ELV: Embodied Learning and Vision

Mengye Ren

NYU

Spring 2026

elvcourse.org

Summary: Normalizing Flow

- Requires invertibility and tractability on Jacobian determinant
- Constraint on the functional form
- Or estimation of Jacobian / iterative inverse (numerical approximation).

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Inference as Integration

- Residual Flow: $\phi(x) = x + \delta u(x)$

$$\frac{\phi(x) - x}{\delta} = u(x)$$

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- Can be written in continuous time (continuous normalizing flow).

$$x_t = \phi_t(x_0) = x_0 + \int_0^t \delta u_s(x_s)\ ds.$$

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- Integration: Euler, or directly run an ODE solver
- Backprop solving an ODE without storing intermediate states

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Flow Matching

- Taking integrals during training is cumbersome, and the learning signal from the top latent is weak.

Lipman et al. Flow Matching for Generative Modeling. ICLR 2023.

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Flow Matching

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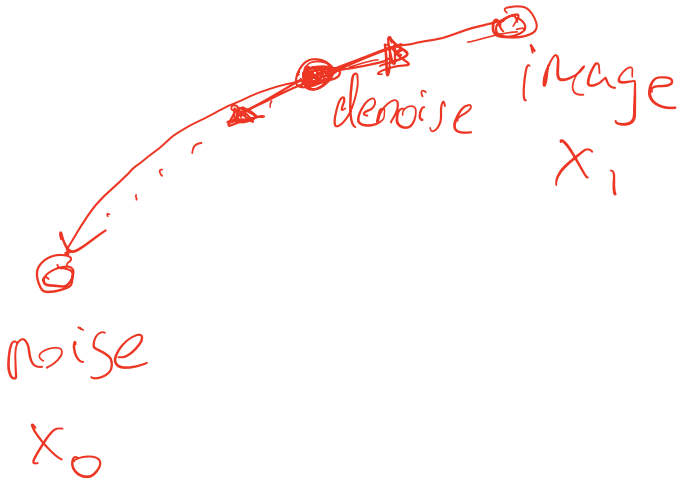
- Given an original input x_1 and a noise x_0, $x_t = (1-t)x_0 + tx_1$



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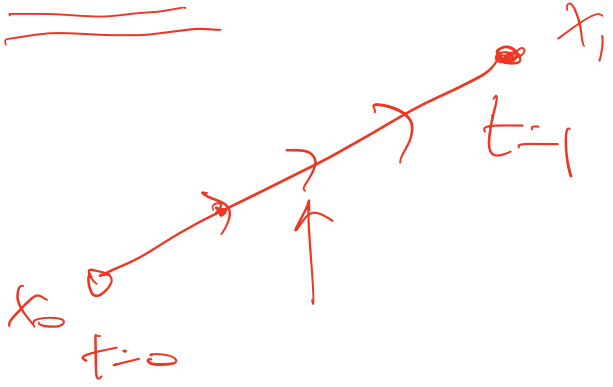

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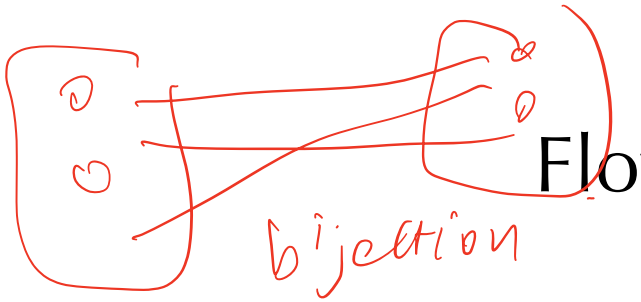

deterministic noise.

Flow Matching

- Taking integrals during training is cumbersome, and the learning signal from the top latent is weak.
- Given an original input x_1 and a noise x_0, $x_t = (1-t)x_0 + tx_1$
- Want to fit a velocity $v_t(x)$ to match $u_t = x_1 - x_0$
- Flow Matching learning objective:

$$\mathcal{L}_{FM}(\theta) = \mathbb{E}_{t,x_0,x_1}[||v(x_t,t) - u_t(x_0,x_1)||^2]$$

targets

ideal velocity.

Euler integration → 10-20 steps.

efficient.

Diffusion Models

- A popular generative model formulation.

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Diffusion Models

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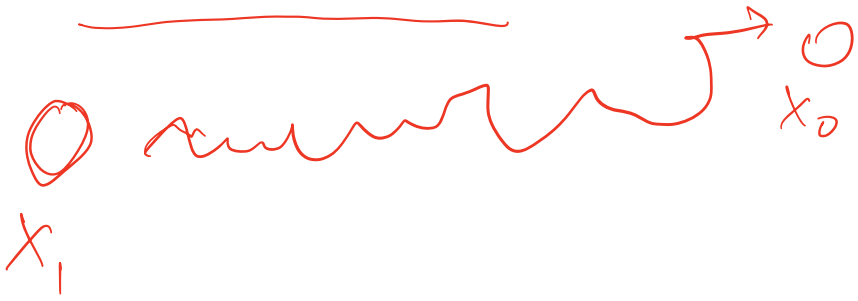
- The intuition is to iteratively denoise from Gaussian random noises into an image.



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Diffusion Models

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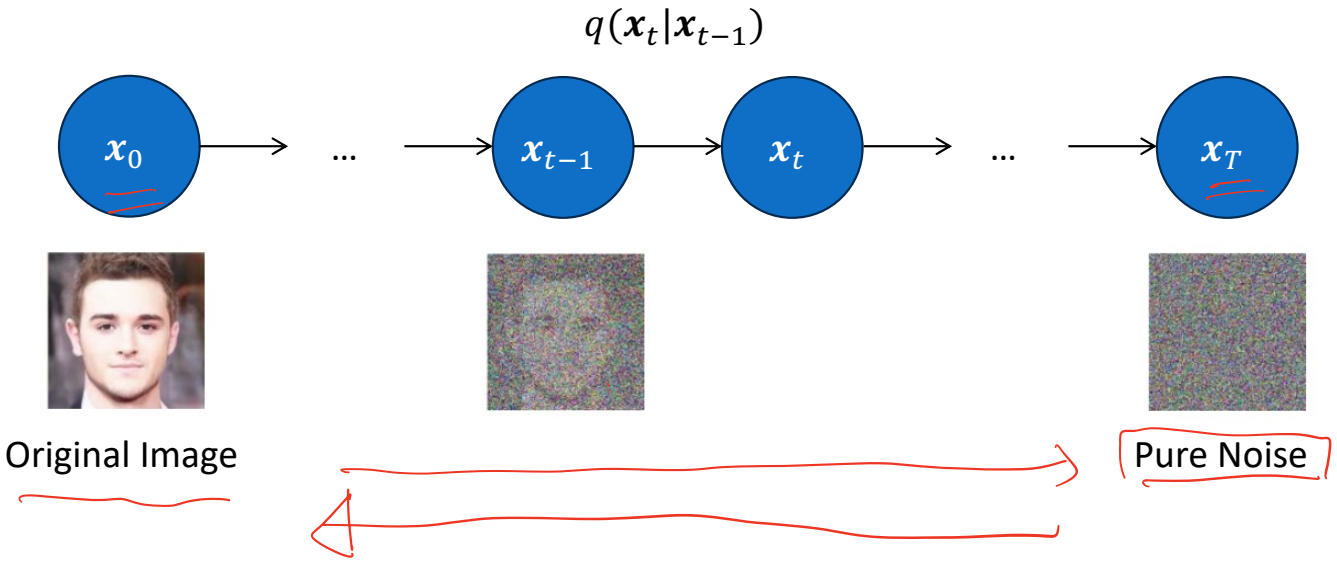
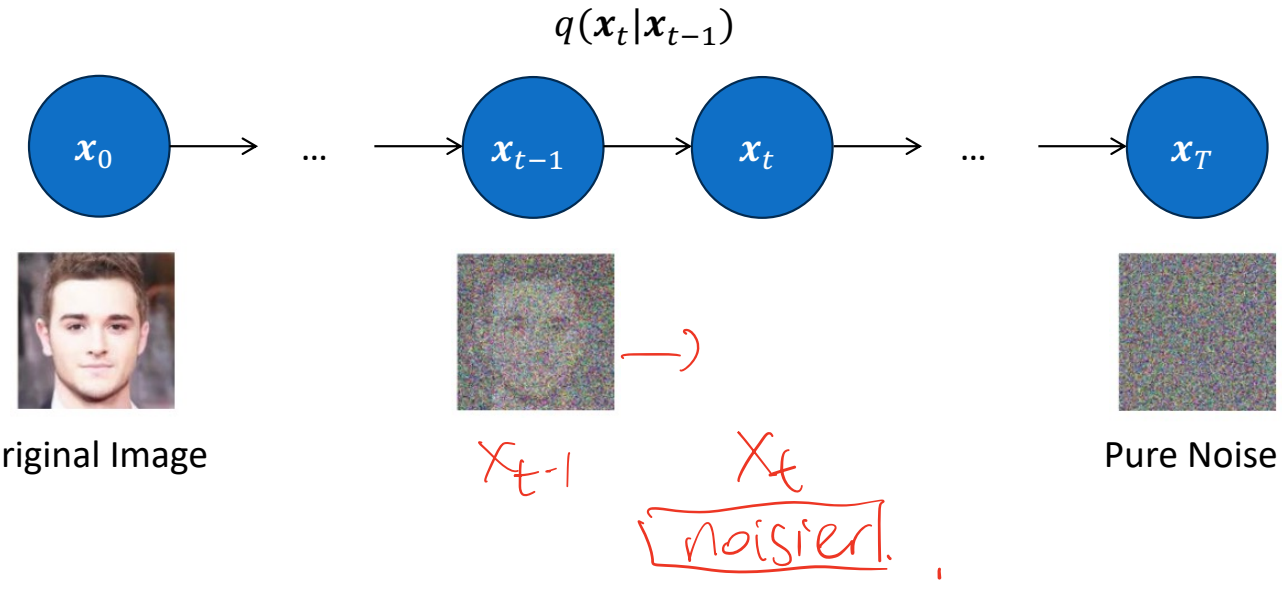


