'}}(f_{\theta})(x)$ .

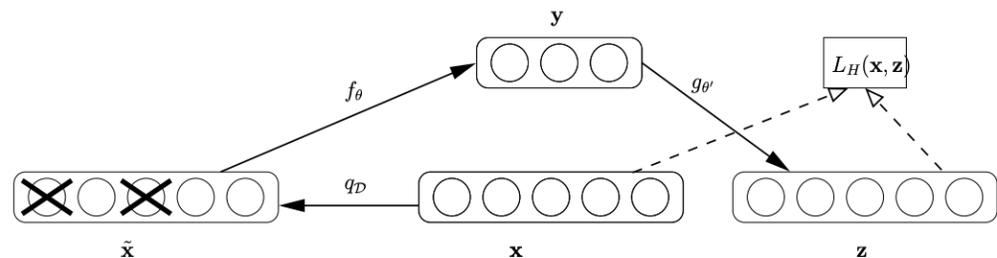
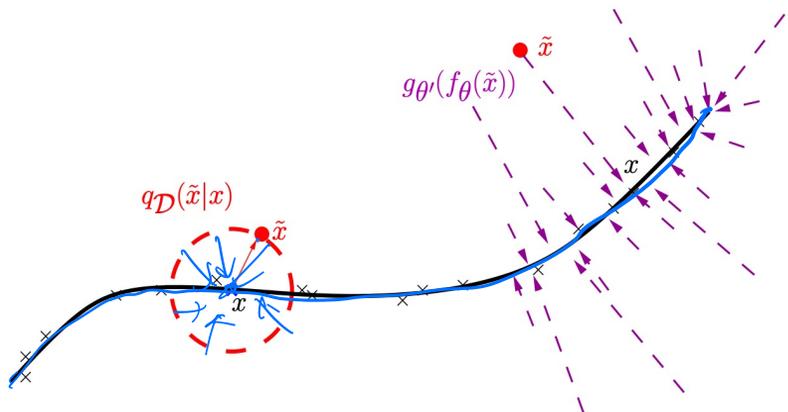
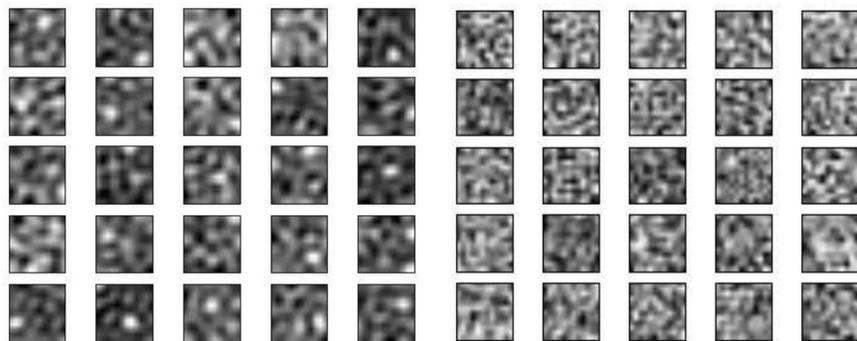


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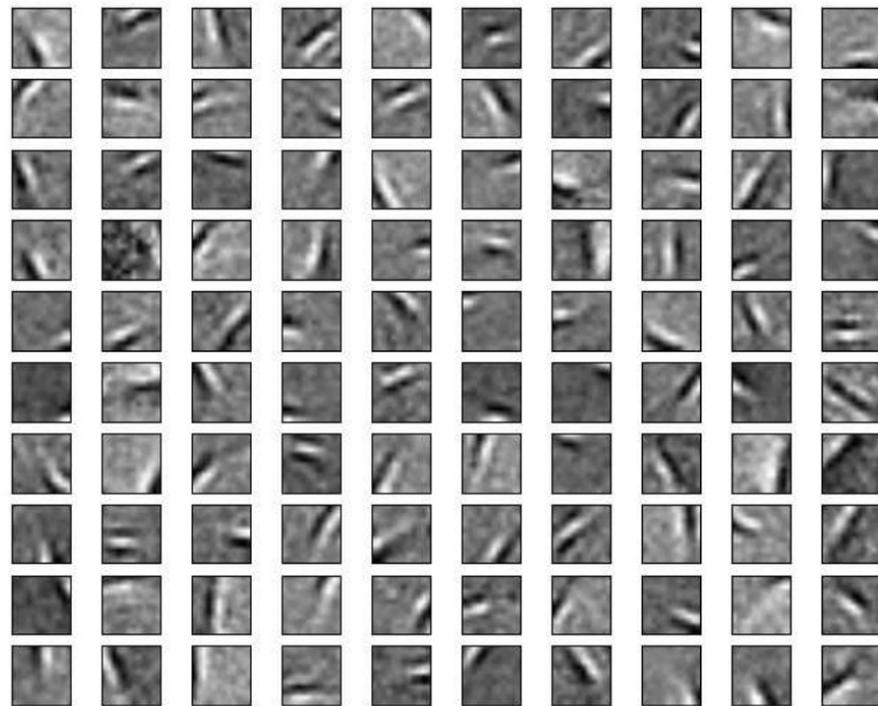


# Denoising Autoencoder (DAE)

- Regular autoencoders do not learn good filters.



preserve all information possible.

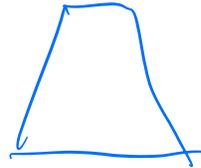


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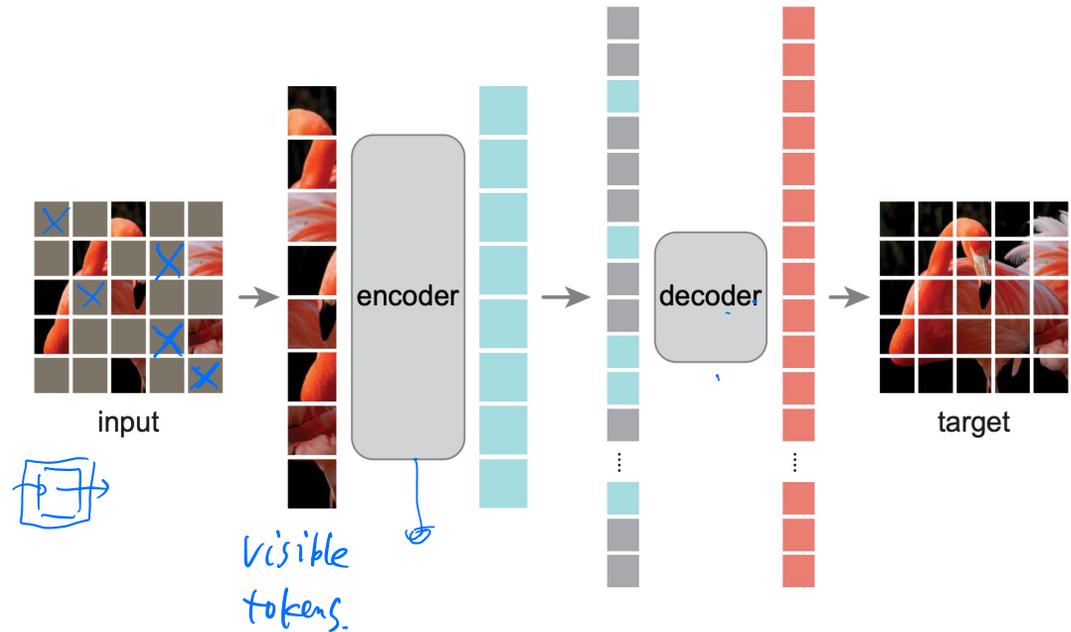
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- Stacked DAE: Stacked layerwise noise-denoise mechanism. Used to  
“pretrain” deep networks.



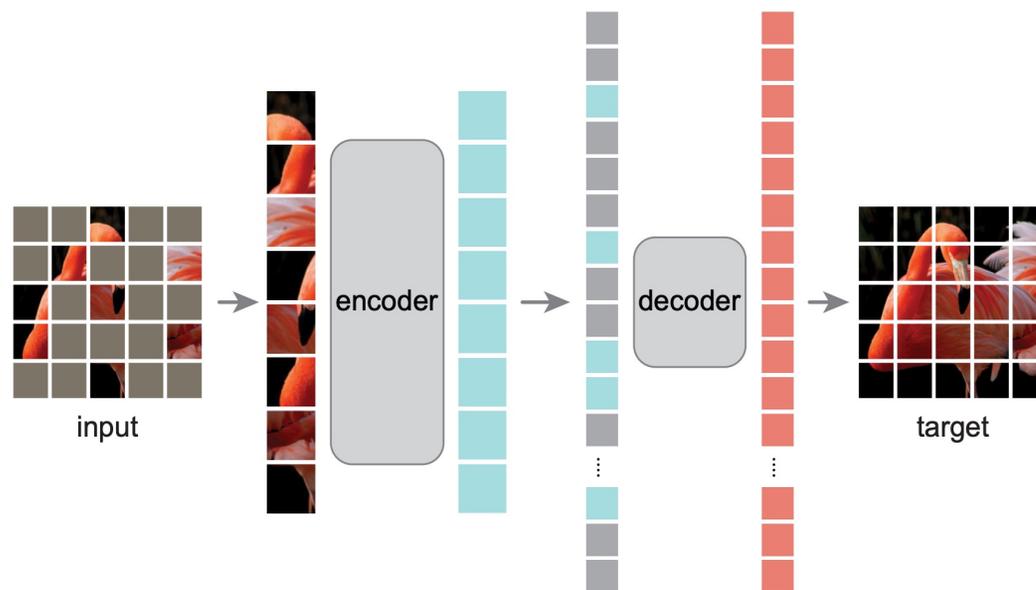
# Masked Autoencoder (MAE)

- Modernized version of denoising autoencoder.



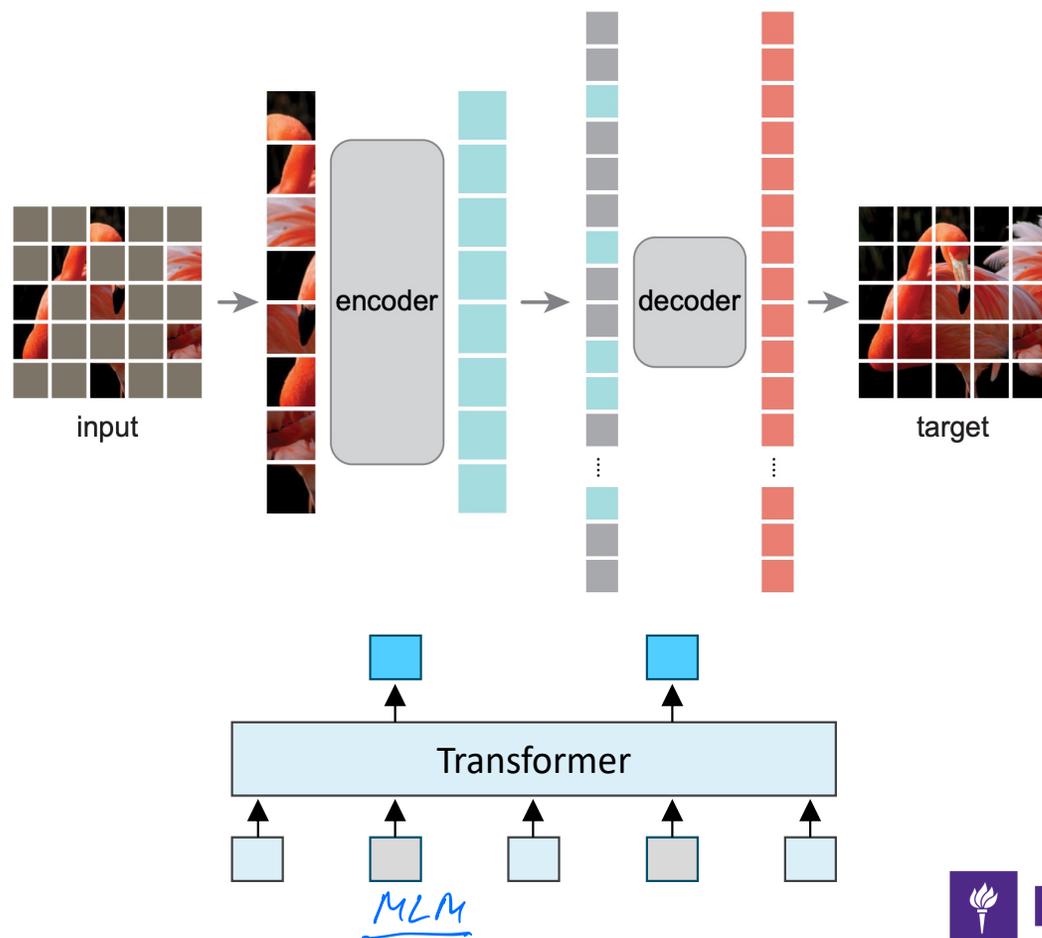
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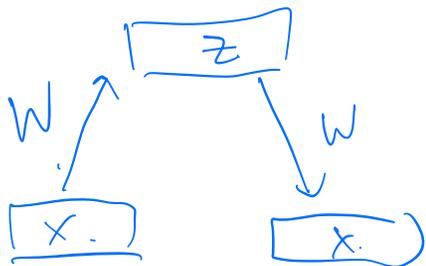
- Modernized version of denoising autoencoder.
- Mask noise: No artifacts
- ViT: No overlapping region, no empty space, no boundary.



# Masked Autoencoder (MAE)

- Modernized version of denoising autoencoder.
- Mask noise: No artifacts
- ViT: No overlapping region, no empty space, no boundary.
- Idea also came from masked language models.





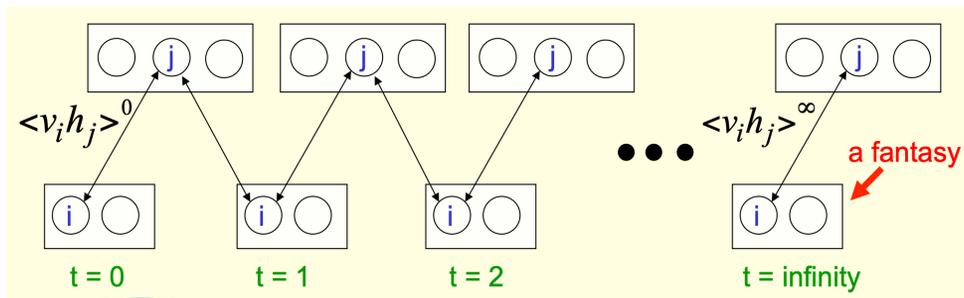
# Energy-Based Learning

- Example: RBMs

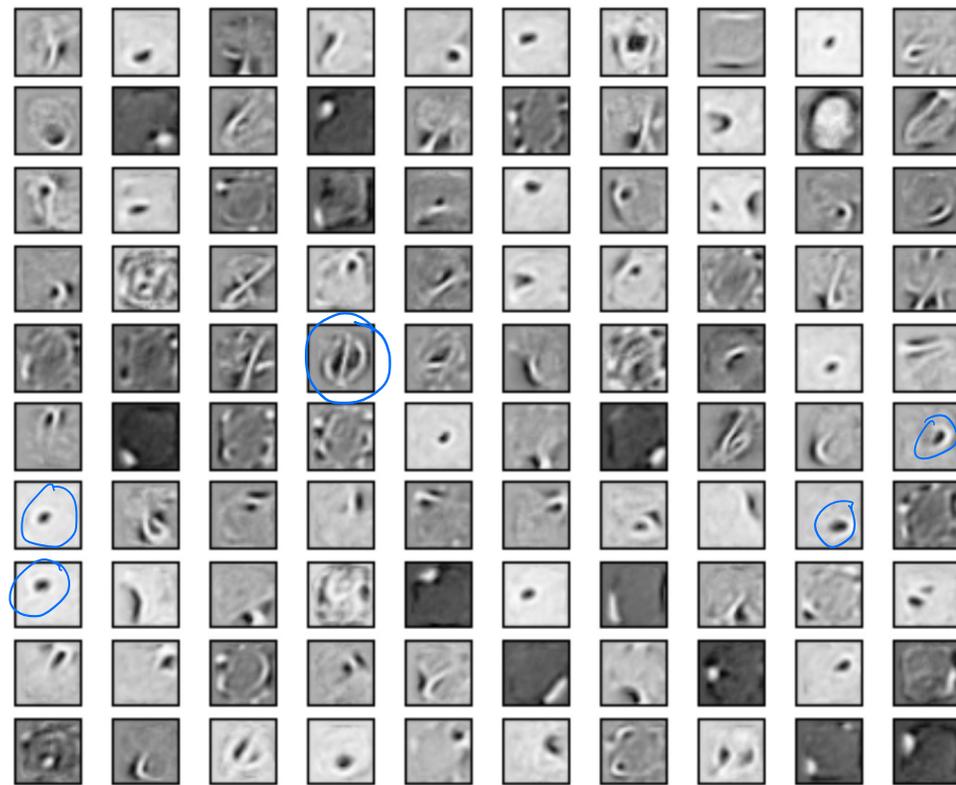
• Energy:  $E(\underline{v}, \underline{h}) = - \sum_{i,j} v_i h_j w_{ij}$ .

$p(h_j = 1 | v_i) = \sigma(\sum_{ij} v_i w_{ij})$ .

$x W(z)$



$$\frac{\partial \log p(v)}{\partial w_{ij}} = \langle v_i h_j \rangle^0 - \langle v_i h_j \rangle^\infty$$



# General EBMs

- Inference requires running gradient descent and MCMC samples.

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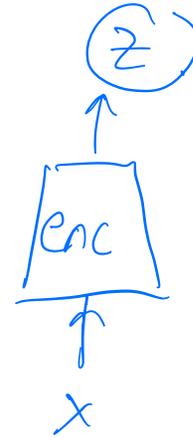
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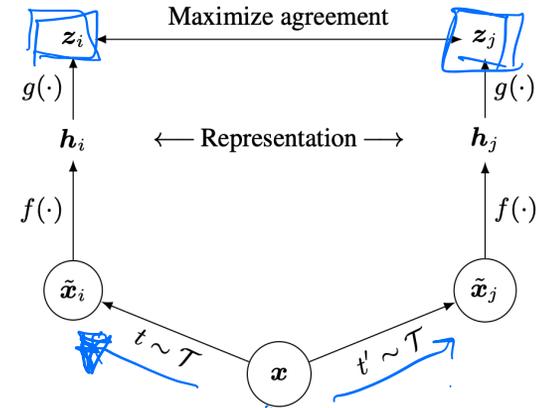
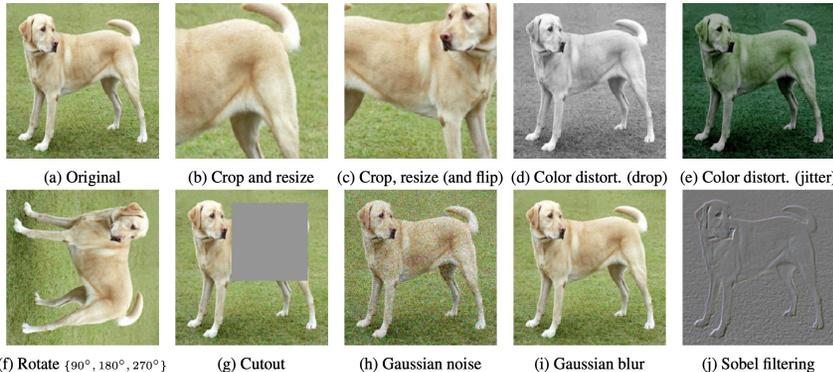
- Can be applied on hand manipulation trajectory generation.
- Good results in generation but still not a generalized representation learning algorithm.