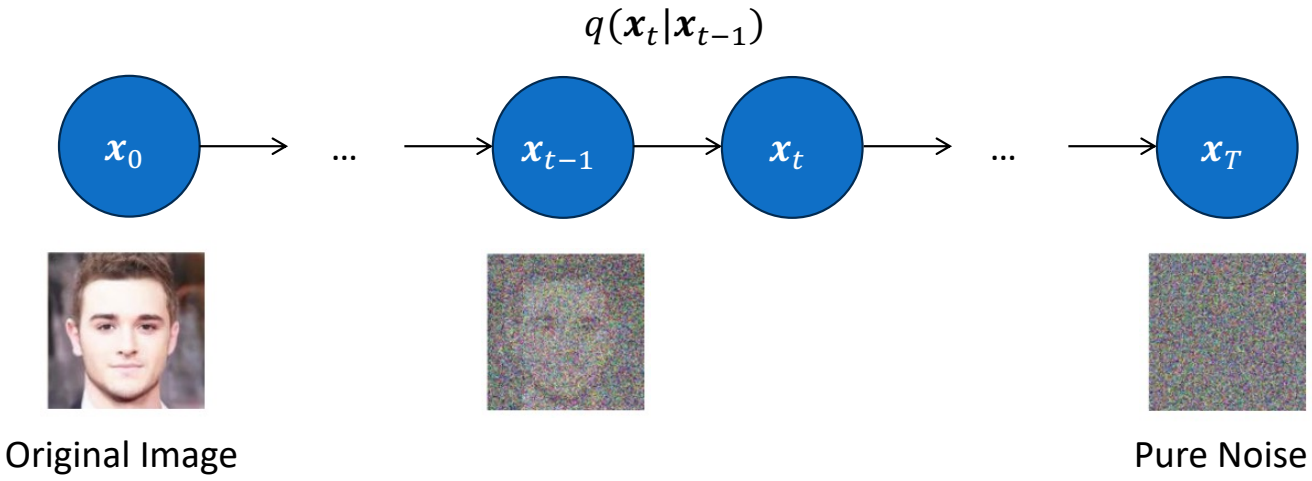
Diffusion Models

- Forward process: $q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}x_{t-1}, \beta_t I)$.



Diffusion Models

- Forward process: $q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}x_{t-1}, \beta_t I)$.
- You can also write: $x_t = \sqrt{1-\beta_t}x_{t-1} + \sqrt{\beta_t}\epsilon_t$, $\epsilon_t \sim \mathcal{N}(0, I)$.



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Properties of the Forward Process

- Forward process: $x_t = \sqrt{1-\beta_t}x_{t-1} + \sqrt{\beta_t}\epsilon_t$

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$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t}x_0, (1-\bar{\alpha}_t)I).$$

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amount of original.

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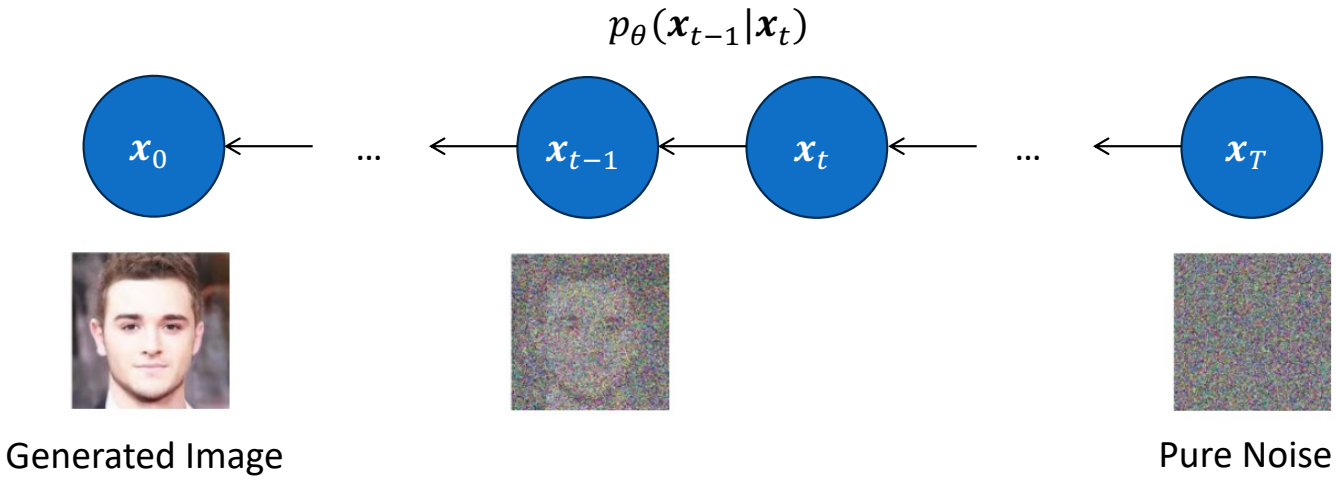
Reverse Process

- A generative model wants to predict x_0 from x_T.



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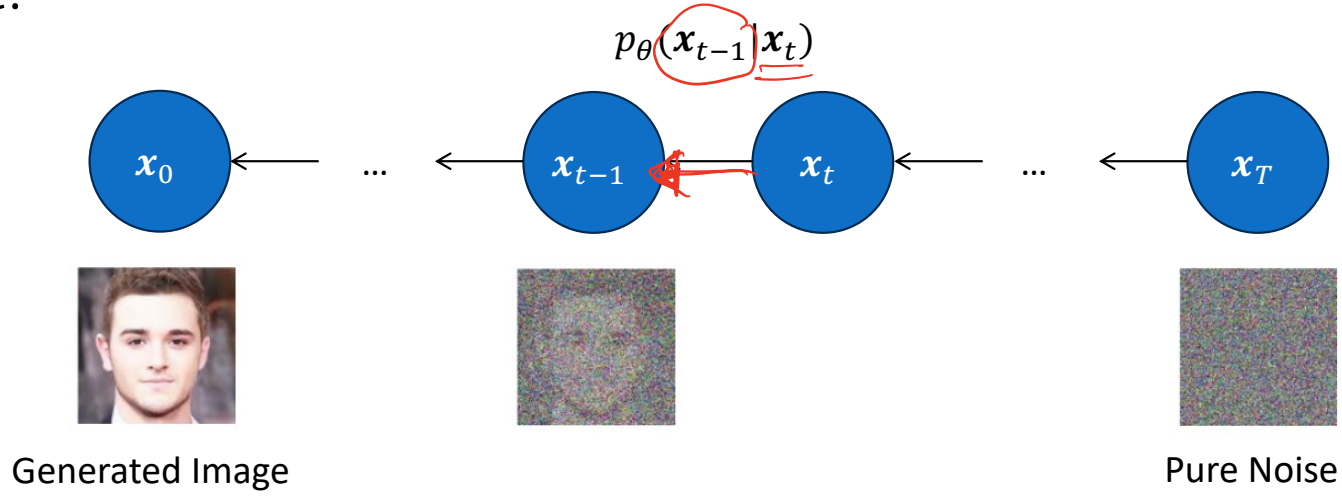
Reverse Process

- A generative model wants to predict x_0 from x_T.
- The reverse process transition is also Gaussian distributed. But we don't know what the transition will be like just by looking at the noisy image!

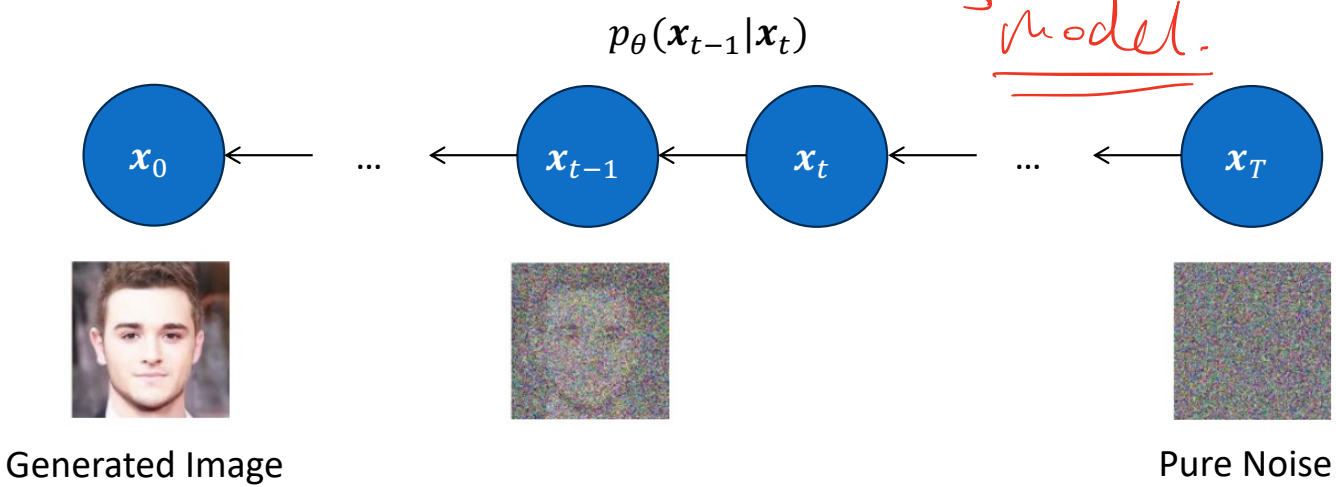


Reverse Process

- So, we need to learn a "model":

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t)).$$

model.

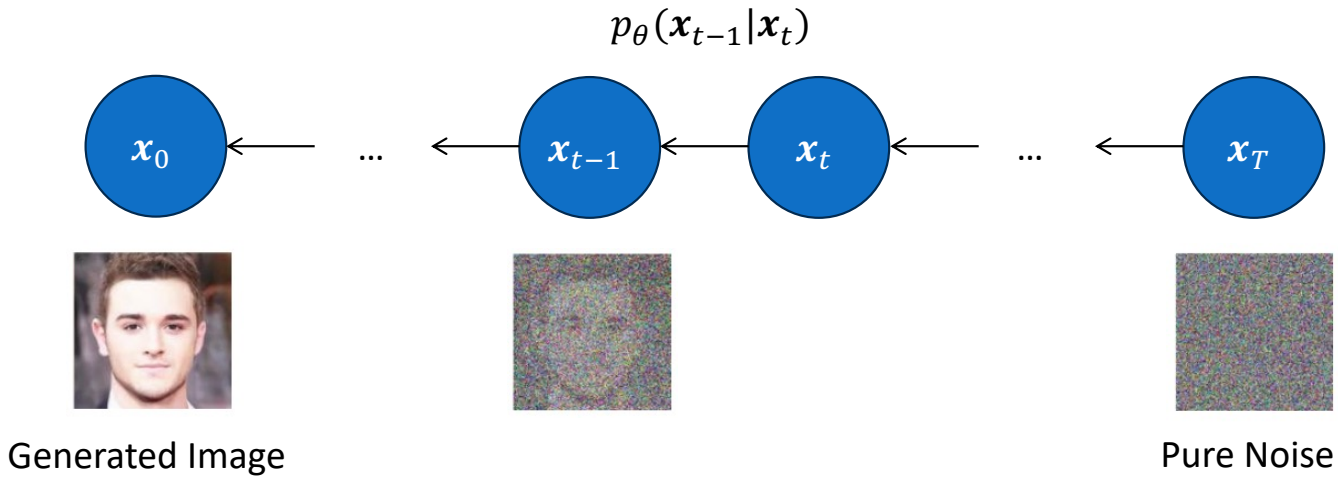


Reverse Process

- So, we need to learn a "model":

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t)).$$

- μ_θ is the denoising vector.



Reverse Process

- Compute μ_θ? Derive $p(x_{t-1}|x_t)$.

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Reverse Process

- Compute μ_θ? Derive $p(x_{t-1}|x_t)$.
- Bayes rule:

$$q(x_{t-1}|x_t) = \frac{q(x_t|x_{t-1})q(x_{t-1})}{q(x_t)}.$$

forward process.

Reverse Process

- Compute μ_θ? Derive $p(x_{t-1}|x_t)$.
- Bayes rule:

$$q(x_{t-1}|x_t) = \frac{q(x_t|x_{t-1})q(x_{t-1})}{q(x_t)}.$$

- But we don't know the marginal distribution $q(x_{t-1})$. We only know q_T and $q(x_t|x_{t-1})$.

learning approx. $p(x_{t-1}|x_t) \rightarrow \mu_\theta$

↳ training data $\rightarrow$ x_0's: learn.

Reverse Process

- Compute μ_θ? Derive $p(x_{t-1}|x_t)$.
- Bayes rule:

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- But we don't know the marginal distribution $q(x_{t-1})$. We only know q_T and $q(x_t|x_{t-1})$.
- Solution: Condition on the original input x_0:

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_t|x_{t-1})q(x_{t-1}|x_0)}{q(x_t|x_0)}.$$

— during training

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Reverse Process

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_t|x_{t-1})q(x_{t-1}|x_0)}{q(x_t|x_0)}.$$

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Reverse Process

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_t|x_{t-1})q(x_{t-1}|x_0)}{q(x_t|x_0)}.$$

$$q(x_{t-1}|x_t, x_0) = \mathcal{N}(x_{t-1}; \tilde{\mu}_t, \tilde{\beta} I).$$

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Reverse Process

small jump.

big jump.

big jump.

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_t|x_{t-1})q(x_{t-1}|x_0)}{q(x_t|x_0)}.$$

$$q(x_{t-1}|x_t, x_0) = \mathcal{N}(x_{t-1}; \tilde{\mu}_t, \tilde{\beta} I).$$

$$\tilde{\mu}_t = \frac{\sqrt{\alpha_t}\bar{\beta}_{t-1}}{\bar{\beta}_t} x_t + \frac{\sqrt{\alpha_{t-1}}\beta_t}{\bar{\beta}_t} x_0 = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon\right).$$

noisy

orig. img.

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Reverse Process

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_t|x_{t-1})q(x_{t-1}|x_0)}{q(x_t|x_0)}.$$

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- Want: train up a μ_θ to match with $\tilde{\mu}_t$.

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Training

- Sometimes it is more common to predict the denoising vector ϵ instead of μ.

$$\tilde{\mu}_t = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon),$$

$$\mu_\theta(x_t, t) = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t)).$$

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- Sometimes it is more common to predict the denoising vector ϵ instead of μ.
- Randomly pick at a time step and predict the difference between the noisy and the original.

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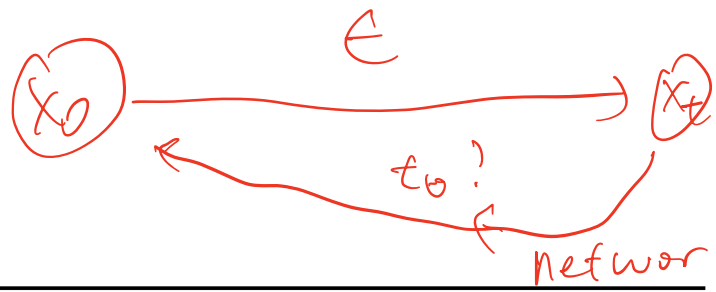

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Algorithm 1 Training

1: **repeat**
2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
3: $t \sim \text{Uniform}(\{1, \dots, T\})$
4: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
5: Take gradient descent step on
$$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$
6: **until** converged

true noise network noisy ver.

$$\tilde{\mu}_t = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon),$$

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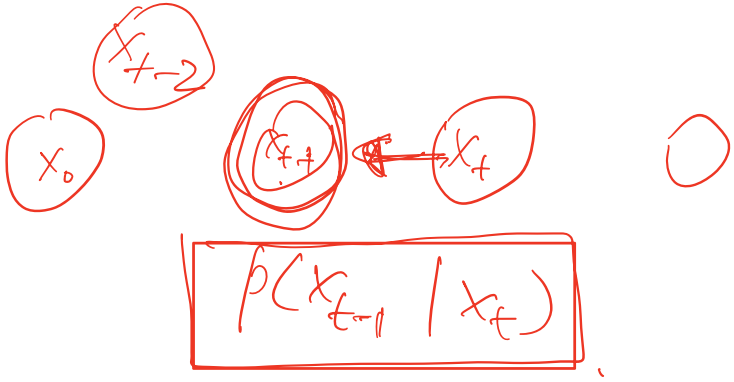
Sampling

- How do we sample an image?

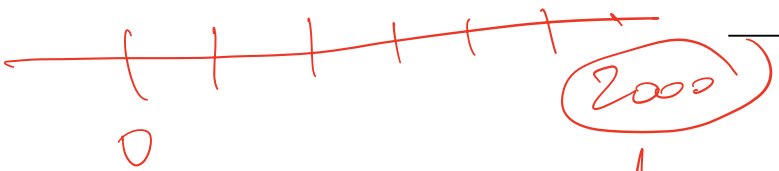


Algorithm 2 Sampling

1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
2: **for** $t = T, \ldots, 1$ **do**
3: $\quad \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
4: $\quad \mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
5: **end for**
6: **return** $\mathbf{x}_0$

add noise

Sampler



denoising

Schedule.

mean

Current image

Sampling

- How do we sample an image?
- We know μ_θ which will help us transition from x_t to x_{t-1}.

$$\mu_\theta(x_t, t) = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t)).$$

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Sampling

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$$\mu_\theta(x_t, t) = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t)).$$

- Sample from $\mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \sigma_t^2)$. σ_t can either be β_t or $\tilde{\beta}_t$ derived from the posterior.