# Self-Supervised Visual Learning

- Match the same image (with severe augmentation)

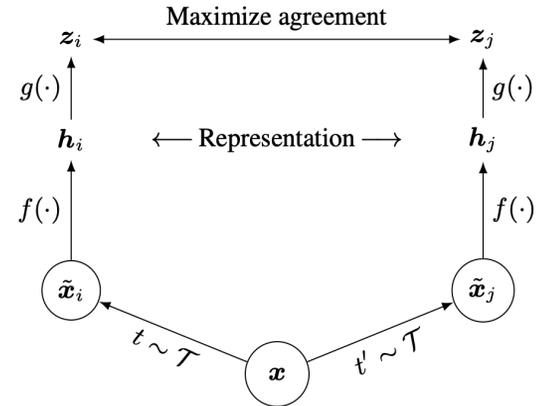
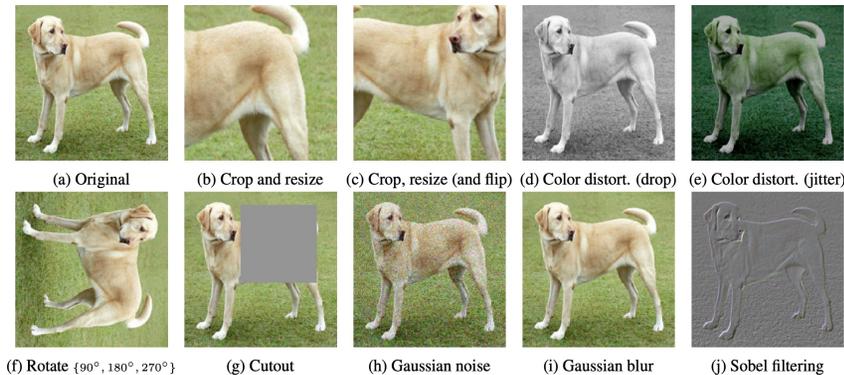


Augmentation



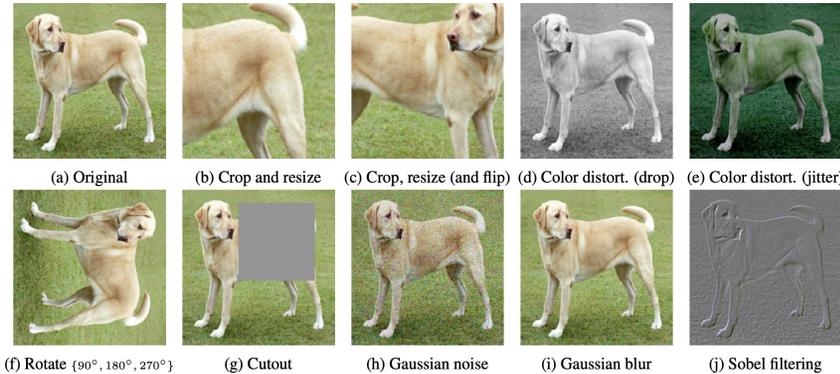
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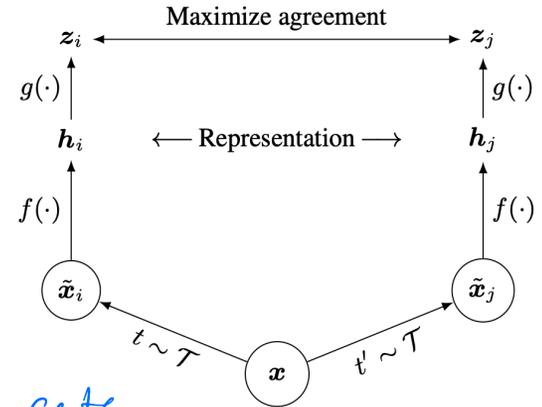


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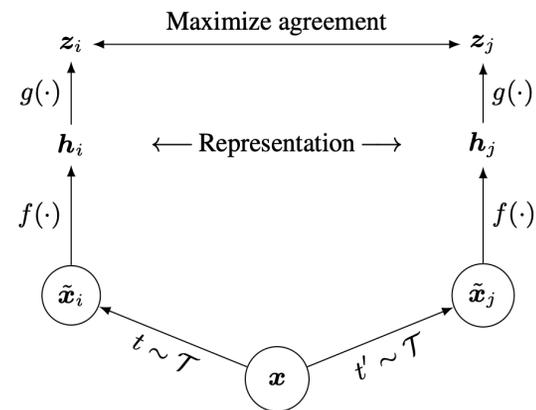
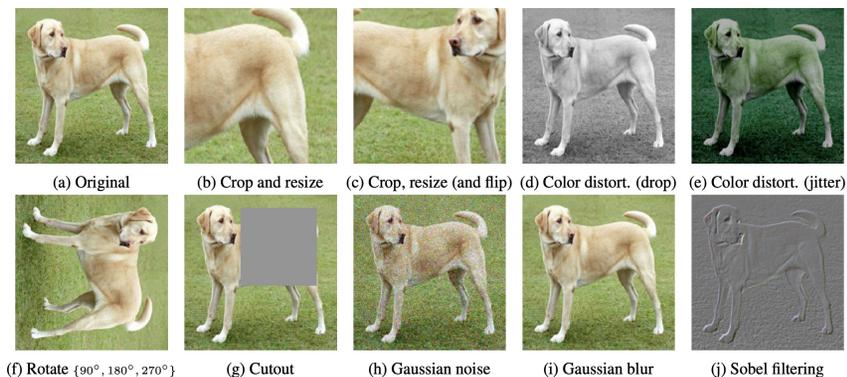


classify,  $z_2$   
based on  $z_1$ .  
among other.  
negative.  $z$ 's  $\rightarrow$  cats  
boats.



# Self-Supervised Visual Learning

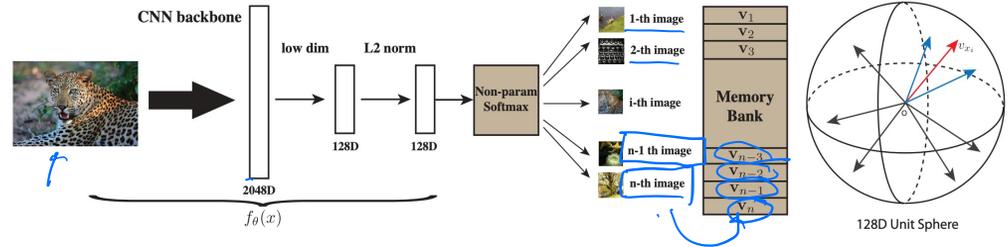
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- Energy is defined between a pair of images.



# Several Embedding Loss Formulations

Wu et al., 2018

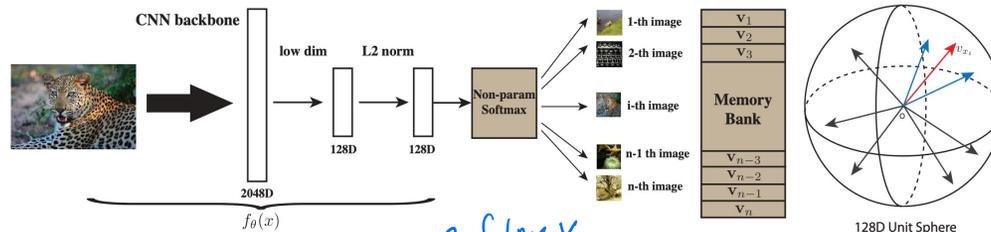
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Wu et al., 2018

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- Contrastive Learning: Cross entropy on pairs

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Chen et al. 2020

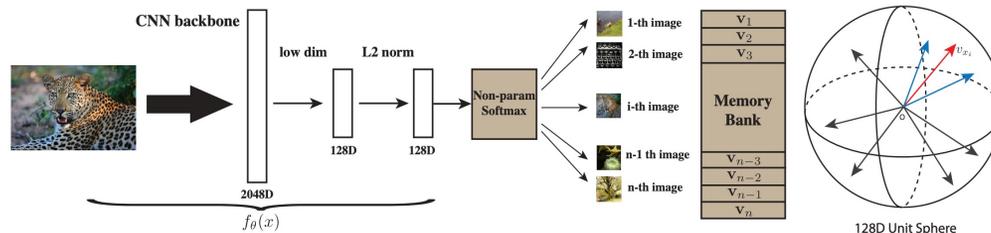
*softmax.*  
*positives.*

*Cross-entropy.*

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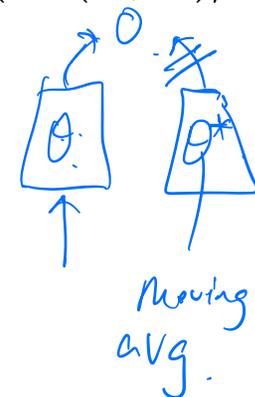
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Chen et al. 2020

- Non-contrastive Learning (Positive Only)

- Moving Average [Grill et al., 2020]
- Stop Gradient [Chen & He, 2020]

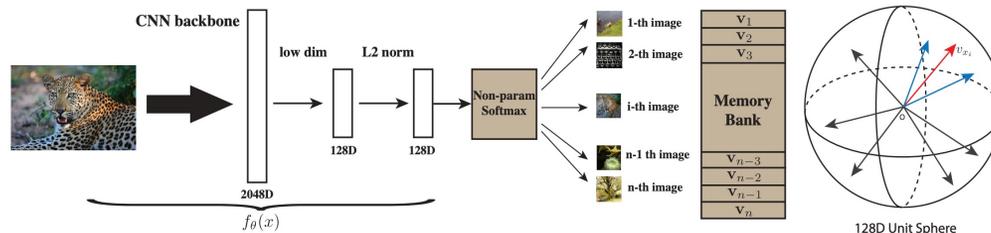
$\|z_1 - z_2\|_2^2$   
 $\hookrightarrow$  collapse.



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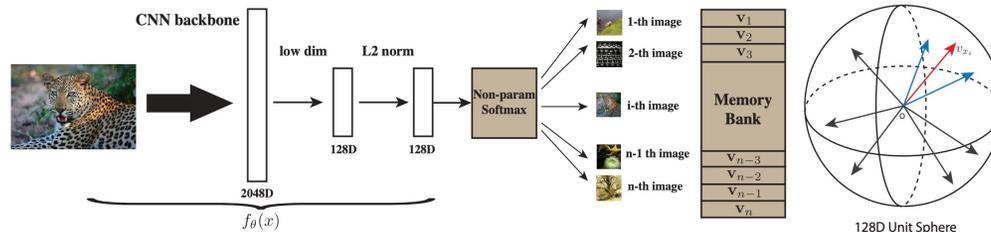
Chen et al. 2020

- Non-contrastive Learning (Positive Only)
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# Several Embedding Loss Formulations

Wu et al., 2018

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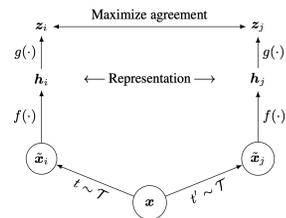
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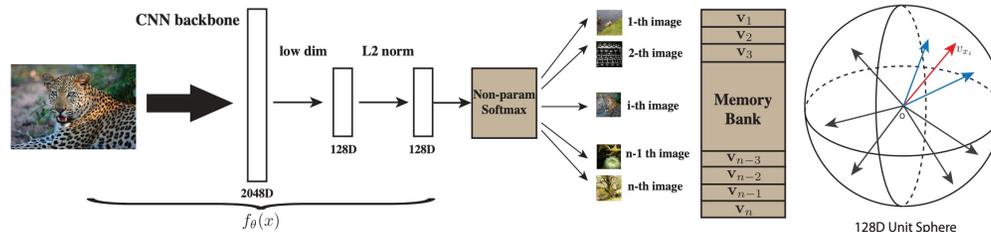


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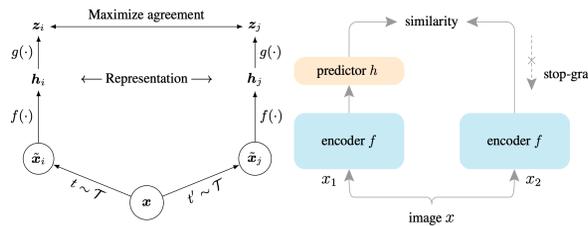
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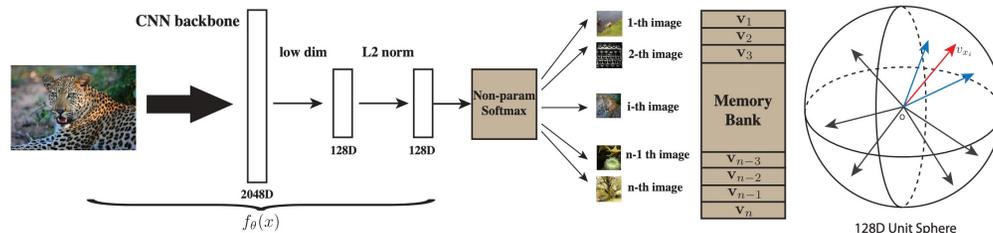


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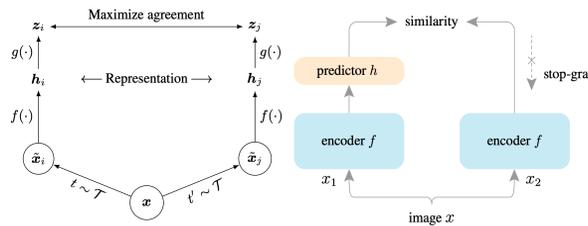
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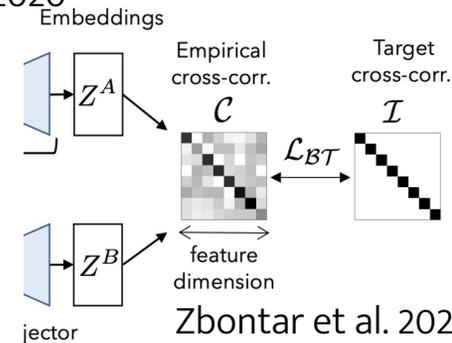
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- Use of projectors and predictors
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Chen et al. 2020    Chen & He, 2021

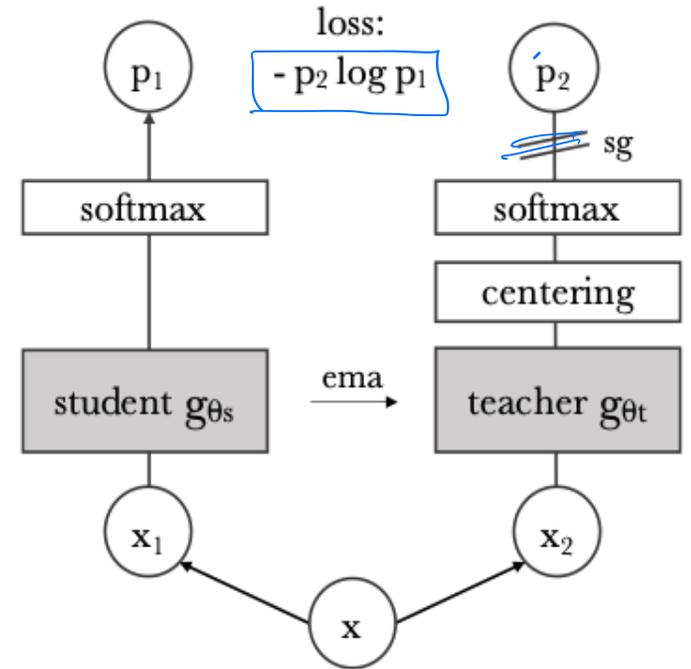
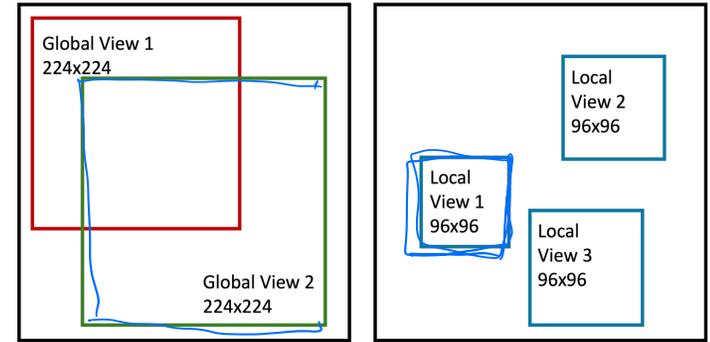


Zbontar et al. 2021  
Bardes et al. 2021

*Handwritten note:* Covariance regularization. feature dimensions.