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5: **end for**
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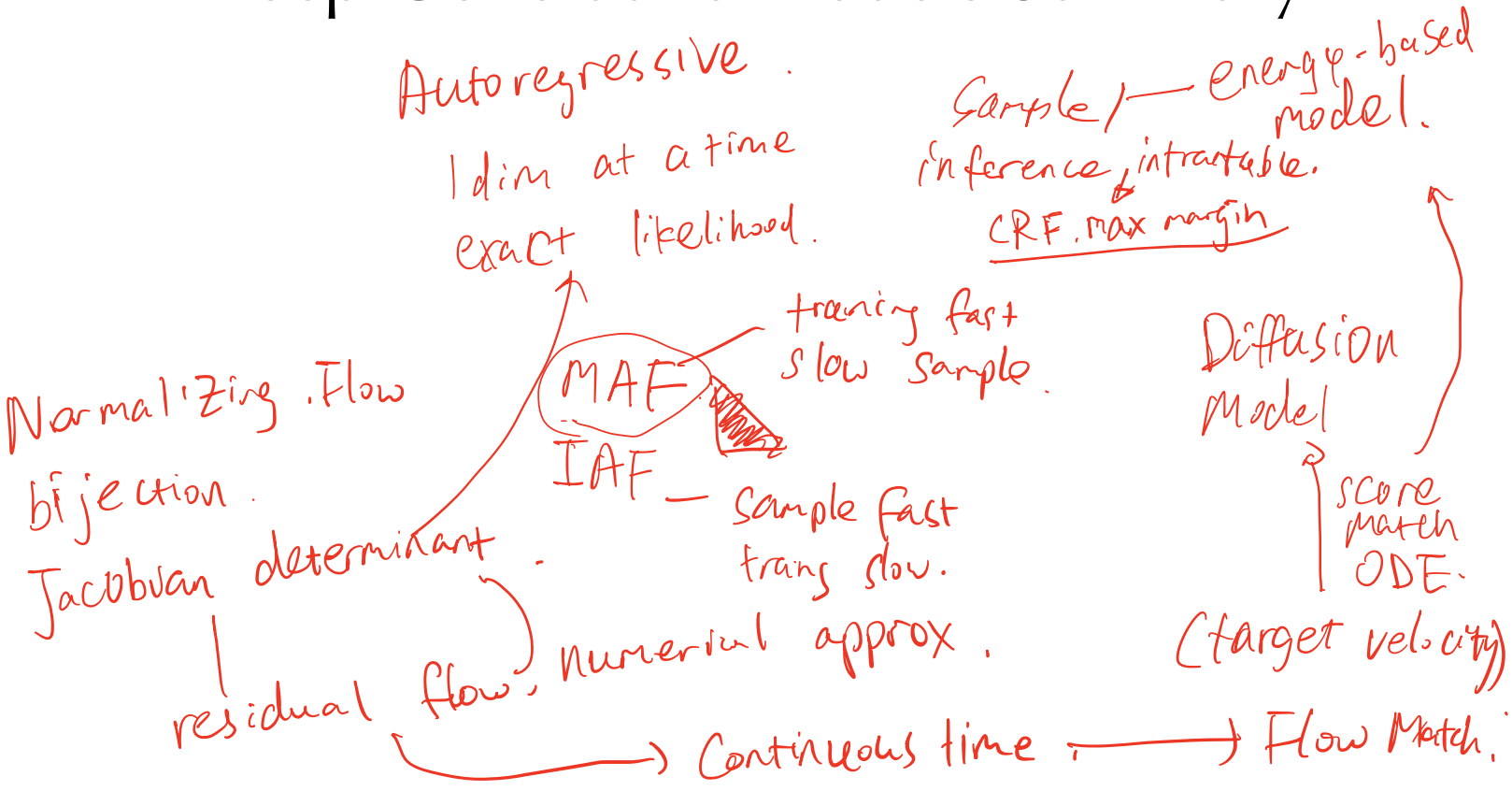
Deep Generative Models Summary



Guided Diffusion

- We can add guidance on the diffusion updates at inference time.

Classifier Guidance / External Score Model

$$x_{t-1} \leftarrow \text{sample from } \mathcal{N}(\mu + s\Sigma \nabla_{x_t} \log p_\phi(y|x_t), \Sigma)$$

$$\hat{\epsilon} \leftarrow \epsilon_\theta(x_t) - \sqrt{1-\bar{\alpha}_t}\, \nabla_{x_t} \log p_\phi(y|x_t)$$
$$x_{t-1} \leftarrow \sqrt{\bar{\alpha}_{t-1}} \left(\frac{x_t - \sqrt{1-\bar{\alpha}_t}\hat{\epsilon}}{\sqrt{\bar{\alpha}_t}} \right) + \sqrt{1-\bar{\alpha}_{t-1}}\hat{\epsilon}$$

gradient from another criterion

Dhariwal & Nichol. Diffusion Models Beat GANs on Image Synthesis. NeurIPS 2021.
Ho & Salimans. Classifier-Free Diffusion Guidance. 2022.

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- We also can train a conditional diffusion model.

repeat
- $(\mathbf{x}, \mathbf{c}) \sim p(\mathbf{x}, \mathbf{c})$ ▷ Sample data with conditioning from the dataset
- $\mathbf{c} \leftarrow \varnothing$ with probability p_{uncond} ▷ Randomly discard conditioning to train unconditionally
- $\lambda \sim p(\lambda)$ ▷ Sample log SNR value
- $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- $\mathbf{z}_\lambda = \alpha_\lambda \mathbf{x} + \sigma_\lambda \boldsymbol{\epsilon}$ ▷ Corrupt data to the sampled log SNR value
- Take gradient step on $\nabla_\theta \left\| \boldsymbol{\epsilon}_\theta(\mathbf{z}_\lambda, \mathbf{c}) - \boldsymbol{\epsilon} \right\|^2$ ▷ Optimization of denoising model

until converged

Condition.

Dhariwal & Nichol. Diffusion Models Beat GANs on Image Synthesis. NeurIPS 2021.
Ho & Salimans. Classifier-Free Diffusion Guidance. 2022.

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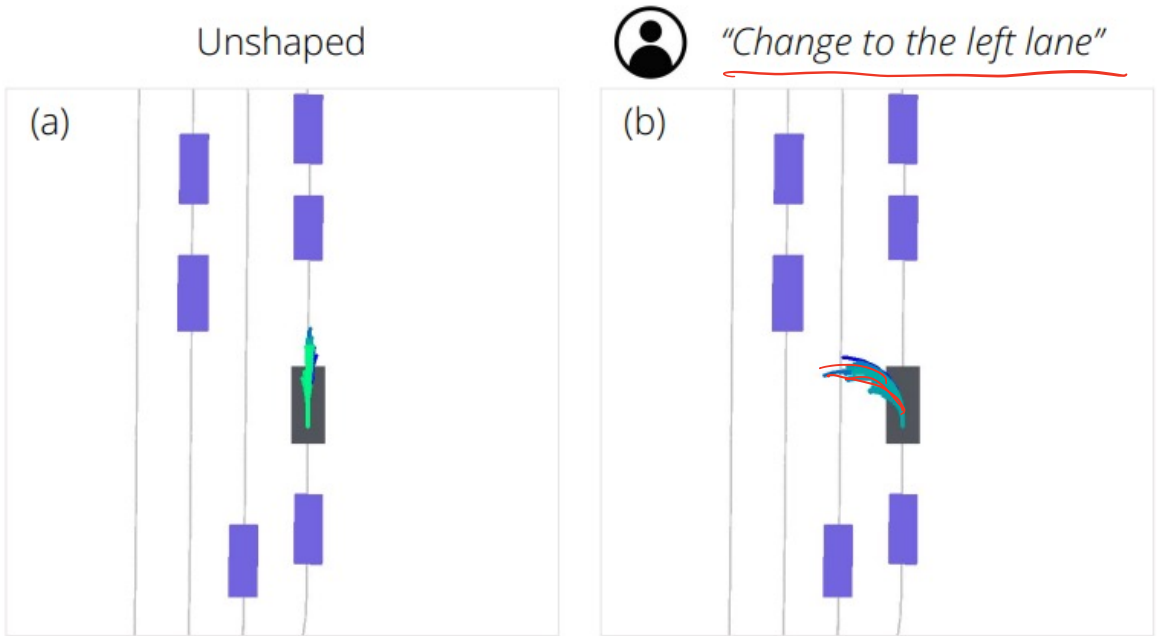
Test-Time Guidance / Adaptation

- Diffusion can be combined / guided at test time.



Yang et al. Diffusion-ES: Gradient-free Planning with Diffusion for Autonomous and Instruction-guided Driving. CVPR 2024.

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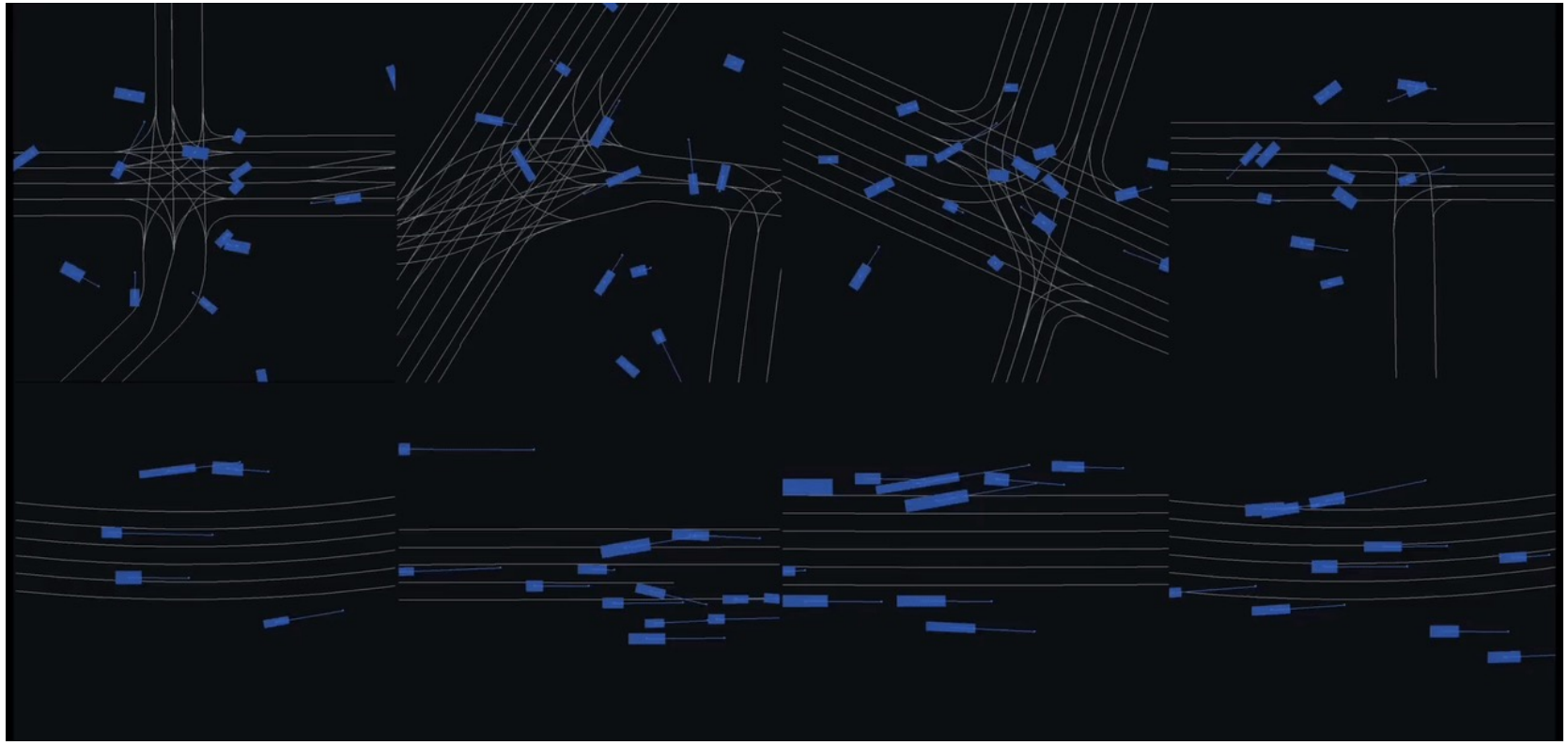
Generating Simulation Scenes



Lu et al. SceneControl: Diffusion for Controllable Traffic Scene Generation. ICRA 2024.