# DINO

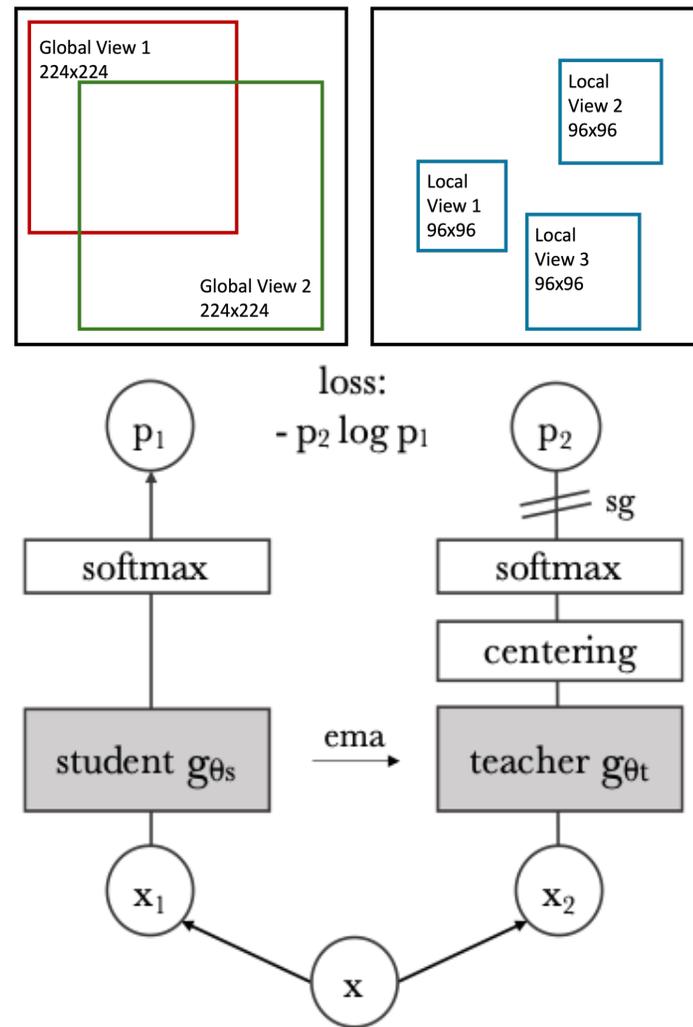
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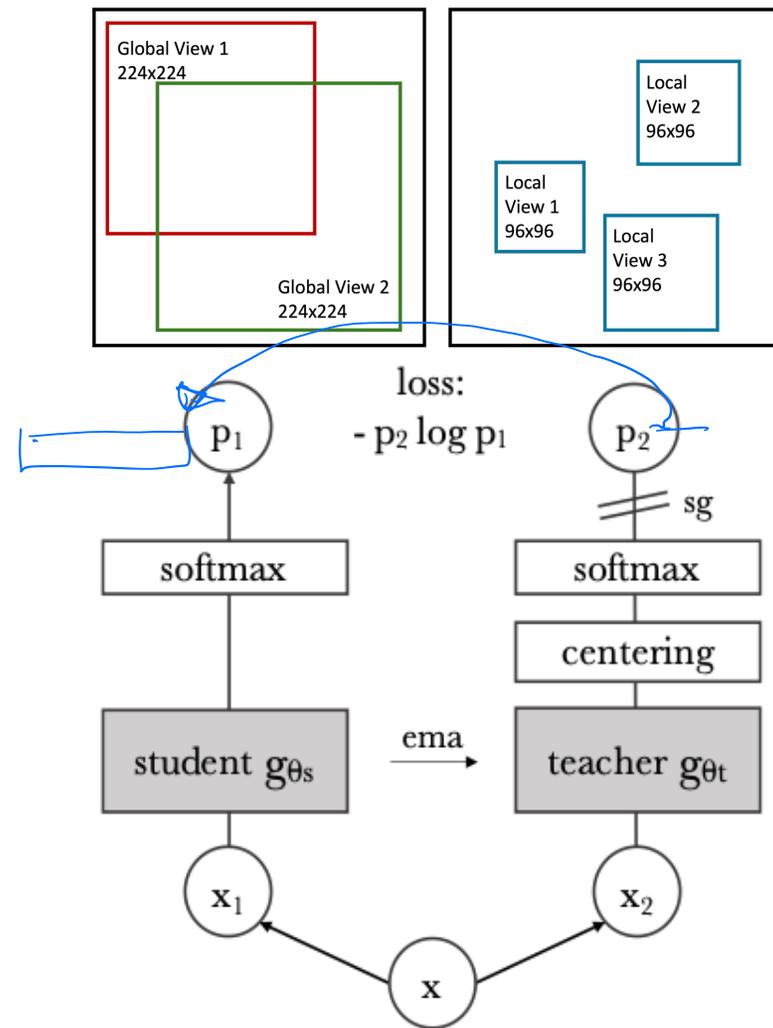


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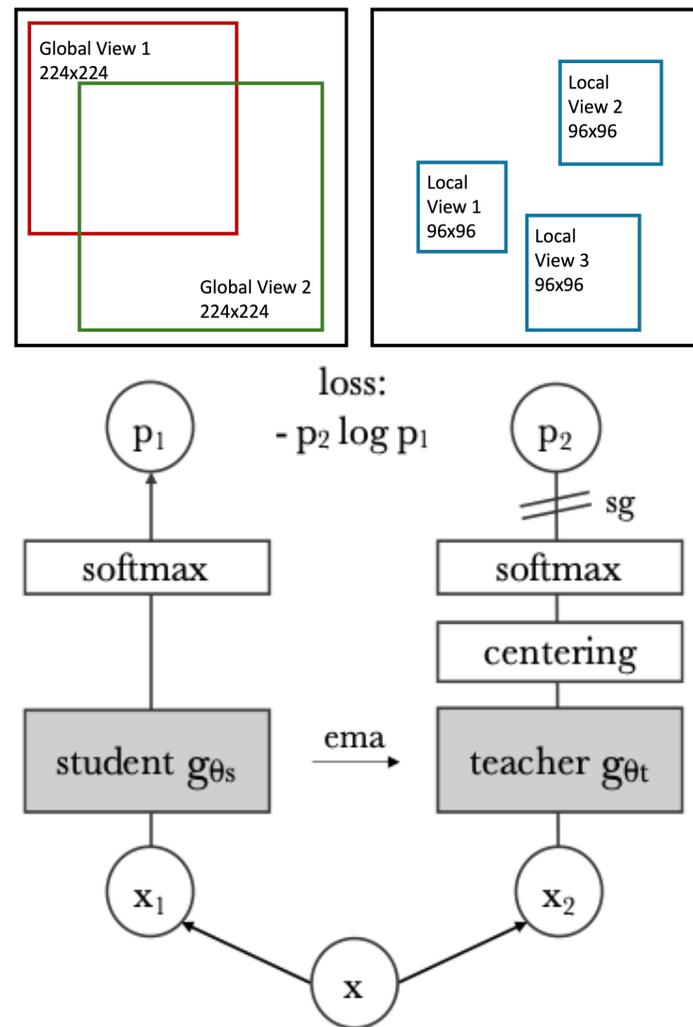
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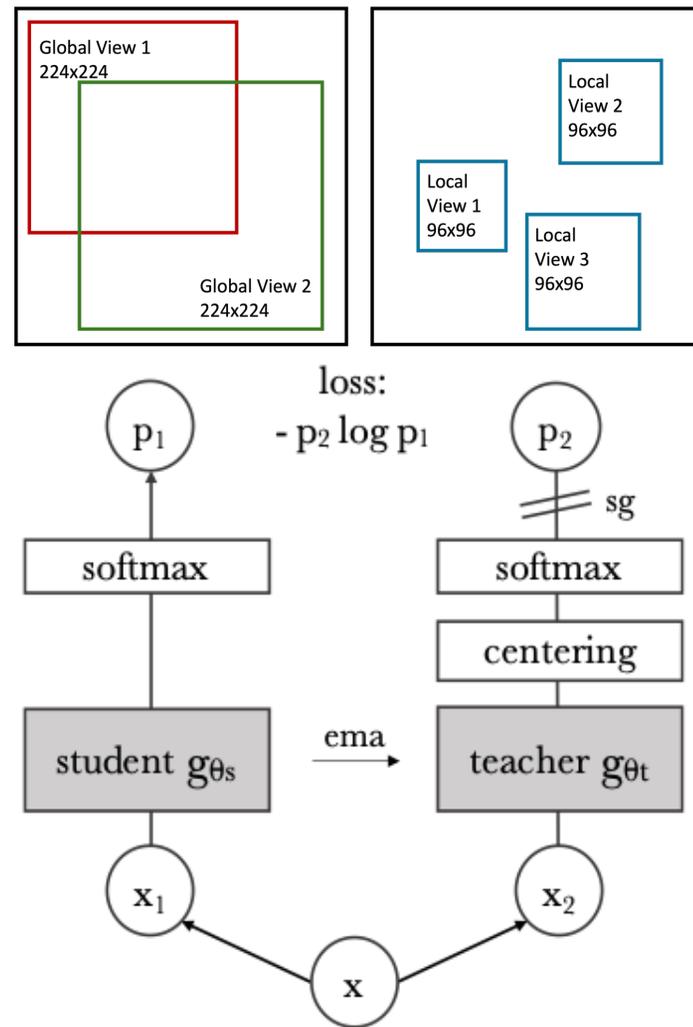
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- Stop gradient on the teacher (no true label).
- Teacher network has EMA weights copied from student (prevent collapse).



# Preventing Collapse

- Cross entropy objective can make both sides collapse to uniform distribution.
  - Apply sharpening, apply a temperature term on both teacher and student.
  - $\text{softmax}(g/\tau)$  The higher the temperature, the more uniform.

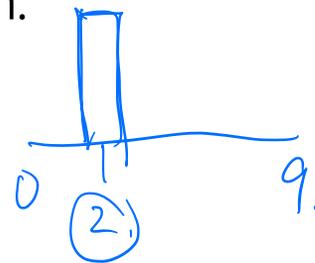
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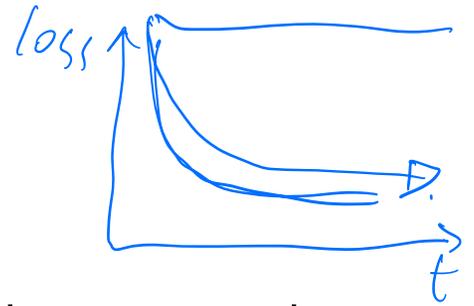
- It can also collapse into always activating a single unit.

- Mean statistics:  $c_t = mc_{t-1} + (1 - m) \frac{1}{B} \sum_{i=1}^B g_{\theta_t}(x_i)$

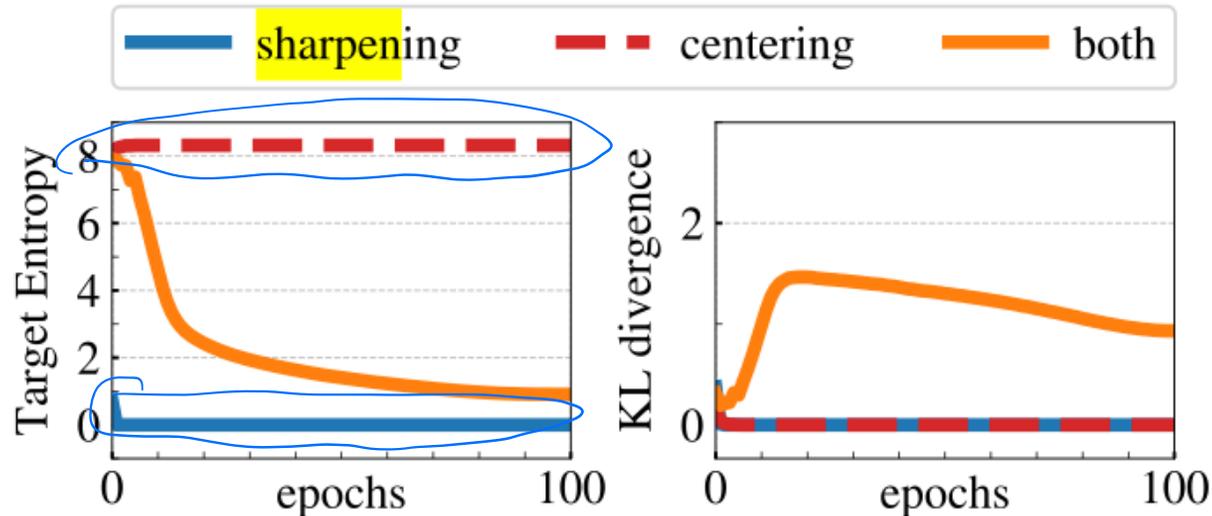


- Center teacher prediction:  $p_t(x) = \frac{\exp((g_{\theta_t}(x)_i - c_t)/\tau_t)}{\sum_k \exp((g_{\theta_t}(x)_k - c_t)/\tau_t)}$

# Centering and Sharpening

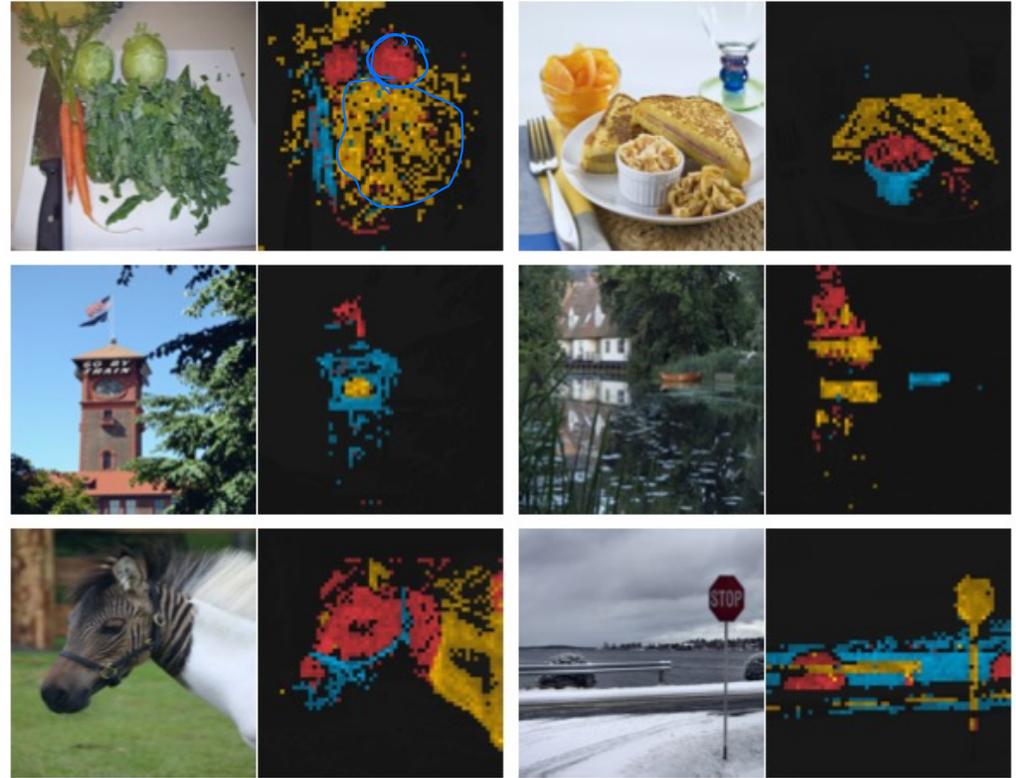


- Only centering: Always uniform distribution, high entropy, easy to guess.
- Only sharpening: Collapsed into one unit, easy to guess, low loss, but no real learning.



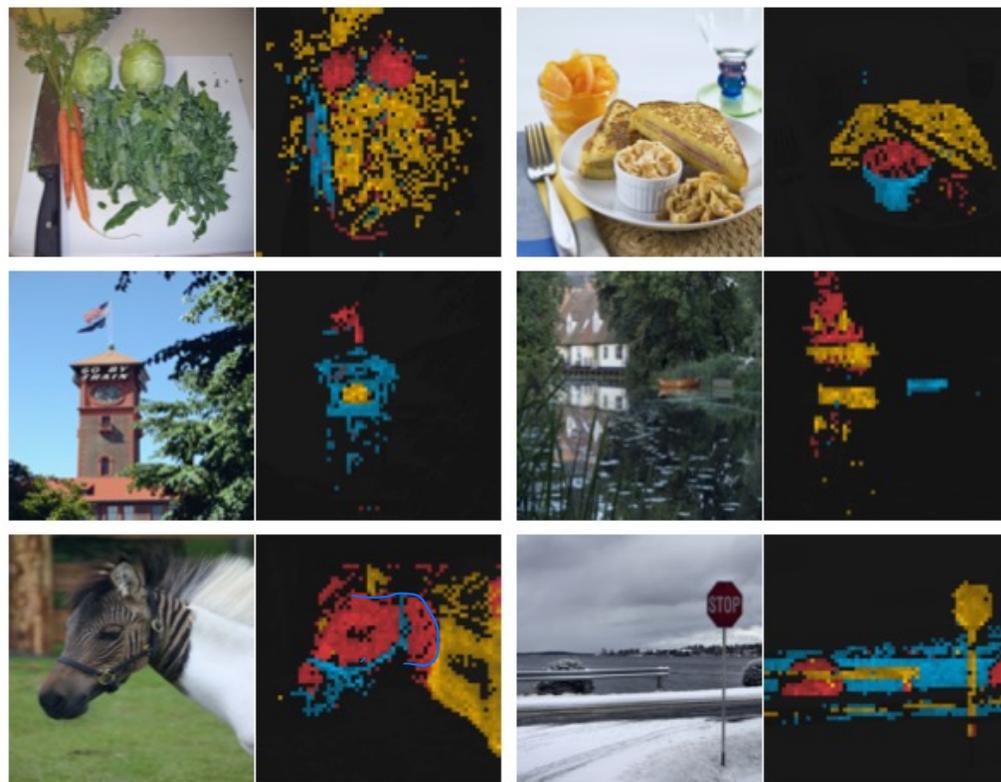
# Visualizing Attention

- The [CLS] token is an extra token added to summarize the whole image into a vector.



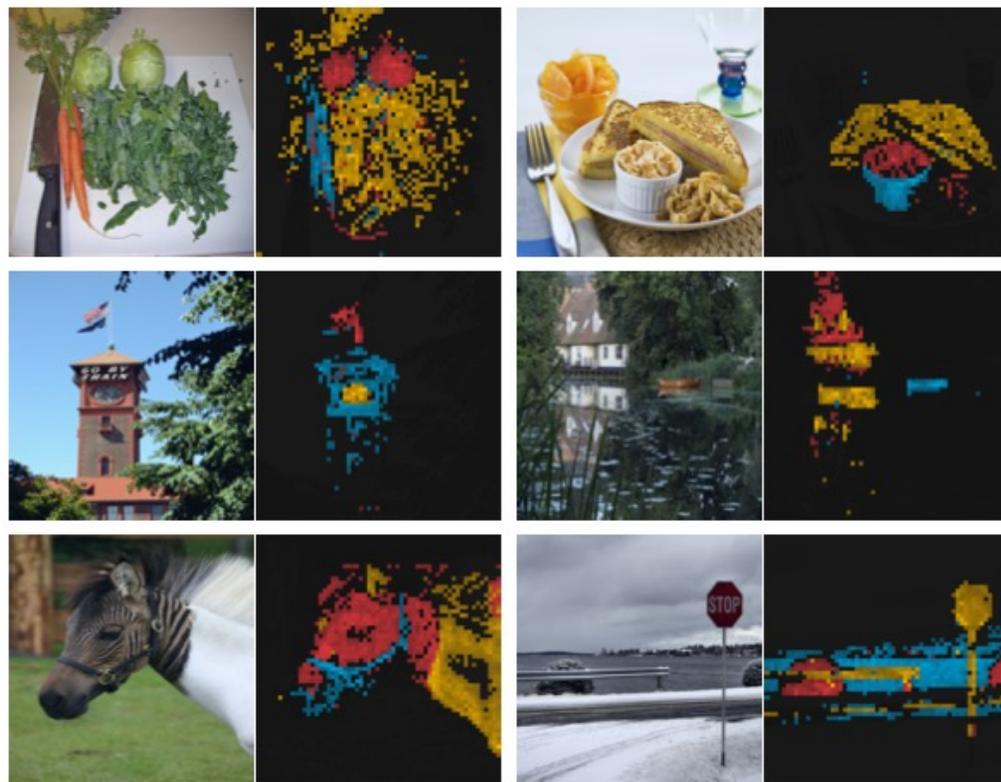
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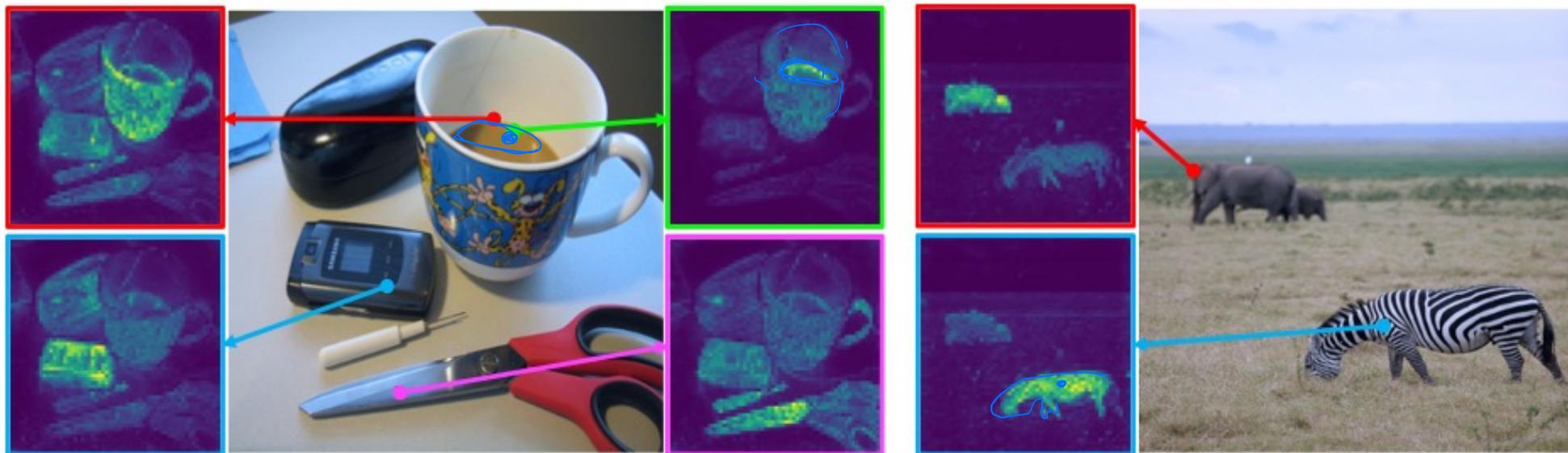
# Visualizing Attention

- The [CLS] token is an extra token added to summarize the whole image into a vector.
- Visualize the attention map of different attention heads using different colors.
- Showing understanding of different objects and parts.



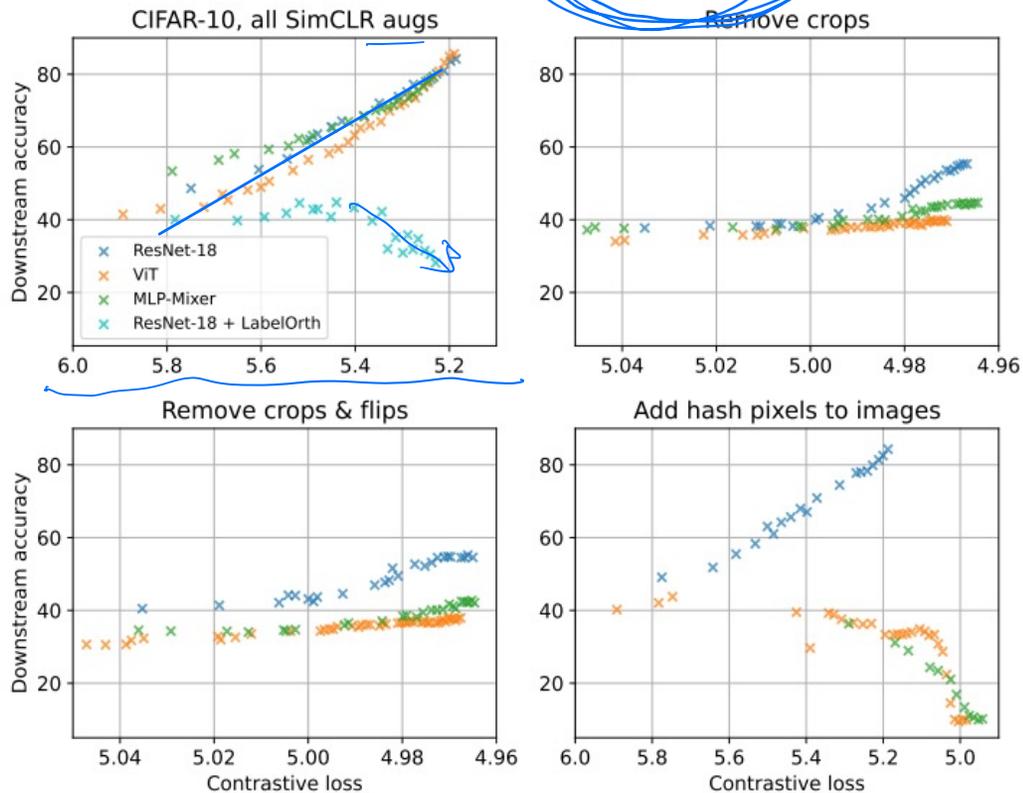
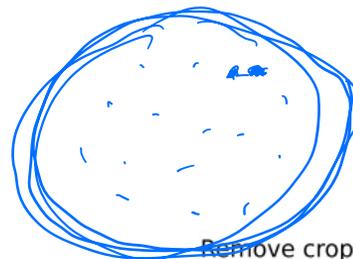
# Visualizing Attention

- We can also visualize the attention by querying from a location.
- Weak separation of objects.



# Why Does SSL Work?

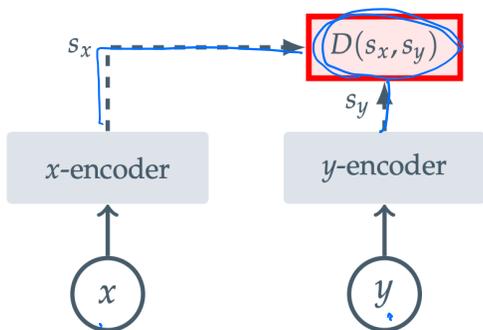
- The unsupervised loss is a surrogate. If an image belongs to a similarity class, it also belongs to the same semantic class.
- The choice of similarity class matters.





# Joint Embedding Predictive Architecture (JEPA)

- Predict what changes from one view to another.

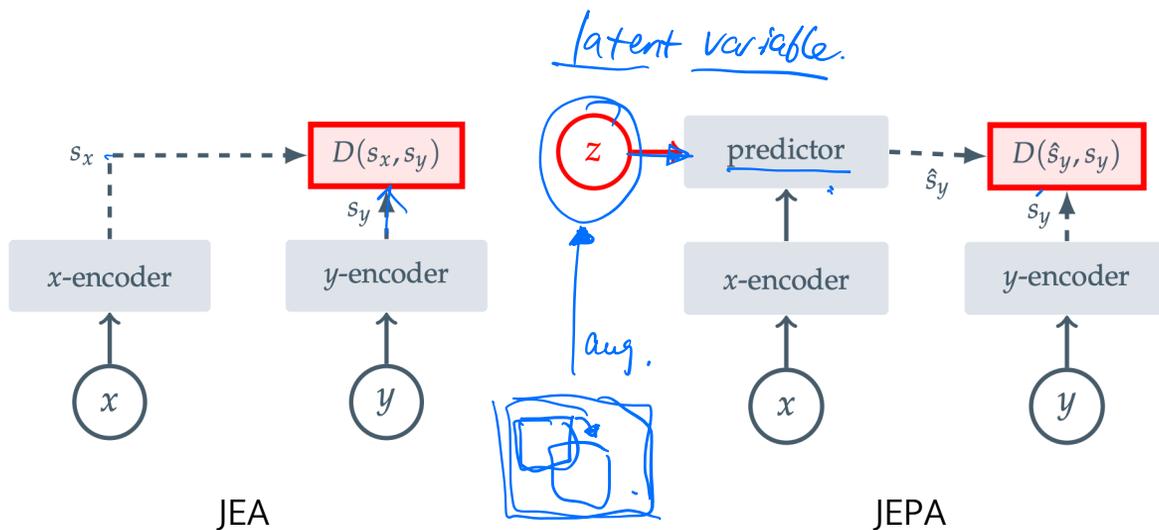


JEPA



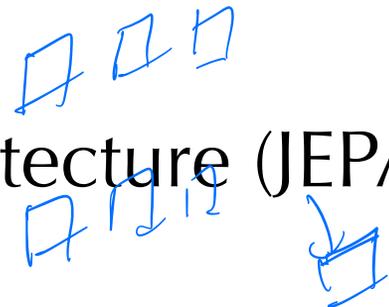
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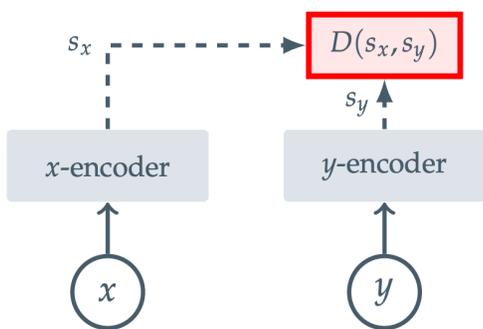




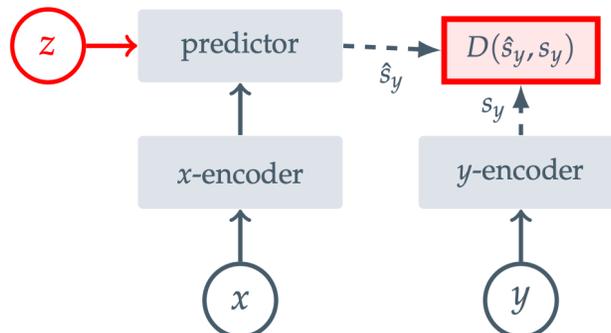
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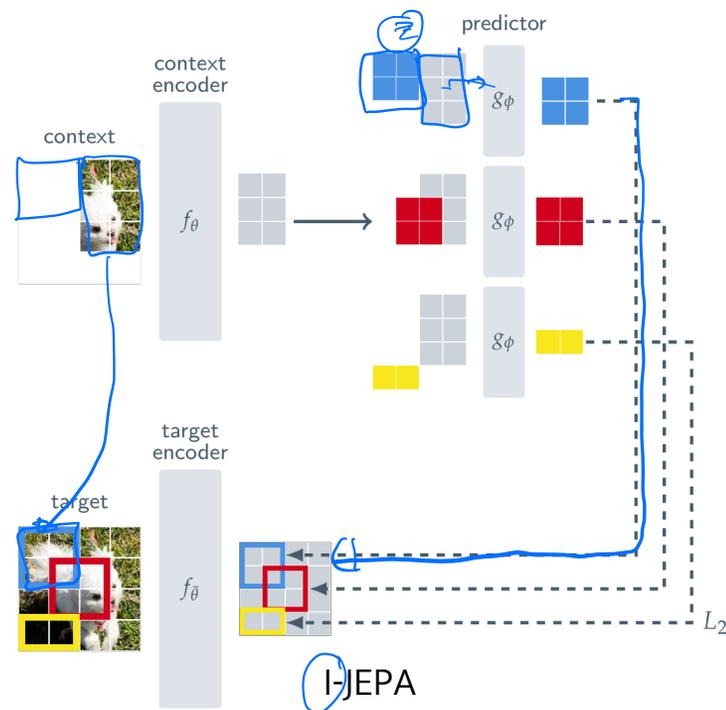
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JEA



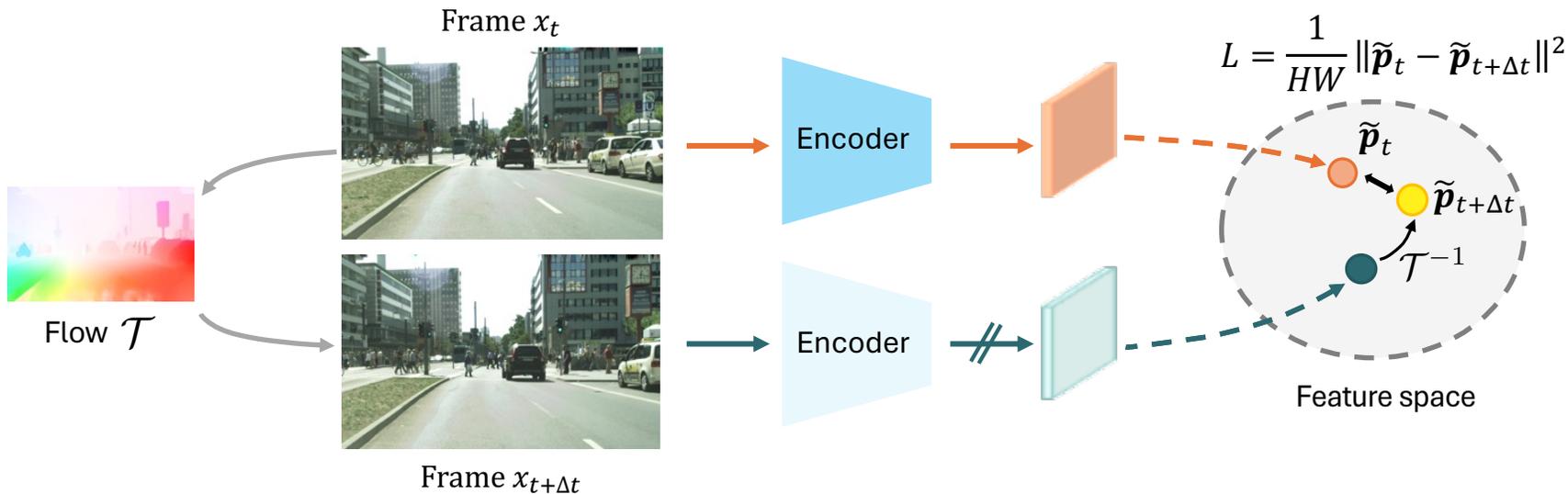
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I-JEPA

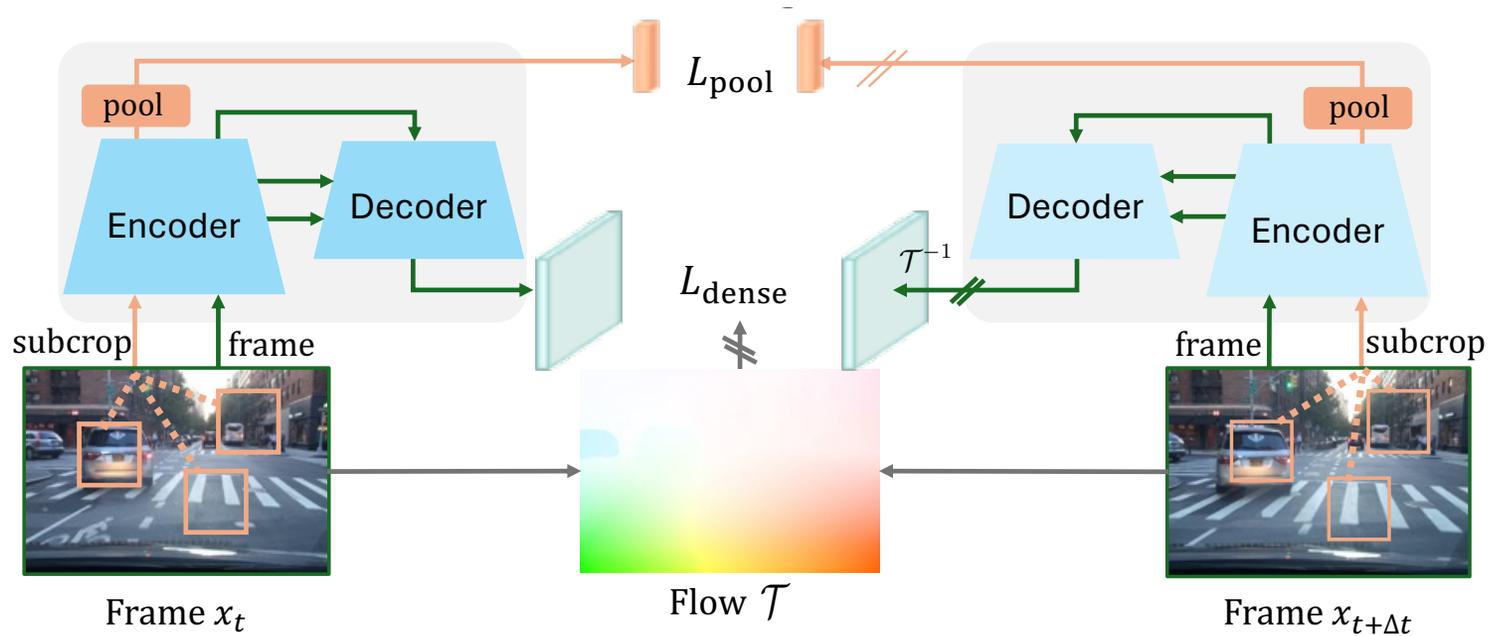
# SSL with Dense Motion

- Can we use adjacent video frames as self-supervision?
- Objects move densely throughout the image.