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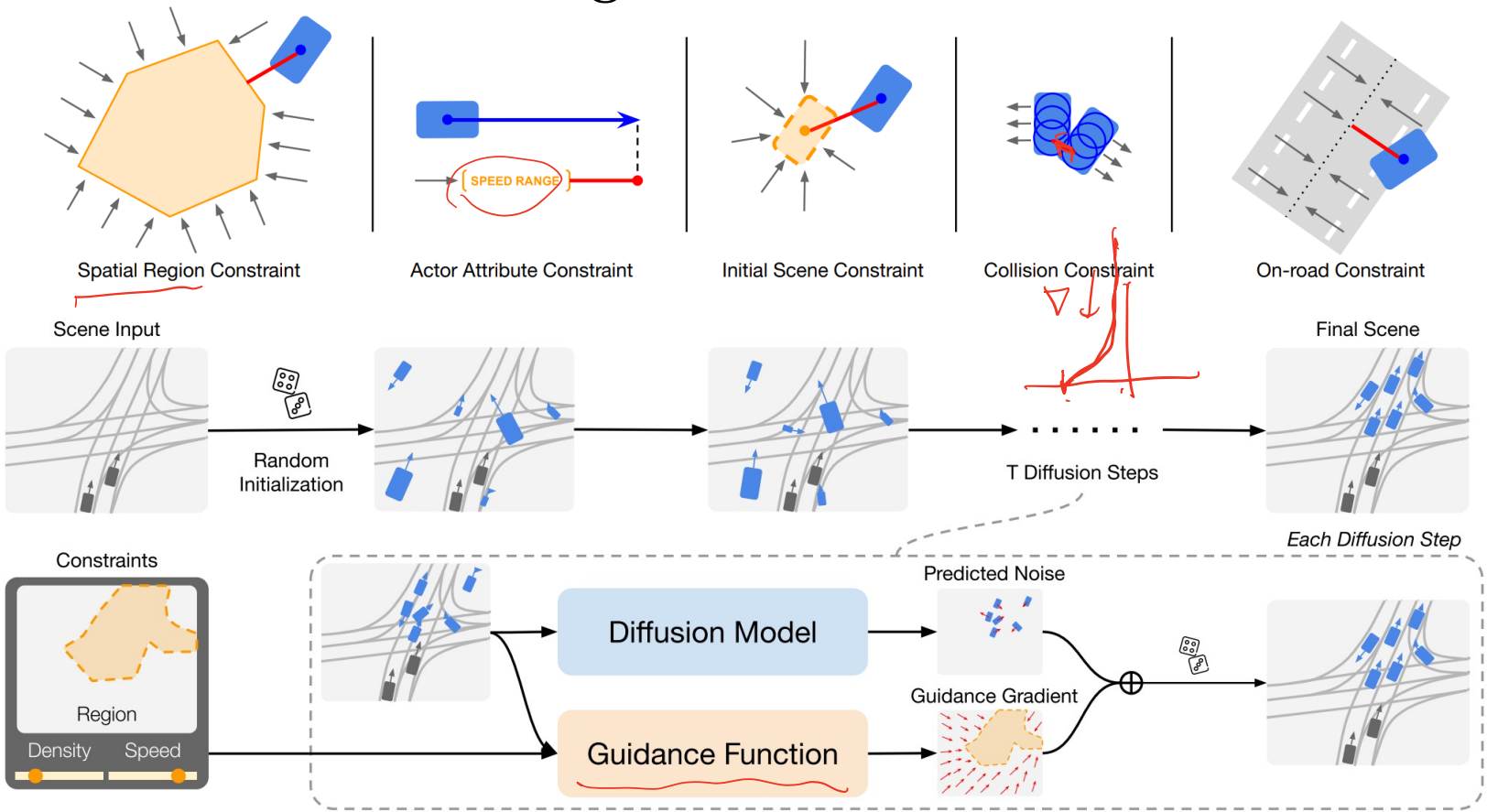
Generating Simulation Scenes



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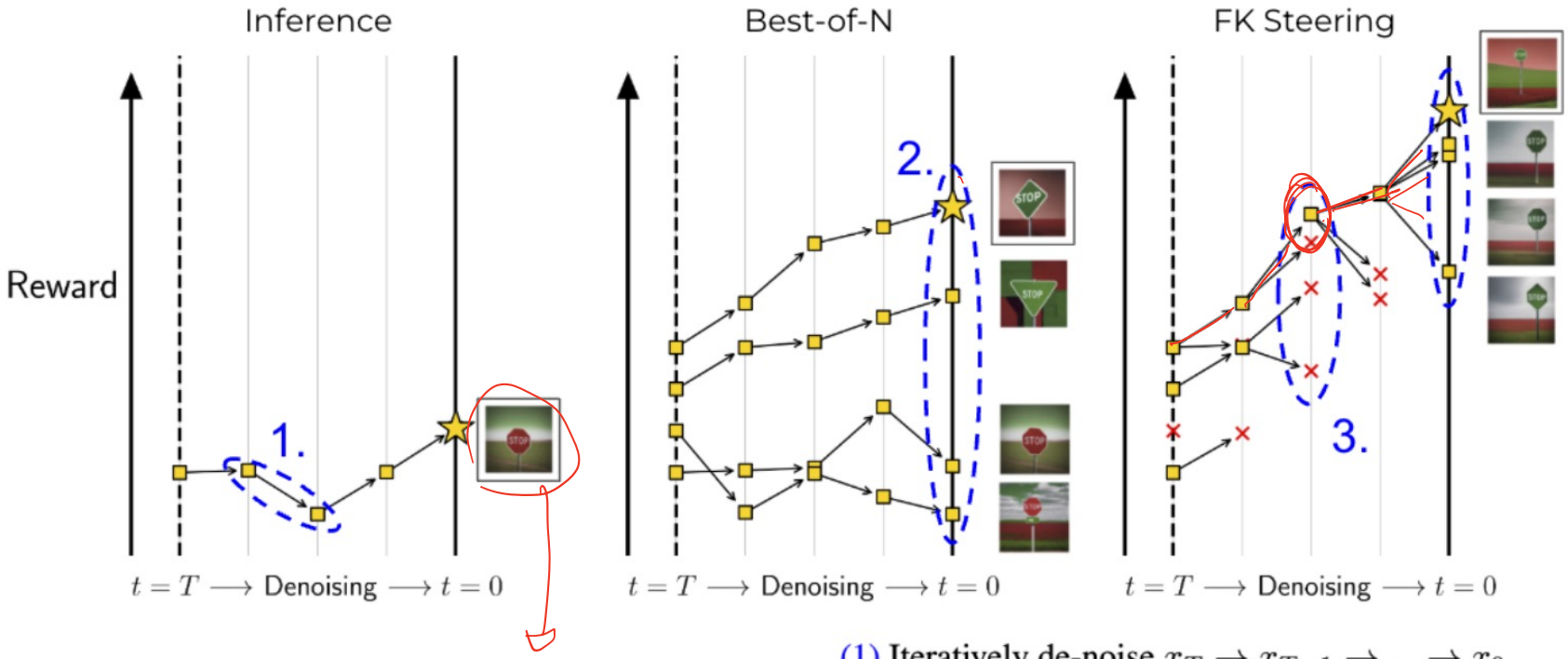
Non-Differentiable Rewards?



Prompt: *"a green stop sign in a red field"*

(1) Iteratively de-noise $x_T \rightarrow x_{T-1} \rightarrow \ldots \rightarrow x_0$.
(2) Generate multiple samples (*particles*).
(3) Resample promising particles at *intermediate* steps.

Singhal et al. A general framework for inference-time scaling and steering of diffusion models. ICML 2025.