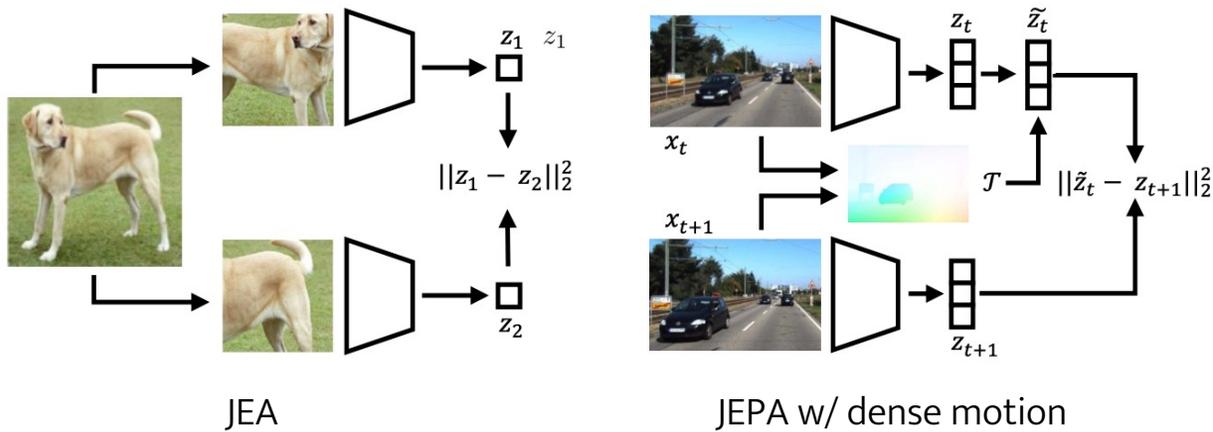
# SSL with Dense Motion

- Perform SSL in multiple scales (small objects vs. big regions).



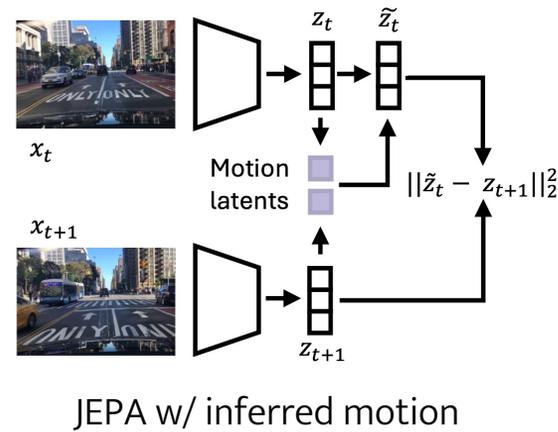
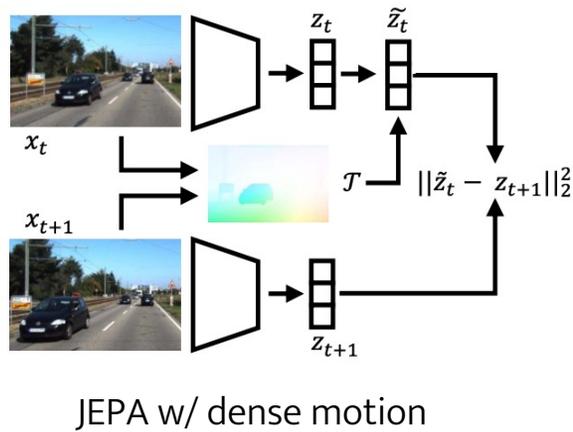
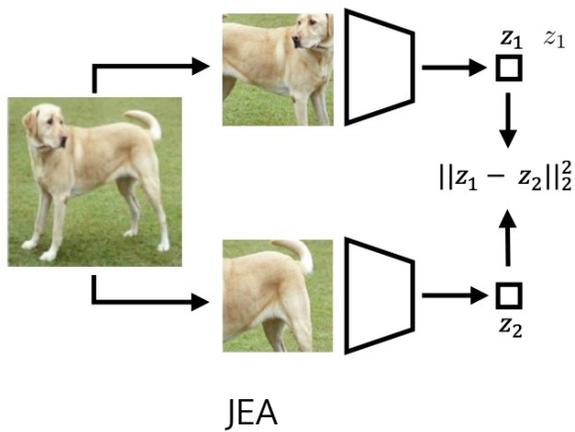
# Joint Motion and Semantic Learning

- We either create or assume some external motion / action information to send to predictors.
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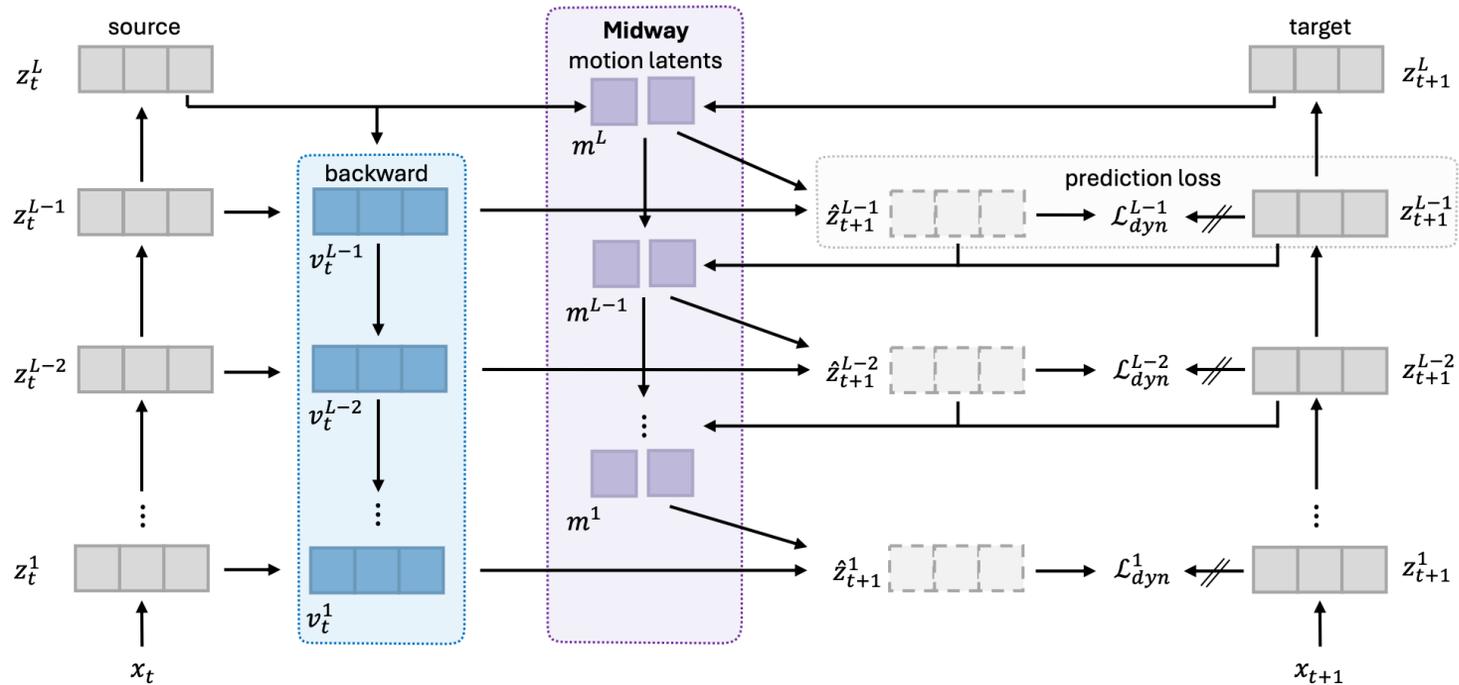
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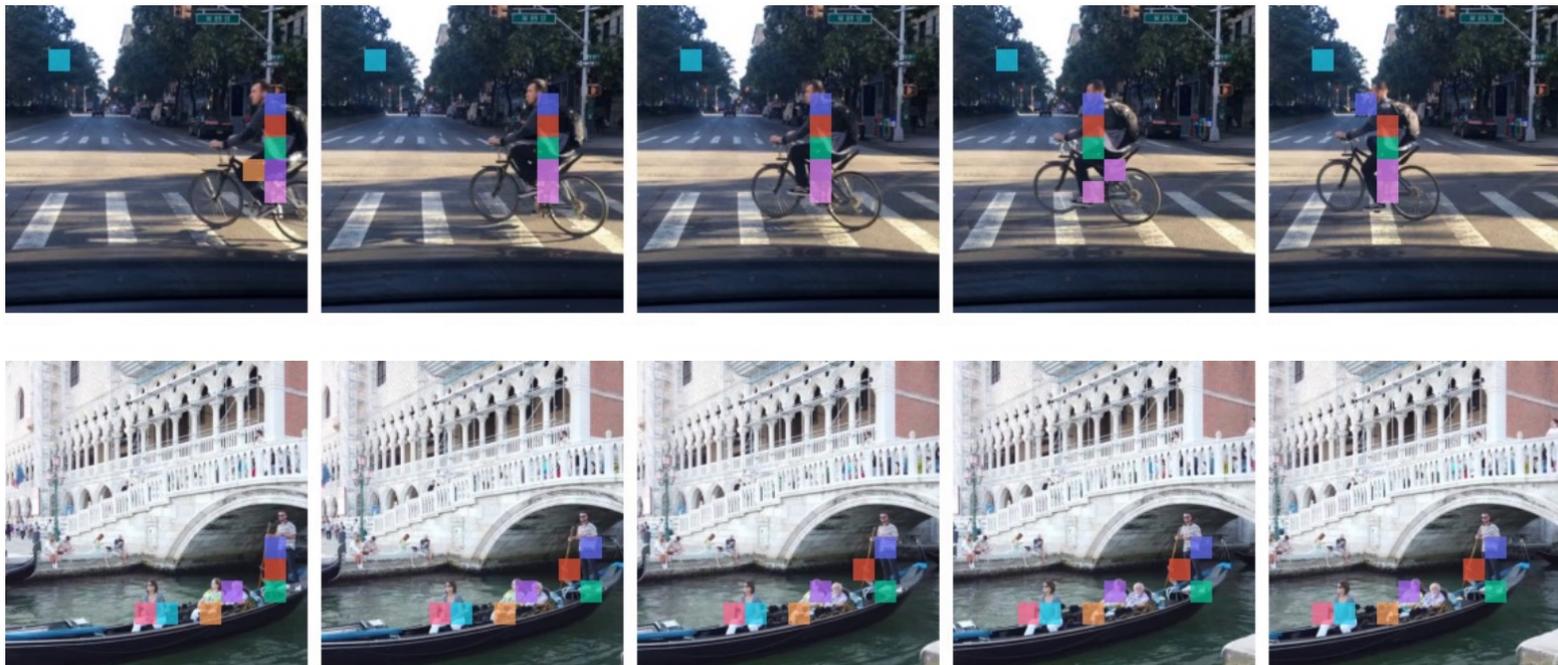
# Midway Networks

- Using backward refinement from Optical Flow networks.



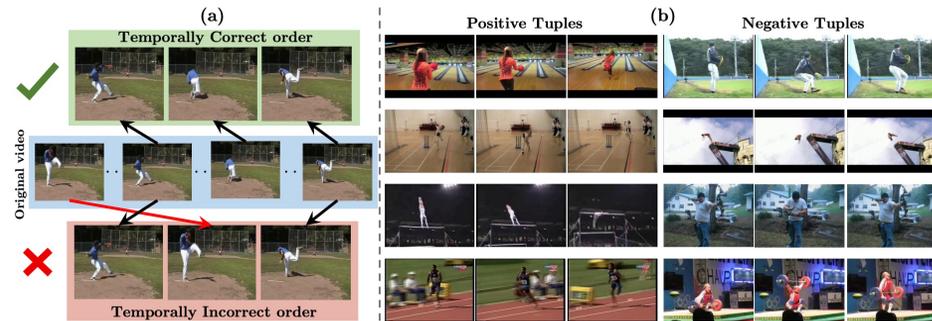
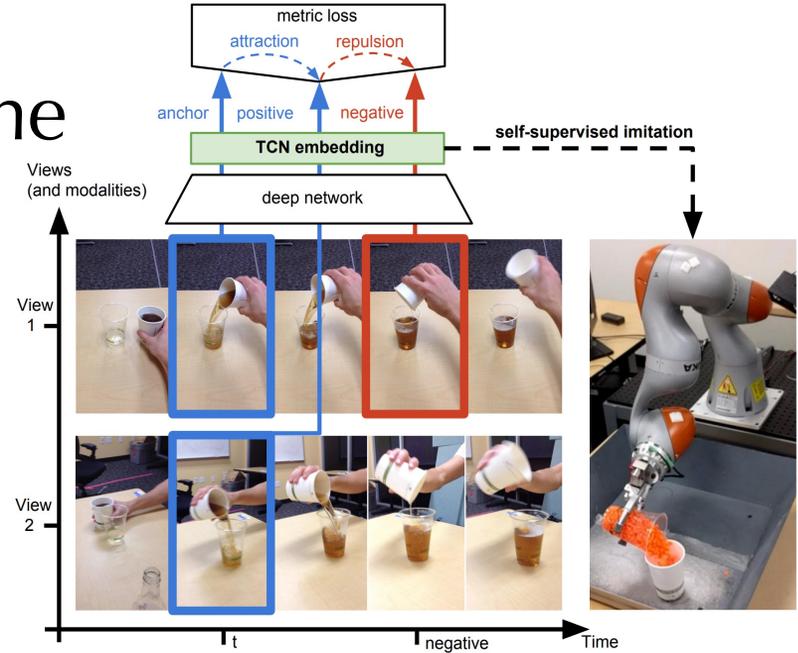
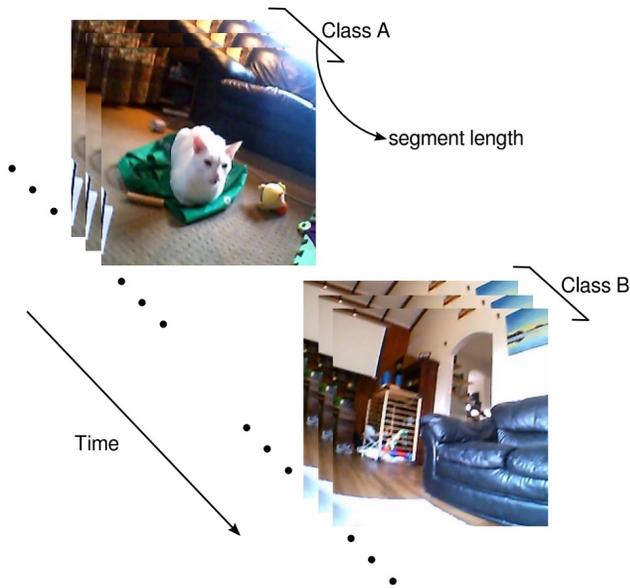
# Midway Networks

- Use perturbation to visualize the motion learned from motion tokens.



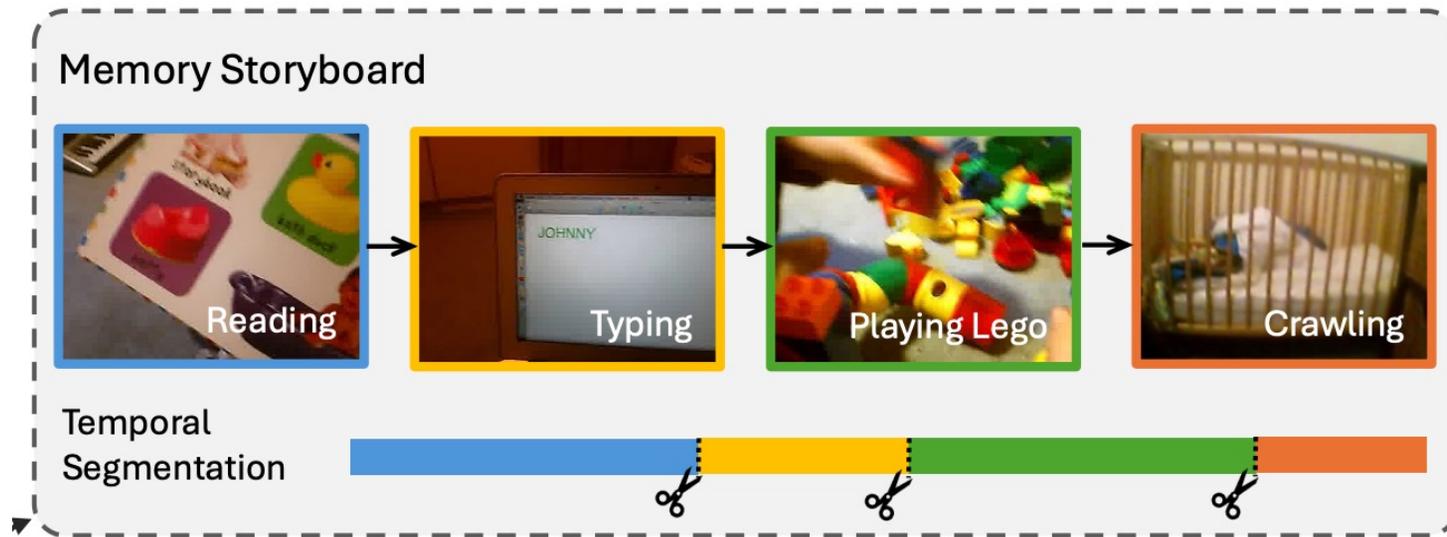
# SSL with Time

- Use time as an additional source of supervision.



# SSL with Time

- We can segment videos into meaningful events.
- Leverage the spatiotemporal continuity structure.



# Summary

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- Possible learning objectives for egocentric videos.
- Incorporate 3D vision and actions for downstream planning.

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- Attending to semantically similar parts facilitates the process.
- The network is a hierarchical information processing pipeline – Lower layers integrate more granular and smaller neighborhood.

# Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.

# Weak-to-Strong Supervision

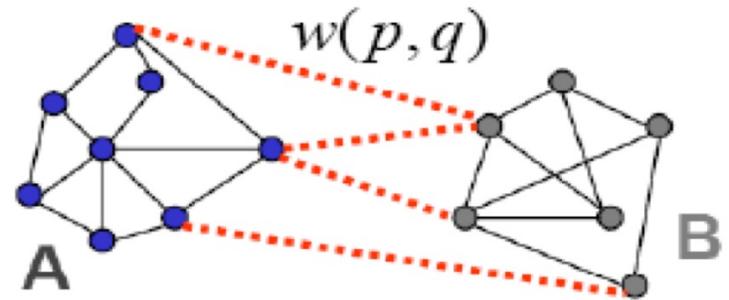
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# Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.
- Then use these masks as pseudo labels and supervise the network to predict these low-quality masks.
- Question: how do we come up with masks? What loss is used to supervise the network?