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Diffusion Models for Detection

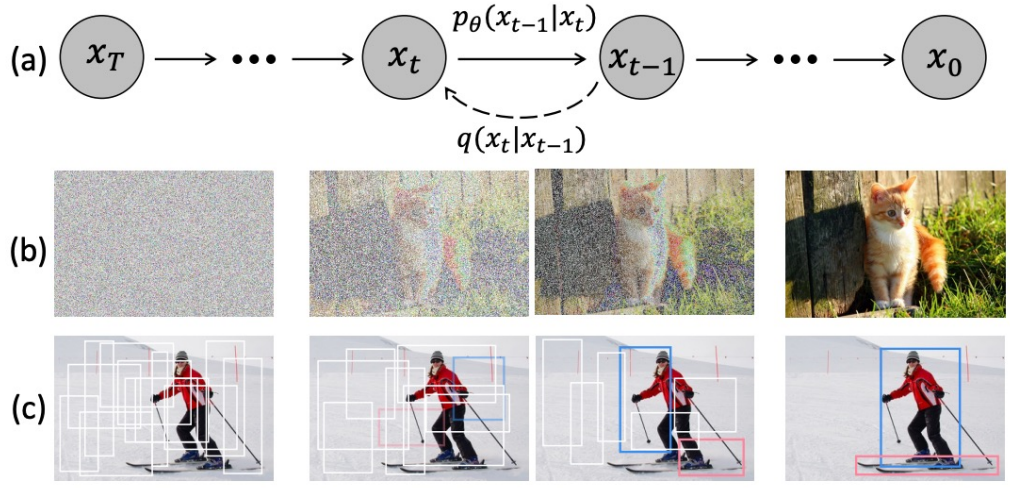
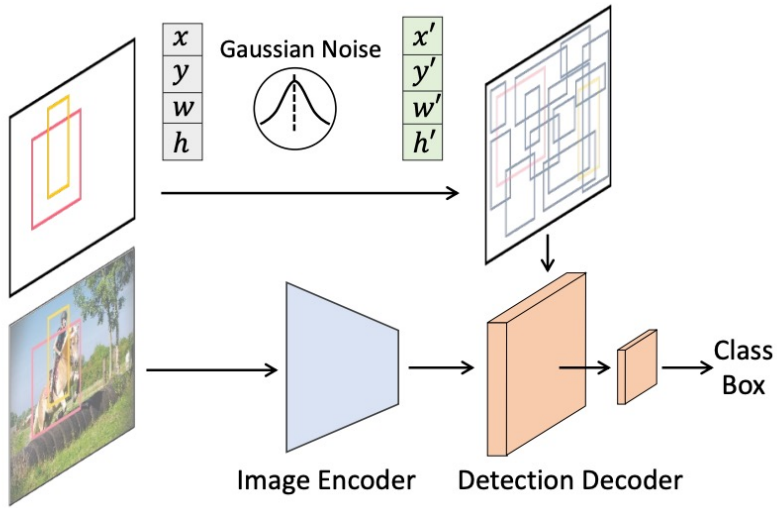


Figure 1. **Diffusion model for object detection**. (a) A diffusion model where q is the diffusion process and p_θ is the reverse process. (b) Diffusion model for image generation task. (c) We propose to formulate object detection as a denoising diffusion process from noisy boxes to object boxes.



Chen et al. DiffusionDet: Diffusion Model for Object Detection. ICCV 2023.

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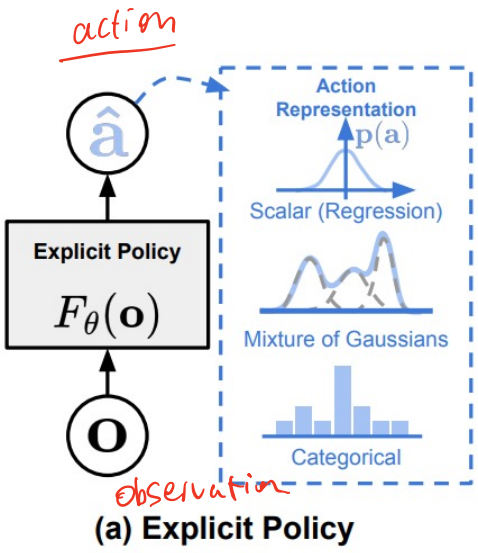
Planning and Control



(a) Explicit Policy

Chi et al. Diffusion Policy: Visuomotor Policy Learning via Action Diffusion. RSS 2023.

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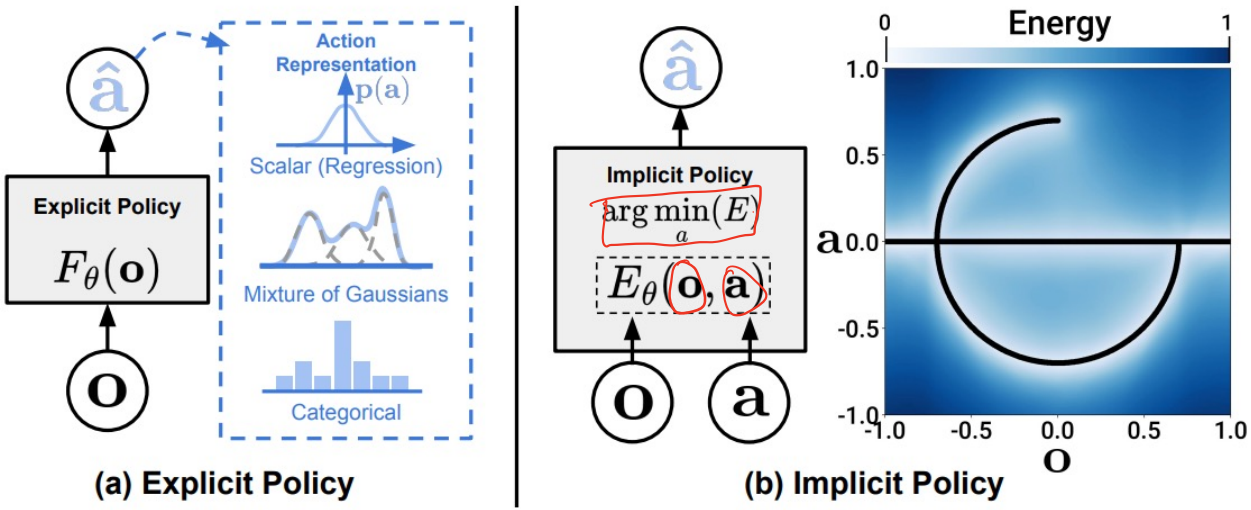

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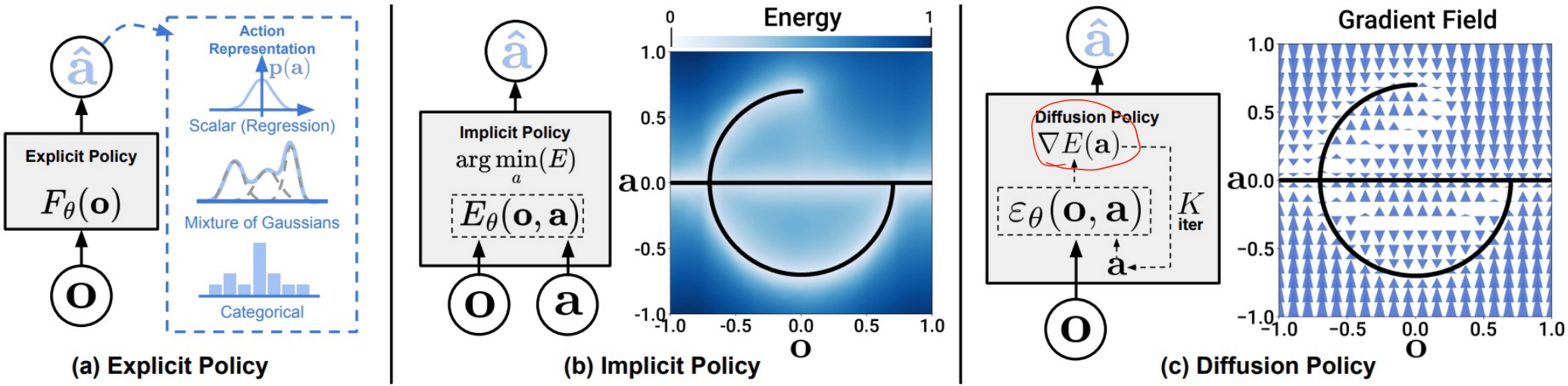
Planning and Control



Chi et al. Diffusion Policy: Visuomotor Policy Learning via Action Diffusion. RSS 2023.

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Summary: DL for Structured Outputs

- Expanding the output dimension has limitations.

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Summary: DL for Structured Outputs

- Expanding the output dimension has limitations.
- Requires us thinking about deep generative models.
  - Graphical models
  - Autoregressive
  - Energy-based
  - Normalizing flow
  - Diffusion

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Summary: DL for Structured Outputs

- Expanding the output dimension has limitations.
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  - Autoregressive
  - Energy-based
  - Normalizing flow
  - Diffusion
- Understand relations, pros and cons.
- Application in embodied environments.

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Module 2:
3D Vision and Mapping

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The World is 3D

- We have previously focused on using 2D images as input.

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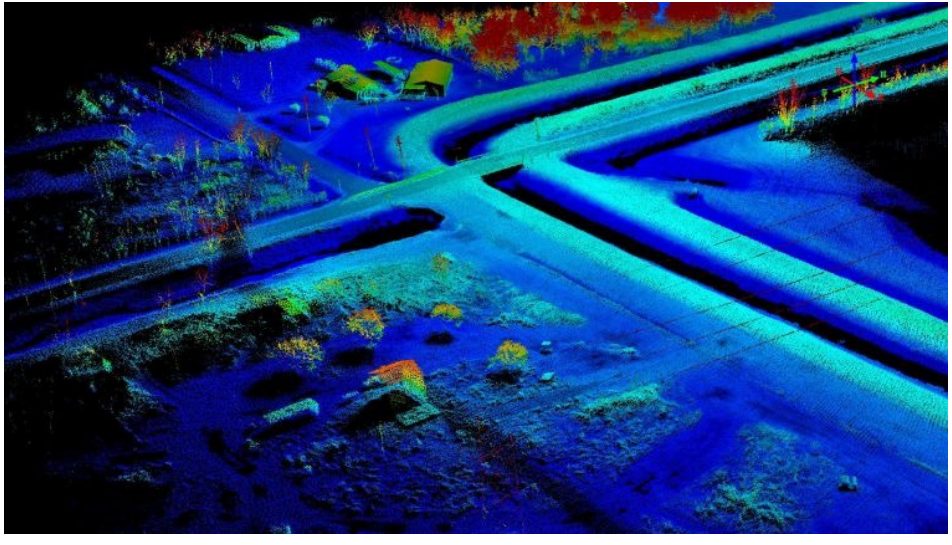
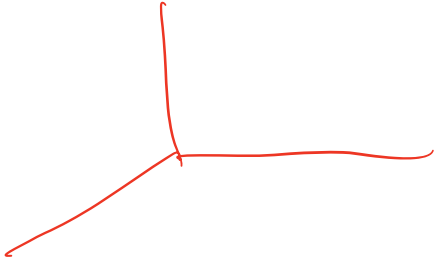
The World is 3D

- We have previously focused on using 2D images as input.
- But, the world is 3D. Many non-rigid in 2D becomes rigid in 3D. There are also a wide range of 3D sensors.
- Stereo (our binocular vision), infrared camera, LiDAR, radar, etc.