# Graph Cut

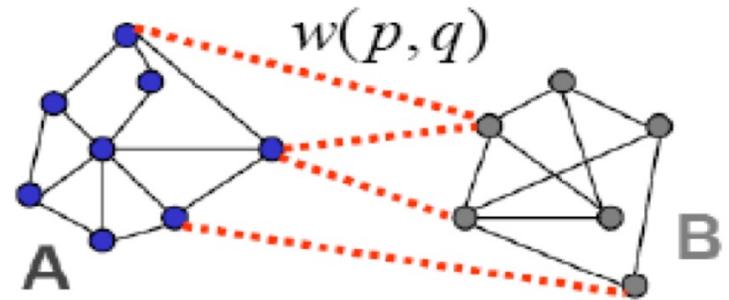
- Segmentation is essentially a clustering problem.



$$\text{cut}(A, B) = \sum_{p \in A, q \in B} w(p, q)$$

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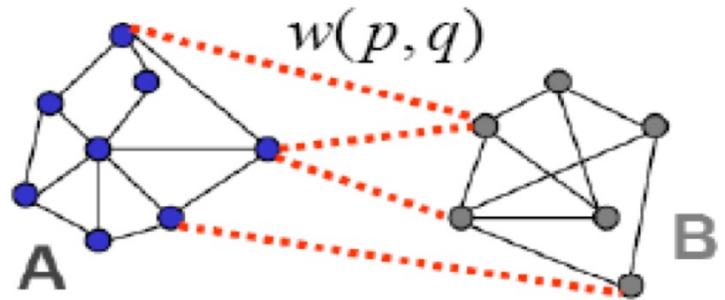
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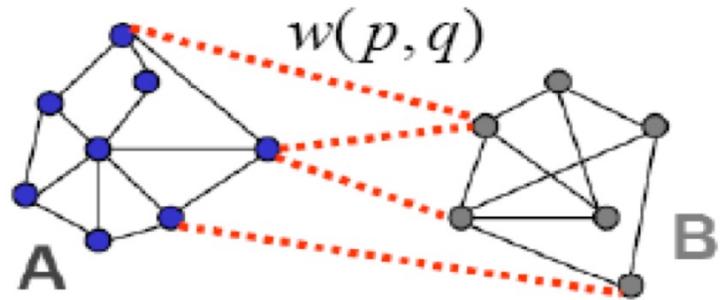
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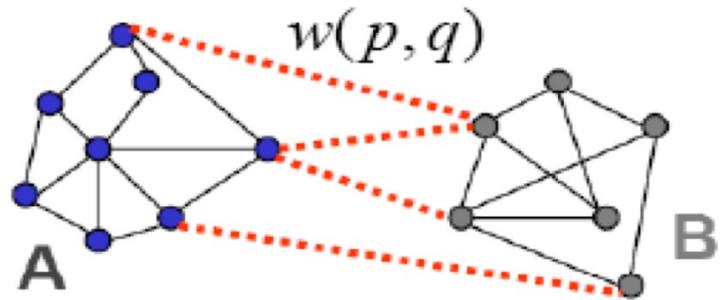
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# Graph Cut

- Segmentation is essentially a clustering problem.
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- Objective: Cut the graph into disconnected components with a minimum sum of edge values.

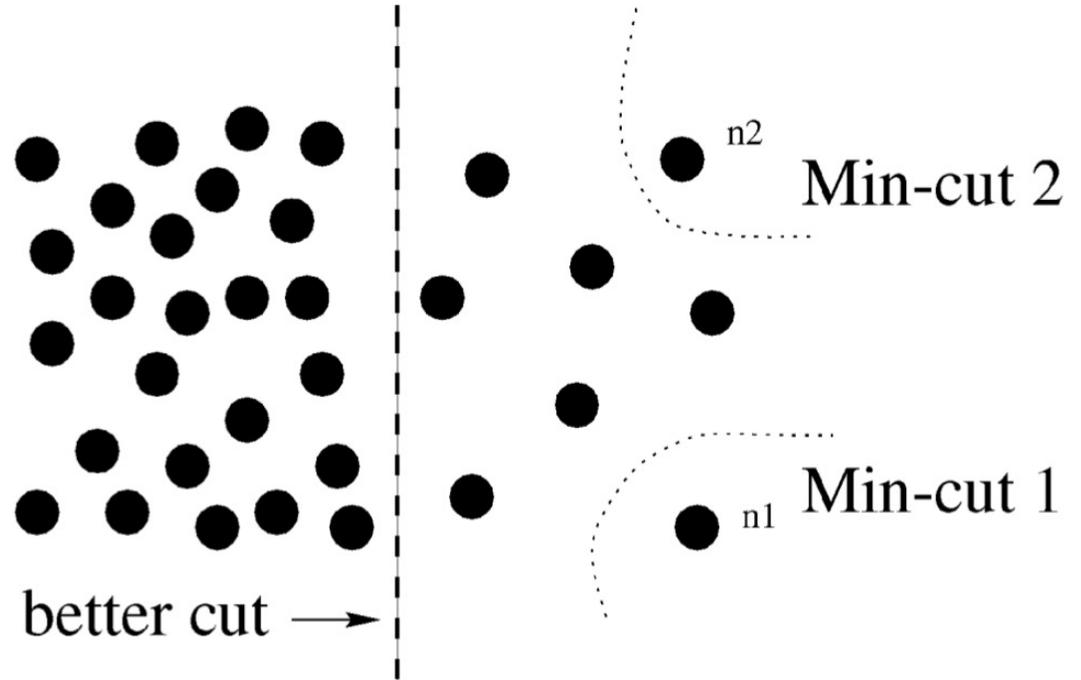


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# Normalized Graph Cut (NCut)

- How to prevent cutting small isolated nodes?

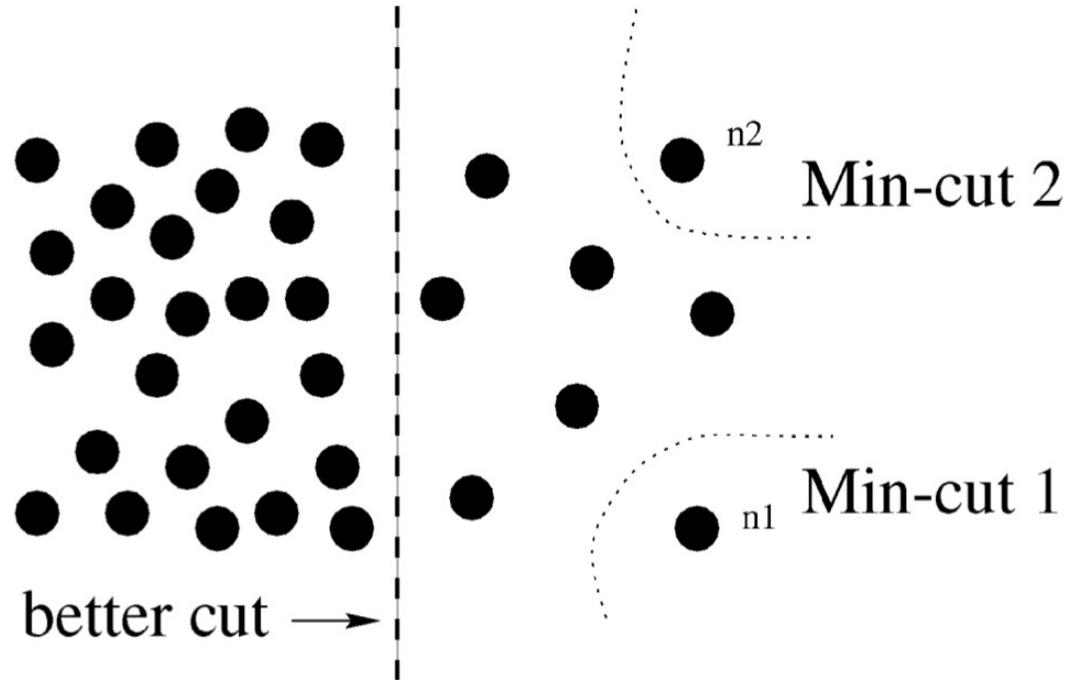
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# Normalized Graph Cut (NCut)

- How to prevent cutting small isolated nodes?
- Normalize by the total edge connections of a group to all the nodes.

$$Ncut(A, B) = \frac{cut(A, B)}{assoc(A, V)} + \frac{cut(A, B)}{assoc(B, V)}$$



# NCut Details (Optional)

- A form of spectral clustering.
- Degree matrix  $D$   $N \times N$  with  $d_i$  on the diagonal.
- Weight matrix  $W$   $N \times N$  symmetric  $w_{ij}$ .
- Selection vector  $x_i = 1$  if  $i \in A$  otherwise  $-1$ .
- Solve the minimization:  $\min_y \frac{y^\top (D - W)y}{y^\top D y}$   $y = (1 + x) - \frac{\sum_{i|x_i > 0} d_i}{\sum_{i|x_i < 0} d_i} (1 - x)$ .
- Generalized eigenvalue system:  $(D - W)y = \lambda D y$ .
- Let  $z = D^{1/2}y$   $D^{-\frac{1}{2}}(D - W)D^{-\frac{1}{2}}z = \lambda z$ .

# NCut

- Sort the eigenvectors from the smallest to the largest.



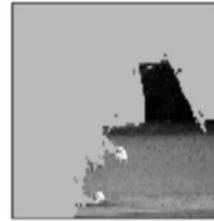
(a)



(b)



(c)



(d)



(e)



(f)



(g)



(h)

# NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.



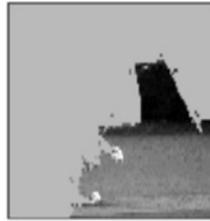
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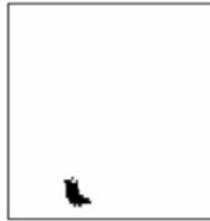
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(f)



(g)



(h)

# NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.
- Now, instead of segmenting pixels, we can directly segment semantically meaningful representations from self-supervision.



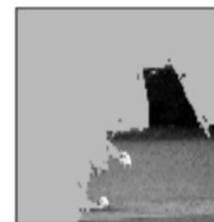
(a)



(b)



(c)



(d)



(e)



(f)



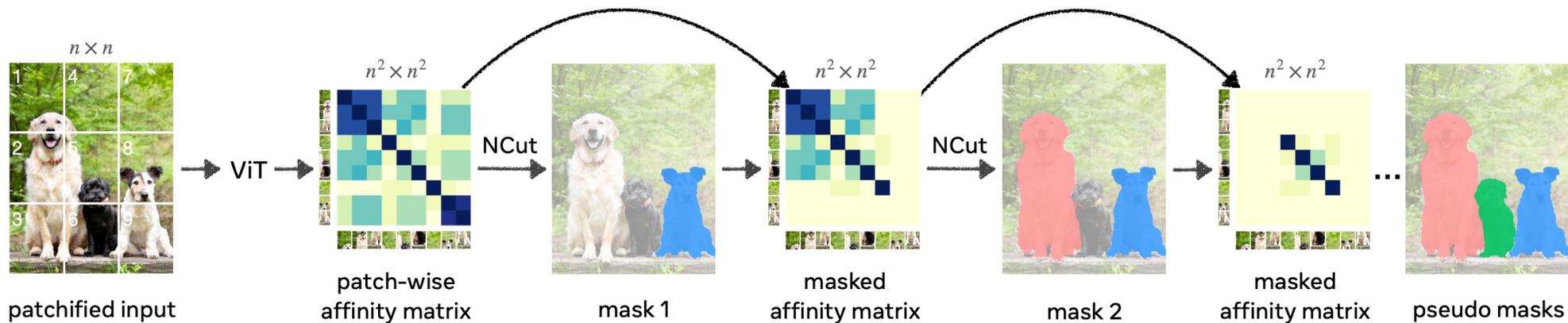
(g)



(h)

# MaskCut

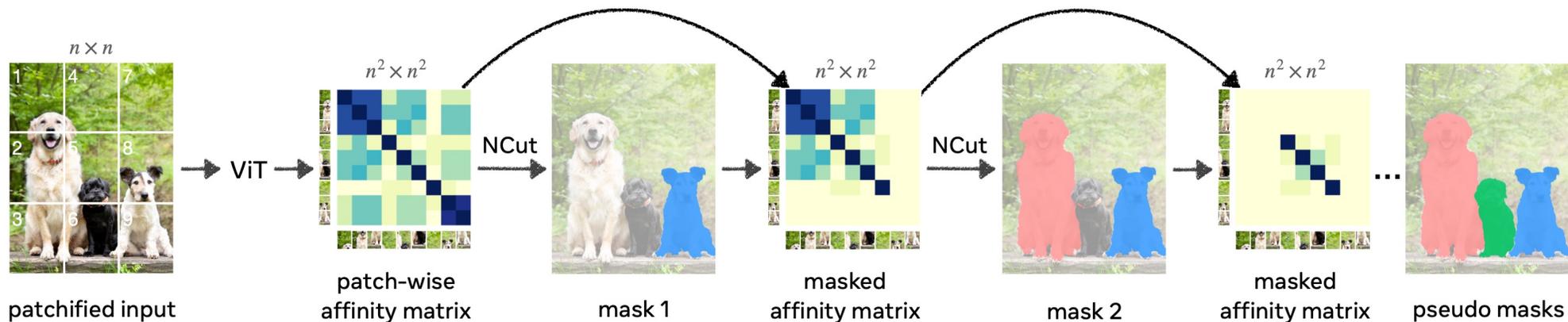
- Use a pretrained DINO ViT network.



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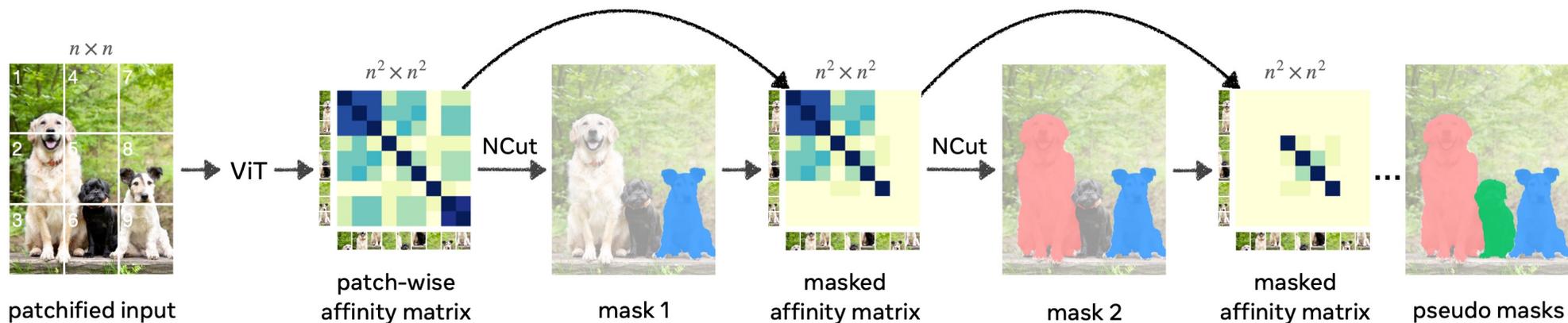
- Use a pretrained DINO ViT network.

- Use the “key” features from the last attention layer:  $W_{ij} = \frac{K_i K_j}{\|K_i\|_2 \|K_j\|_2}$



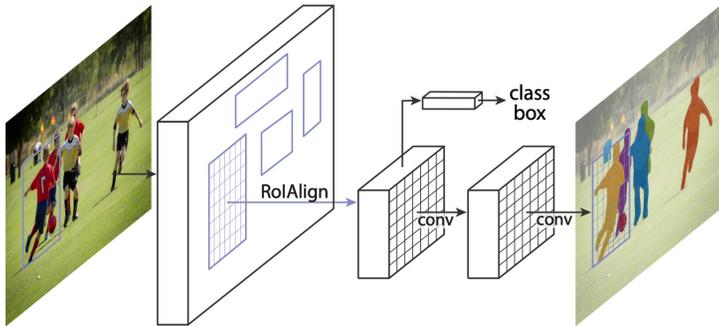
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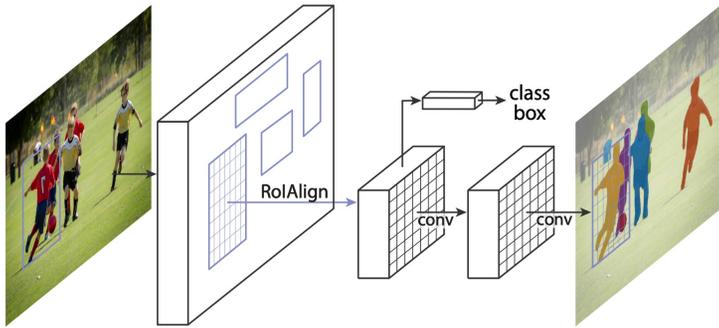
# Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.



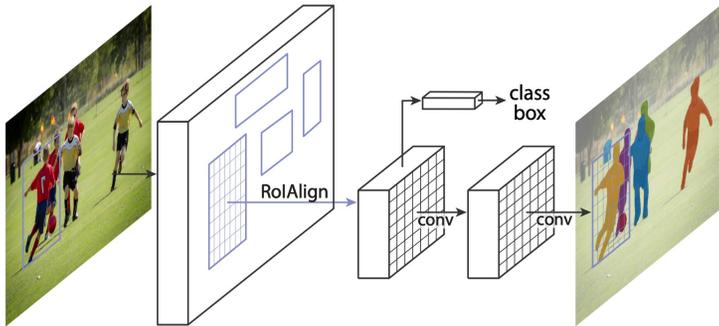
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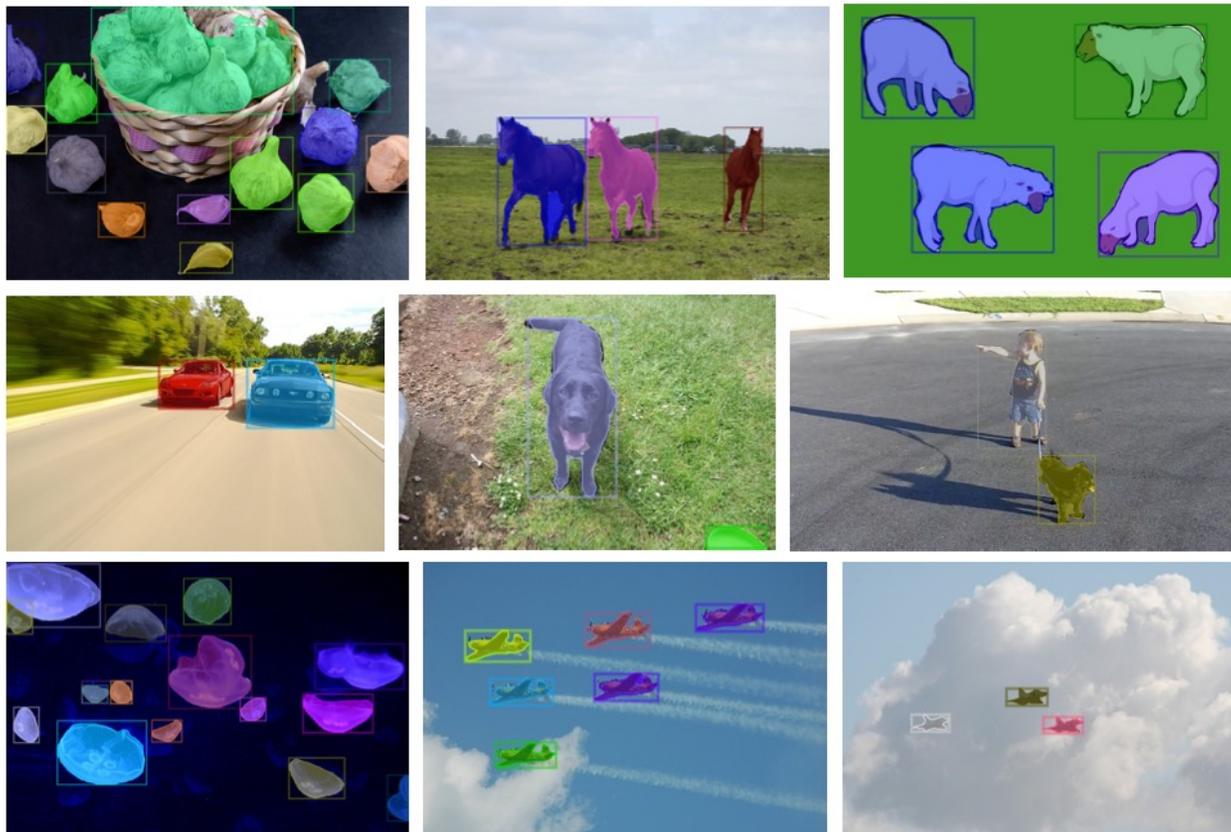


# Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.
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- Neural networks can learn from the noisy labels and output smoother predictions.