LiDAR

10k. (x, y, z) intensity.





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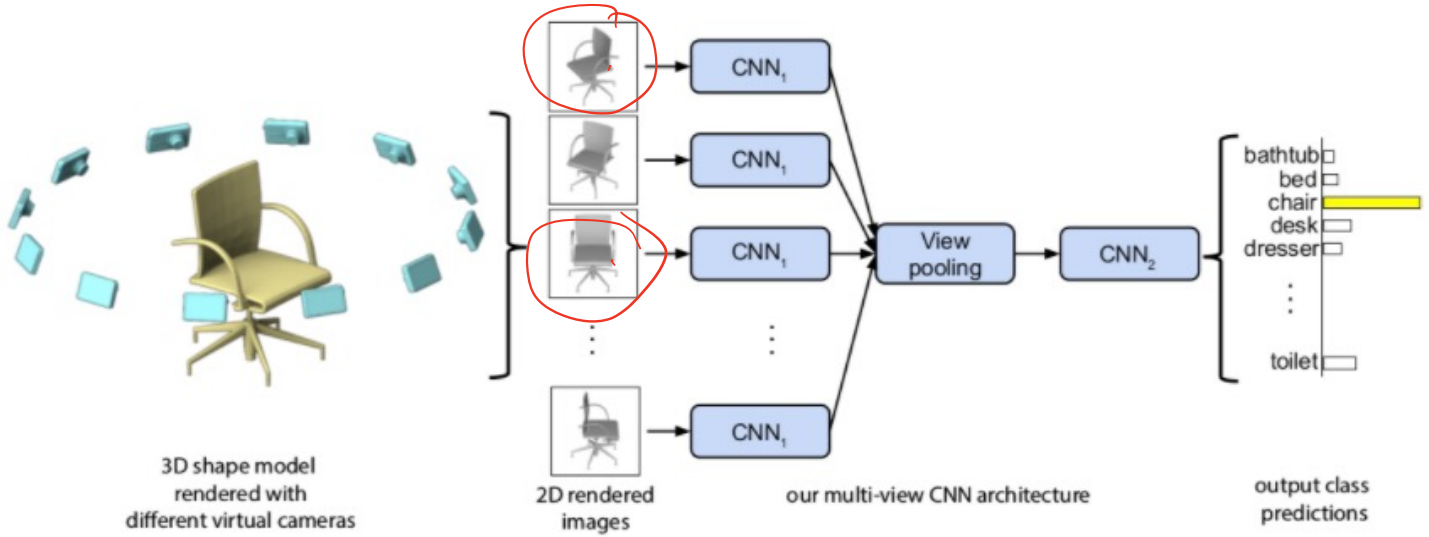
Multi-View CNN

- Treat it as a 2D problem.
- Aggregate the views together with a max-pooling layer.



Su et al., Multi-view convolutional neural networks for 3D shape recognition, ICCV 2015.

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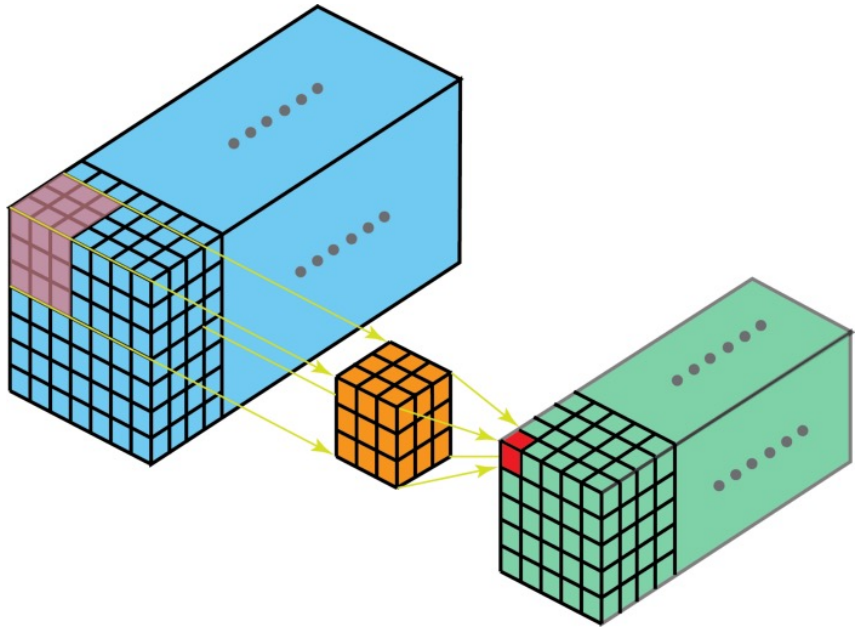
3D Convolution on Voxels

- 3D convolution on occupancy voxels.
- This can be expensive (memory + compute).



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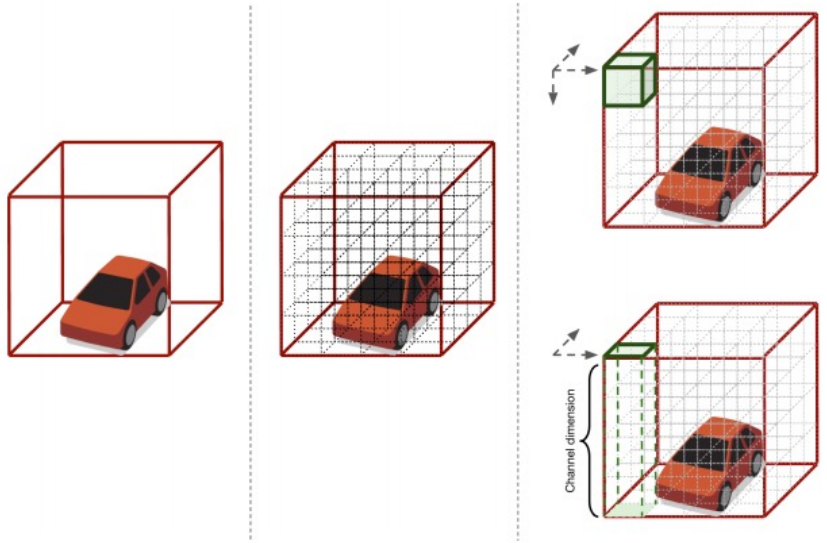
Bird's Eye View (BEV) Voxel

- Treat the z-dimension as different channels.



Zhang et al. Efficient convolutions for real-time semantic segmentation of 3D point clouds, 3DV 2018.1

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Bird's Eye View (BEV) Voxel

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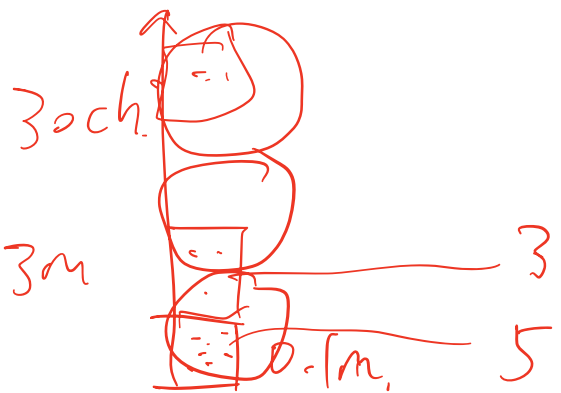