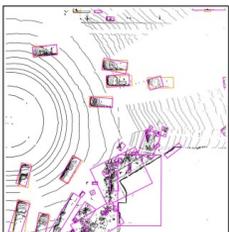


# More Visualization

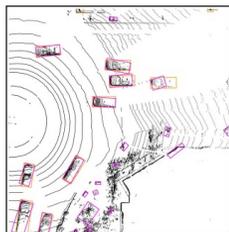


# Pseudo Labels in 3D

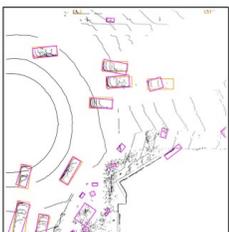
1 Point clustering pseudo-labels



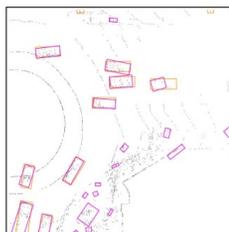
2 Filter out temporally inconsistent tracks



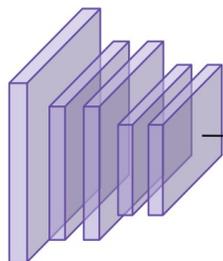
3 Randomly drop lidar beams



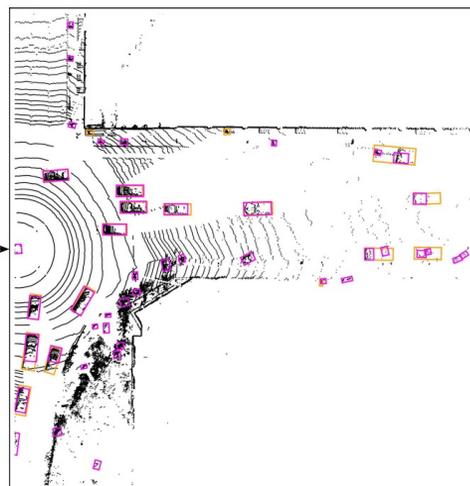
4 Randomly drop spherical rows/cols



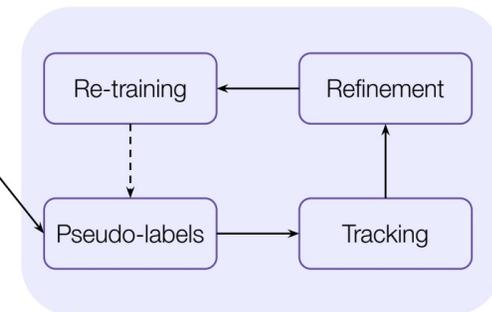
5 Train CNN in short range



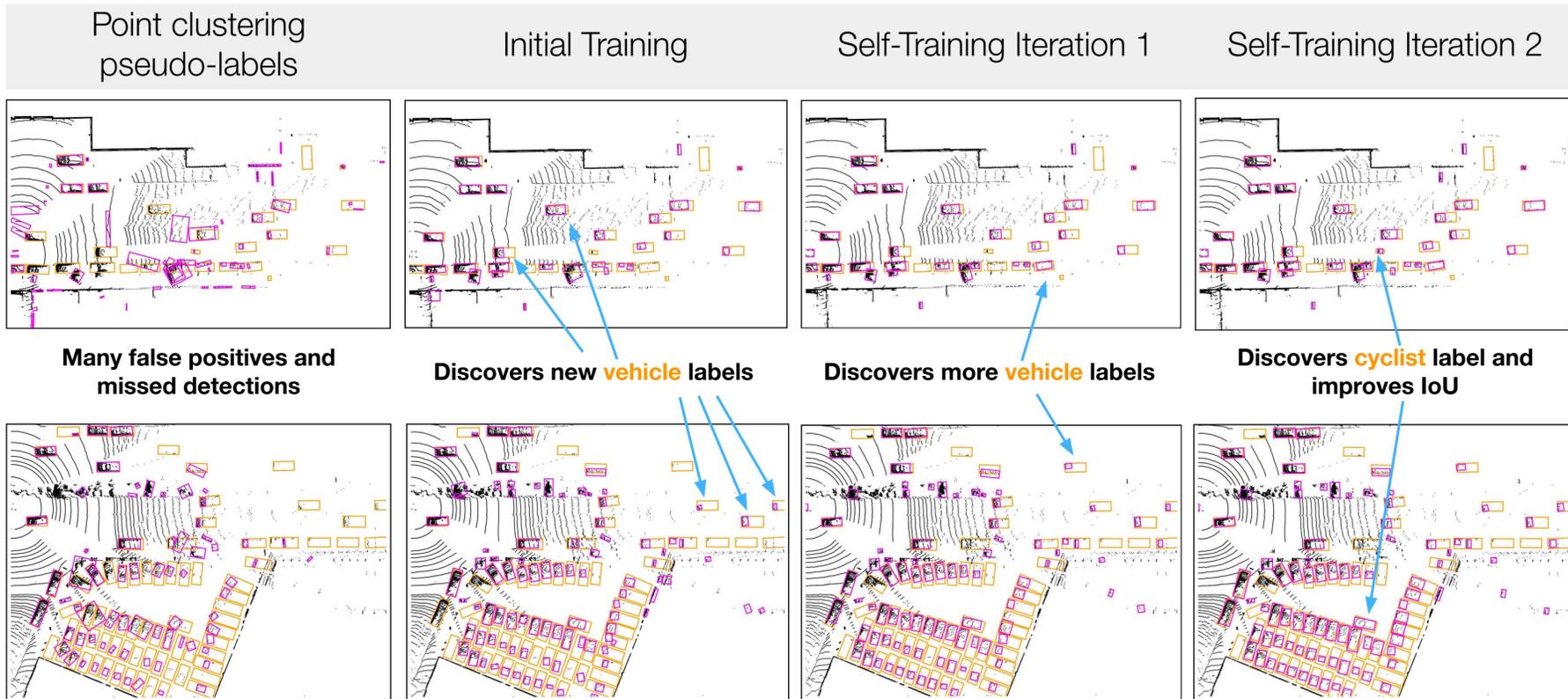
6 Zero-shot generalization to long-range



7 Self-training in long-range

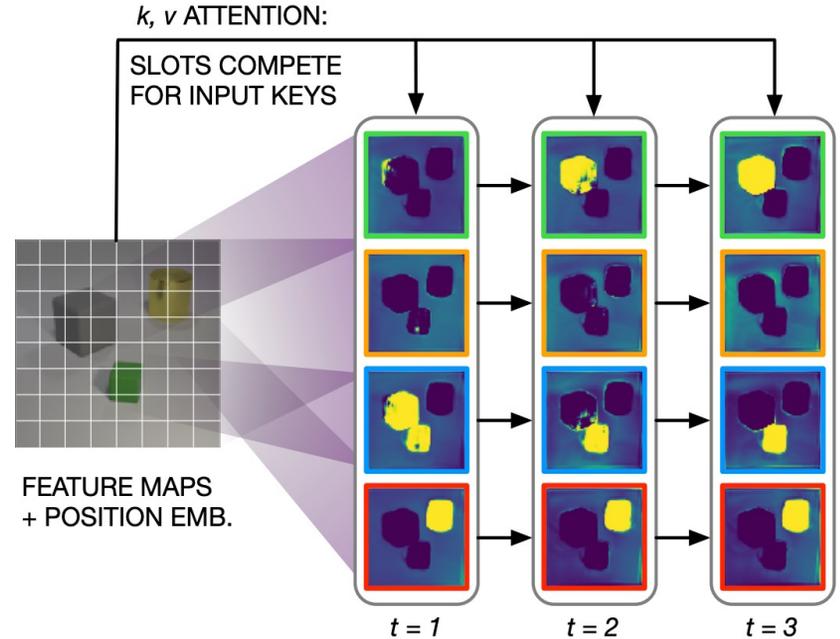


# Iterative Refinement of Pseudo Labels



# Slot Attention Networks

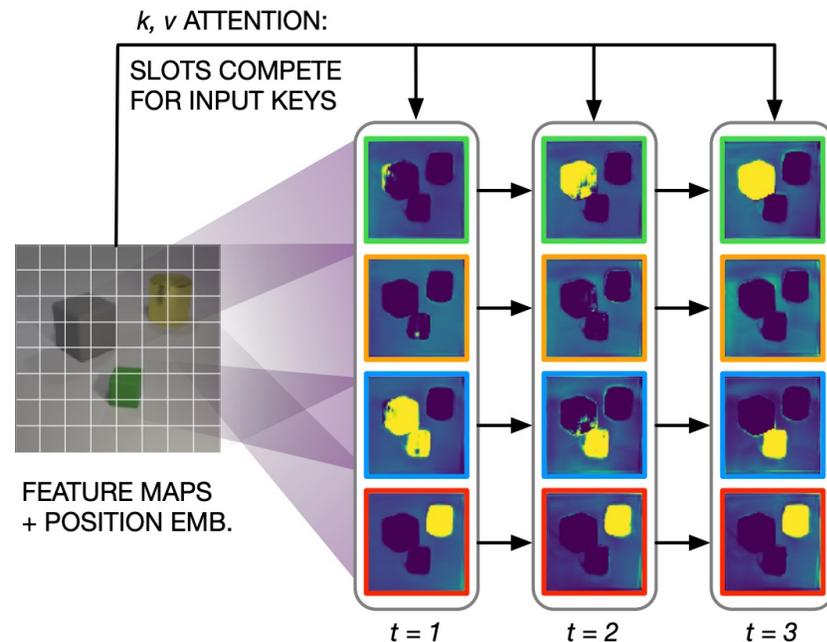
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(a) Slot Attention module.

# Slot Attention Networks

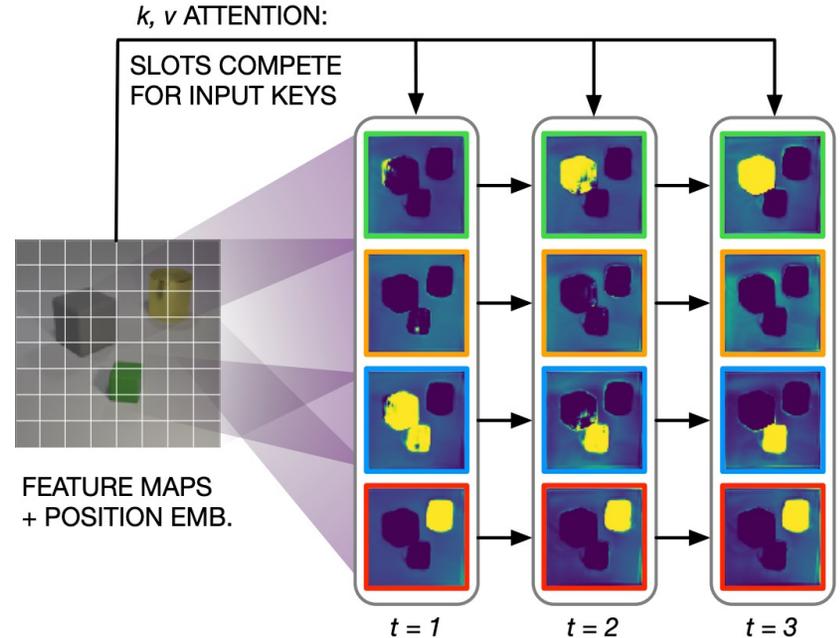
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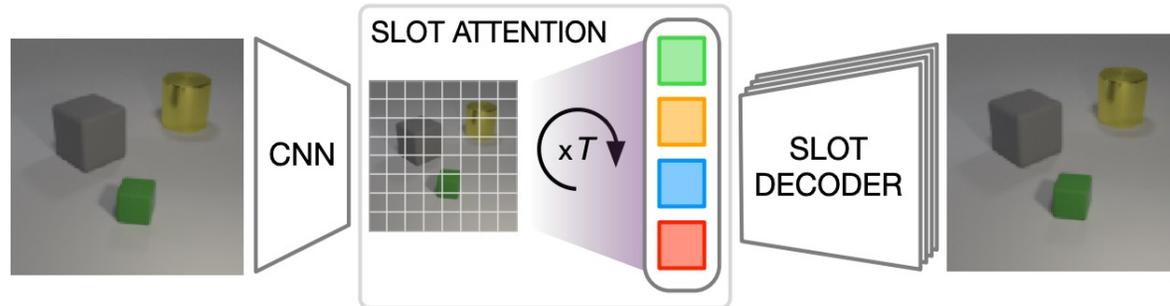
- Can we learn clustering as an end-to-end operation?
- Slot attention is inspired by the success of the attention mechanism.
- Each “slot” attends to a region of the image and stores an object centric representation.



(a) Slot Attention module.

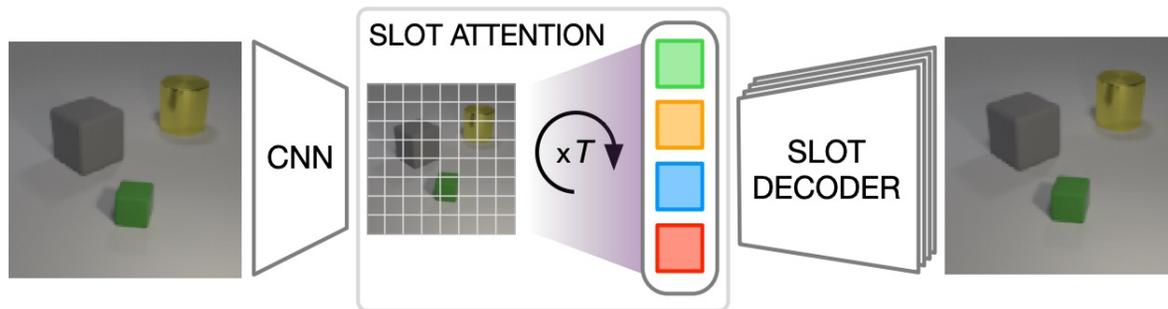
# Slot Attention Networks

- Goal: Reconstruct the image with a concise slot-based representation.



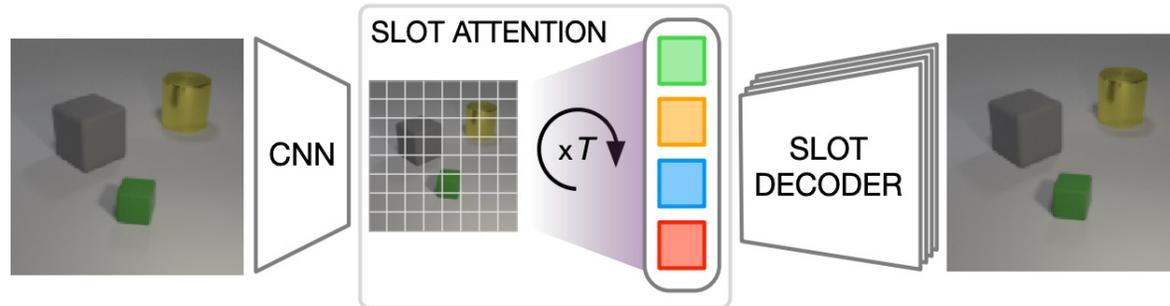
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- Goal: Reconstruct the image with a concise slot-based representation.
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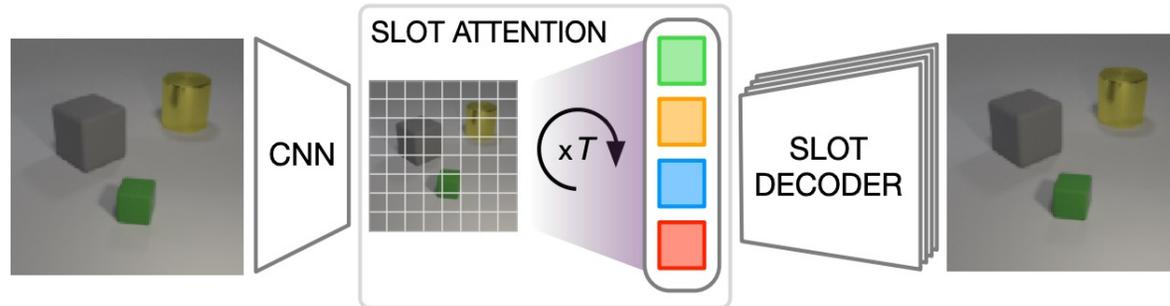
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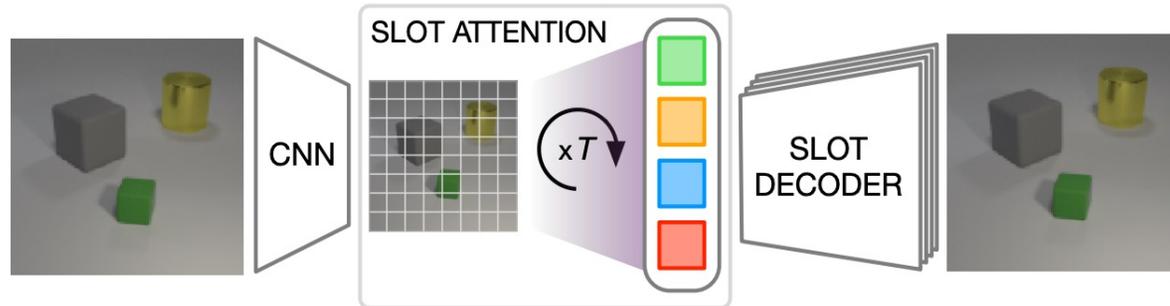
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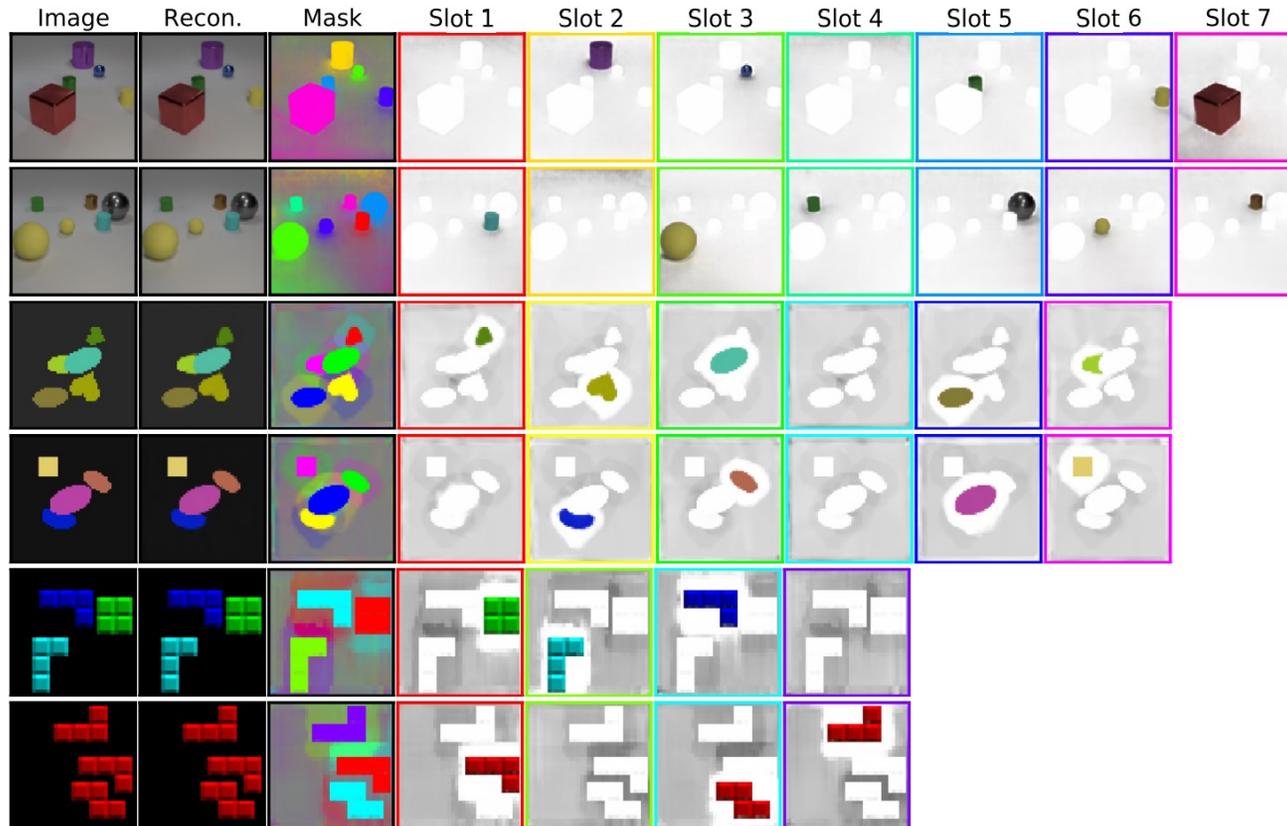


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- Write into slots:  $m_t = GRU(m_{t-1}, u_t) + MLP(\tilde{m}_{t-1})$ .

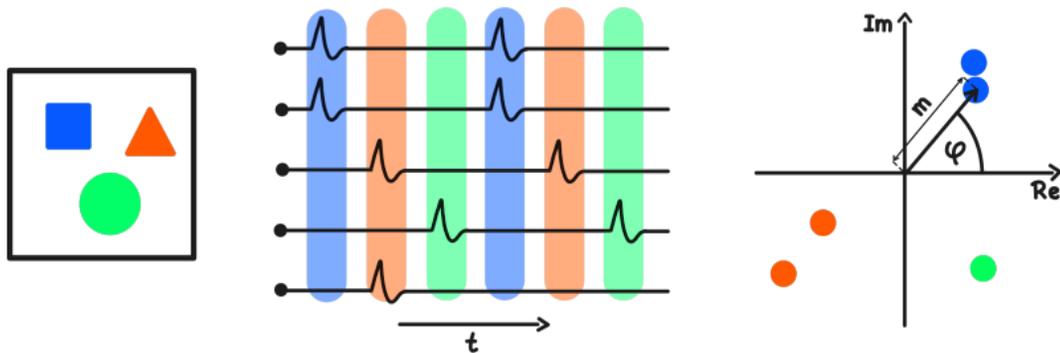
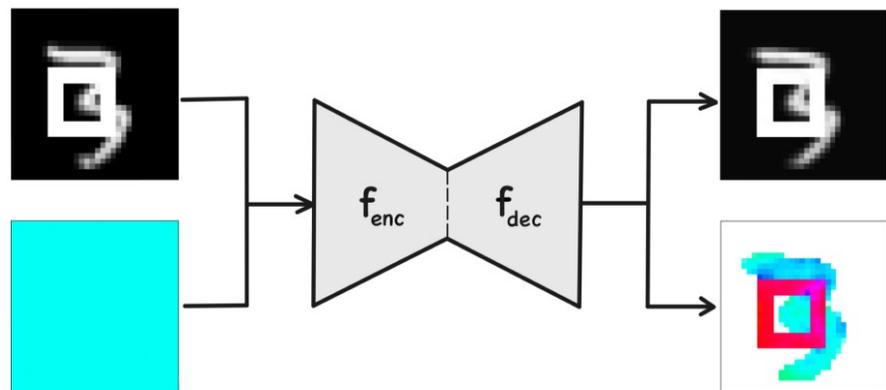


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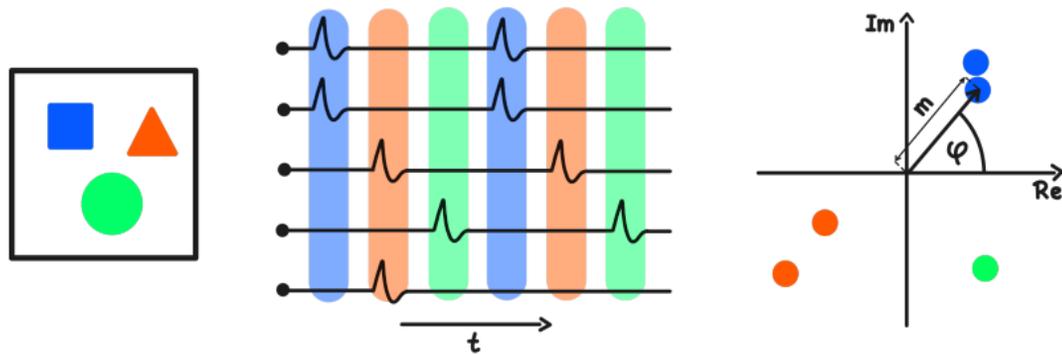
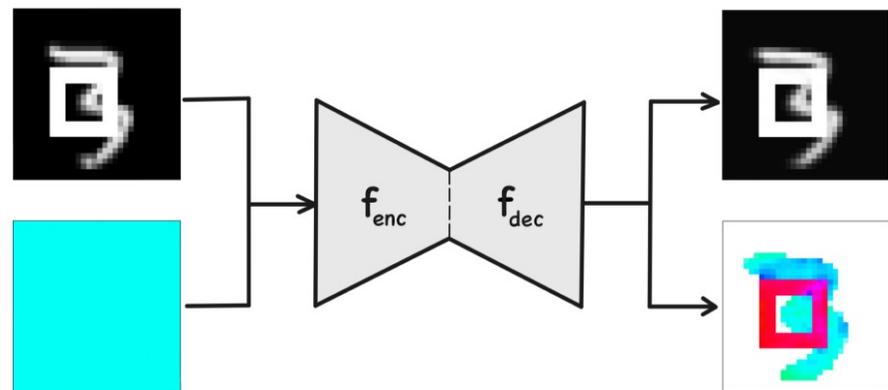
# Complex-Valued Autoencoders (CAEs)

- The complex number can represent magnitude and phase:  $z = m \cdot e^{i\varphi} \in \mathbb{C}$ .



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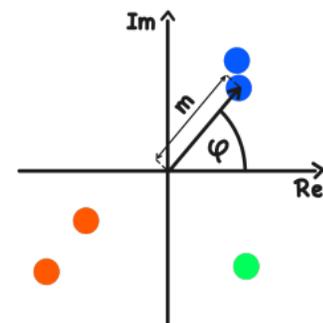
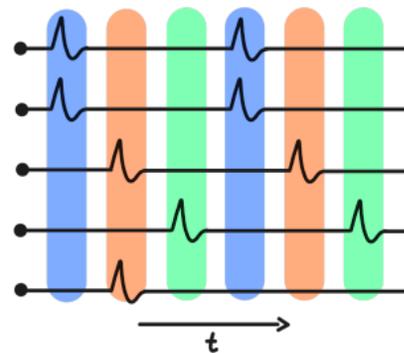
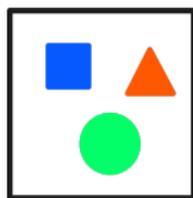
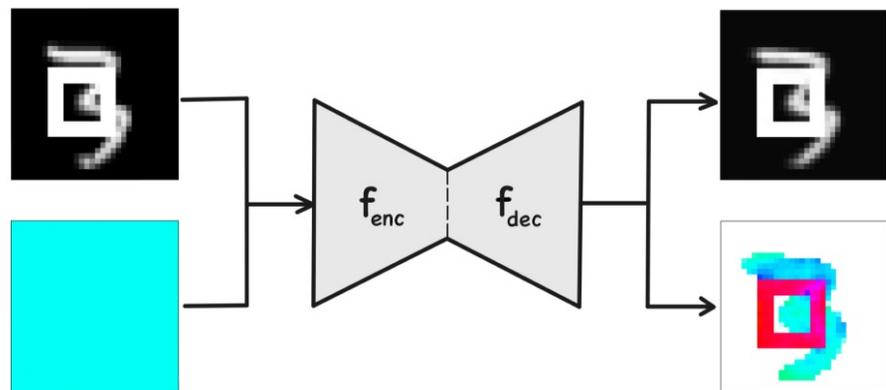
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$$\hat{\mathbf{z}} = f_{\text{dec}}(f_{\text{enc}}(\mathbf{x})) \in \mathbb{C}^{h \times w}.$$

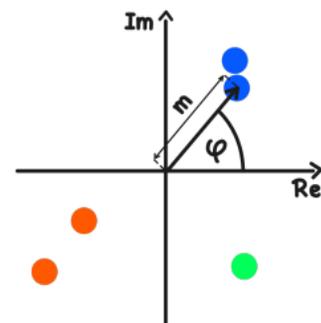
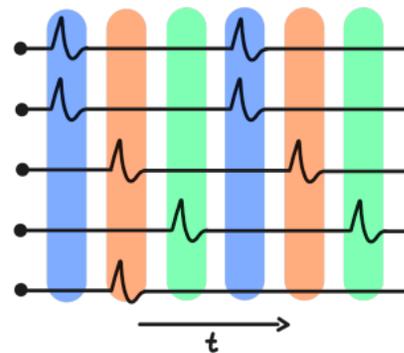
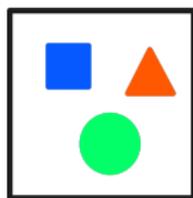
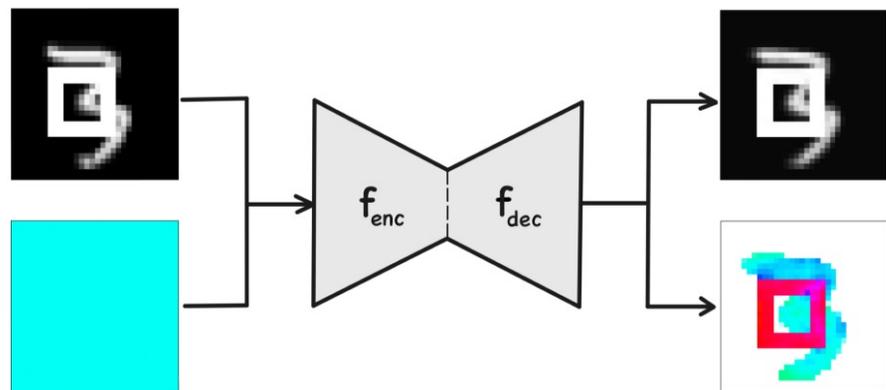


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# CAE: More Details

- Apply weights separately to real and imaginary:

$$\boldsymbol{\psi} = f_{\mathbf{w}}(\mathbf{z}) = f_{\mathbf{w}}(\text{Re}(\mathbf{z})) + f_{\mathbf{w}}(\text{Im}(\mathbf{z})) \cdot i \in \mathbb{C}^{d_{\text{out}}}$$

[Reichert & Serre, 2014]

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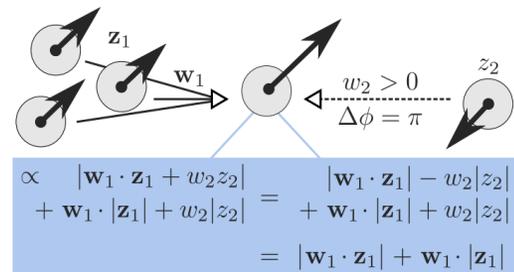
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$$\chi = f_{\mathbf{w}}(|\mathbf{z}|) + \mathbf{b}_m \in \mathbb{R}^{d_{\text{out}}}$$

- Gating:

$$\mathbf{m}_{\mathbf{z}} = 0.5 \cdot \mathbf{m}_{\psi} + 0.5 \cdot \chi \in \mathbb{R}^{d_{\text{out}}}$$

Synchrony Term    Classic Term



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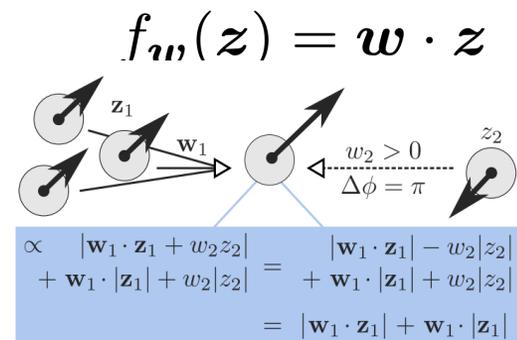
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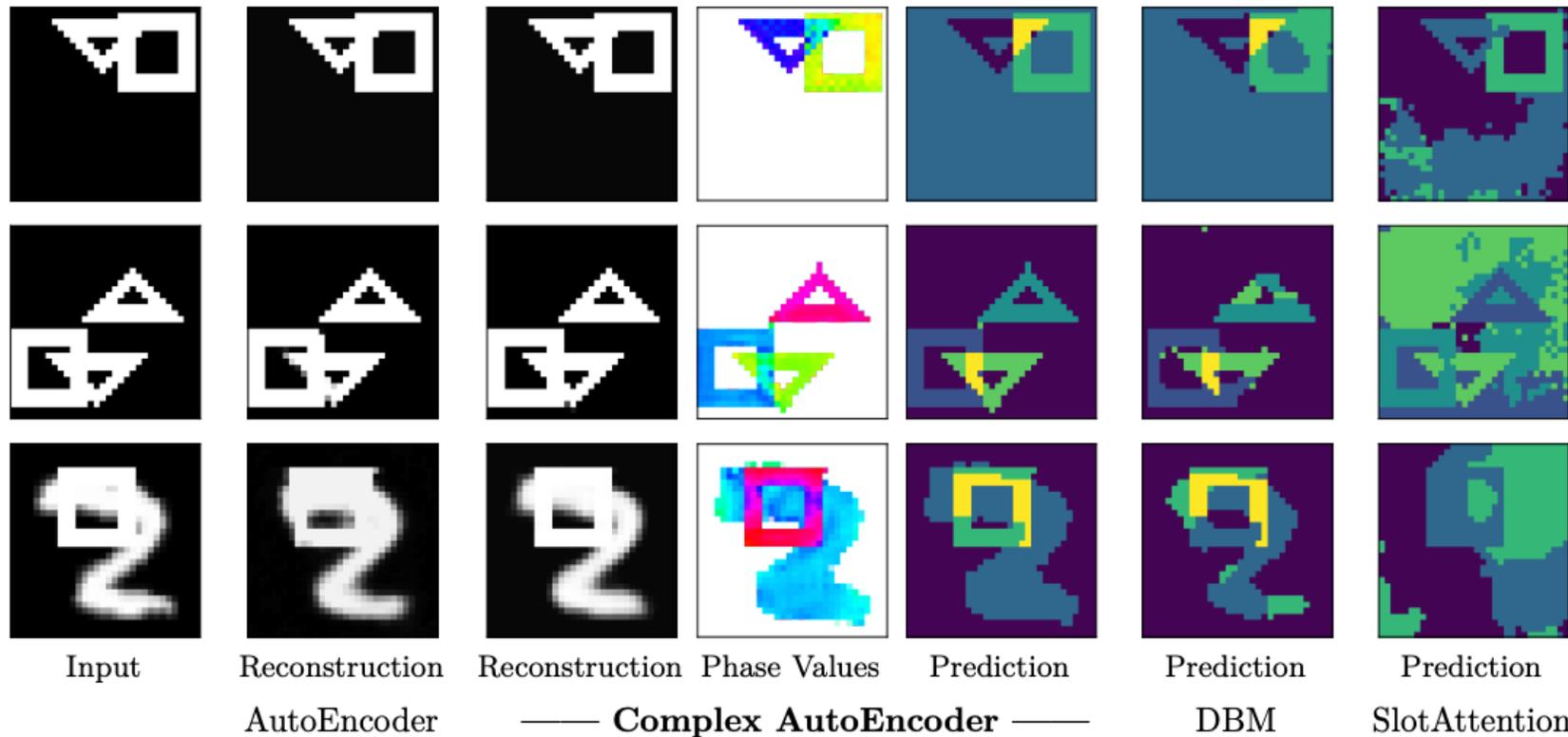
Synchrony Term      Classic Term

- Activation  $\mathbf{z}' = \text{ReLU}(\text{BatchNorm}(\mathbf{m}_{\mathbf{z}})) \circ e^{i\varphi_{\psi}} \in \mathbb{C}^{d_{\text{out}}}$



[Reichert & Serre, 2014]

# Complex-Valued Autoencoders



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- What do we make use of the discovered objects? Is it better to keep the awareness in the latent space?