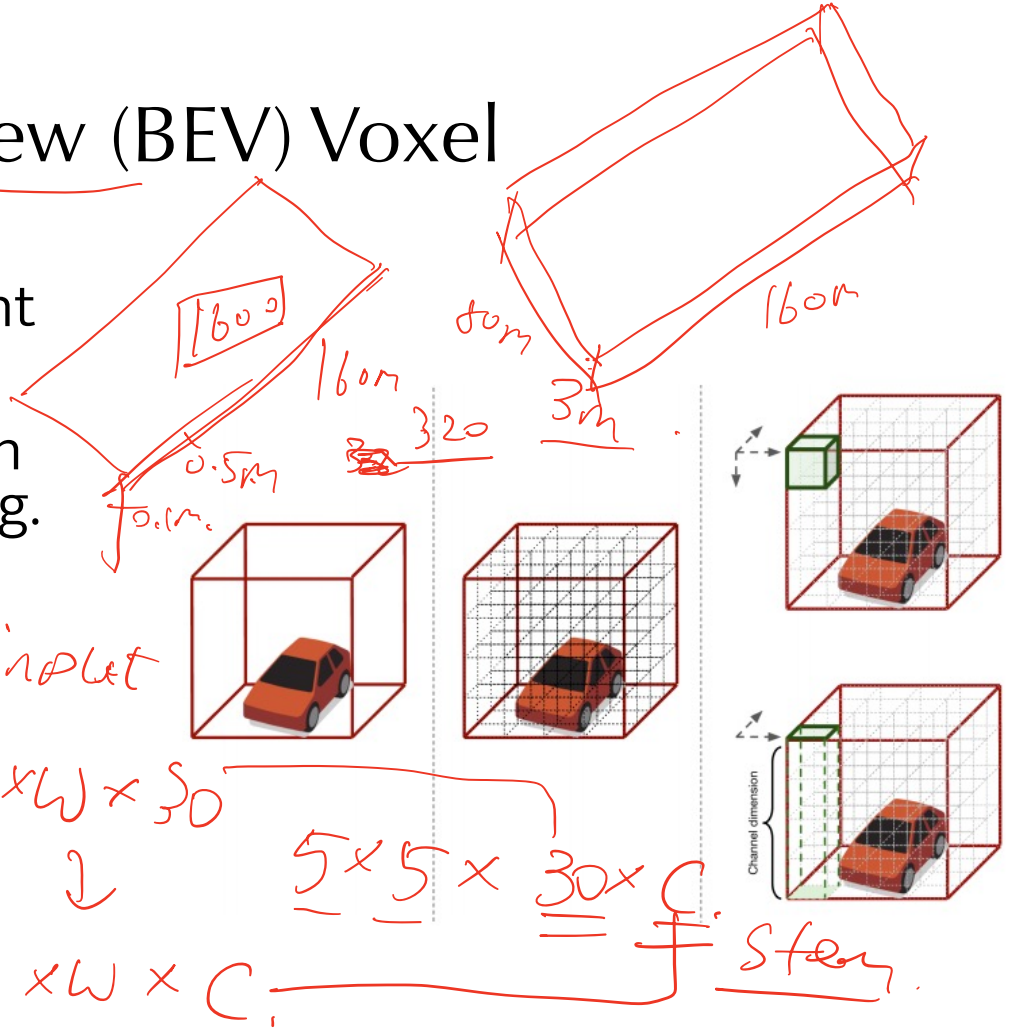

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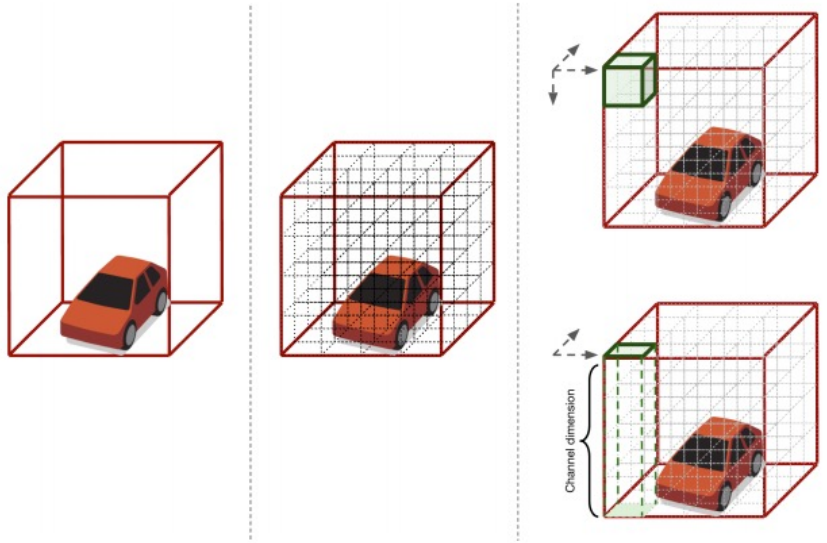

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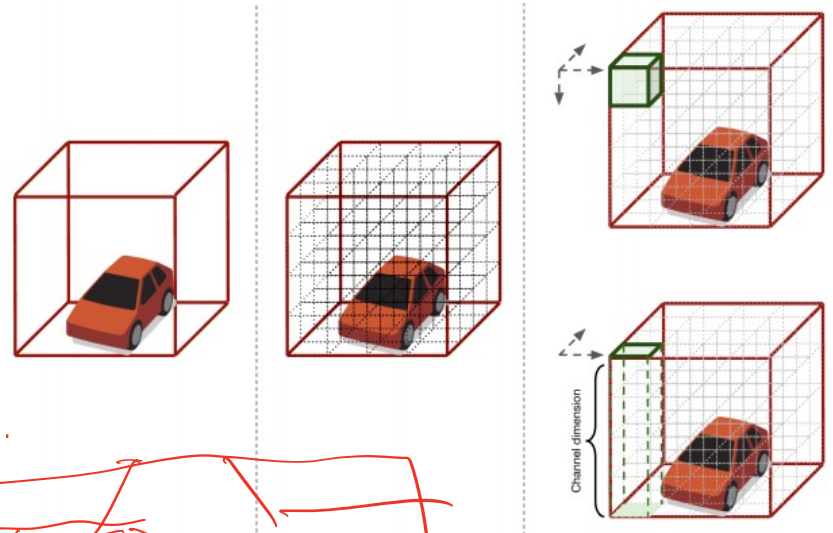

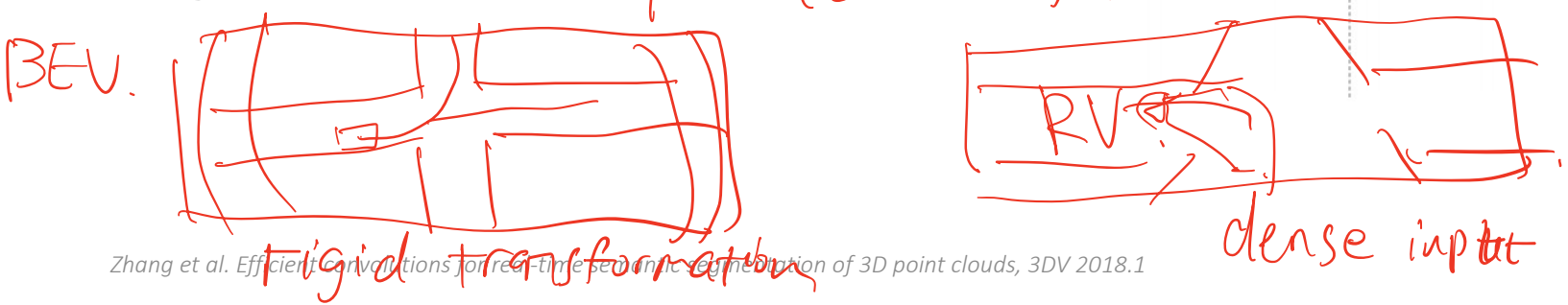

Zhang et al. Efficient convolutions for real-time semantic segmentation of 3D point clouds, 3DV 2018.1

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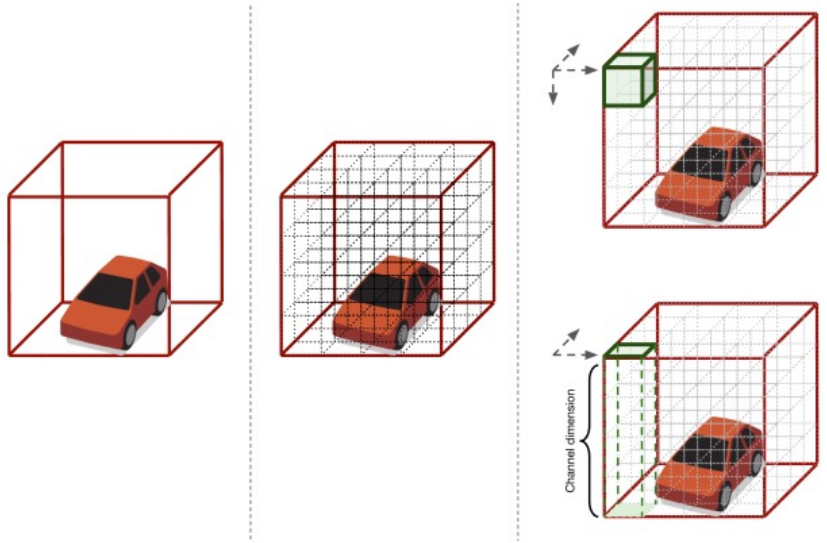
- 140m x 3m (very thin!)
- Transformation in x-y plane is still rigid.
- Bird's eye view: Top down representation of the scene (rigid, sparse) vs. Range view (non-rigid, dense)



Zhang et al. Efficient convolutions for real-time semantic segmentation of 3D point clouds, 3DV 2018.1

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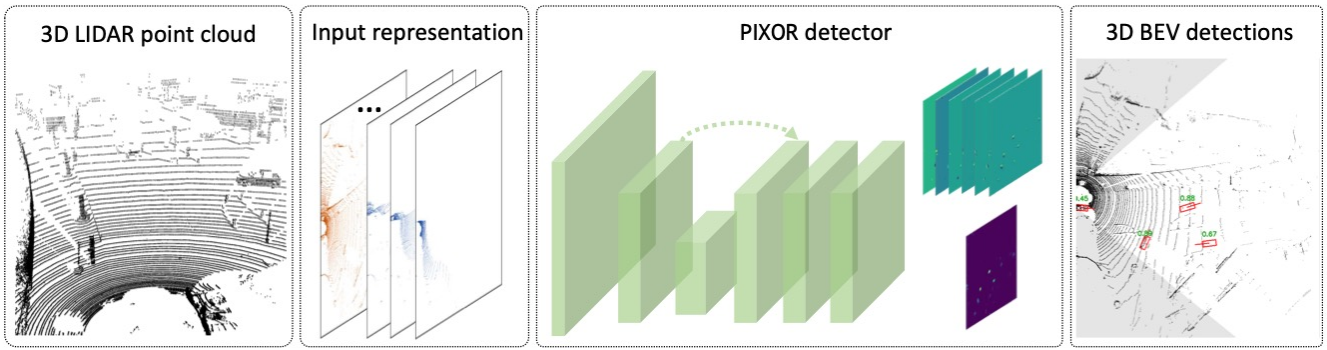
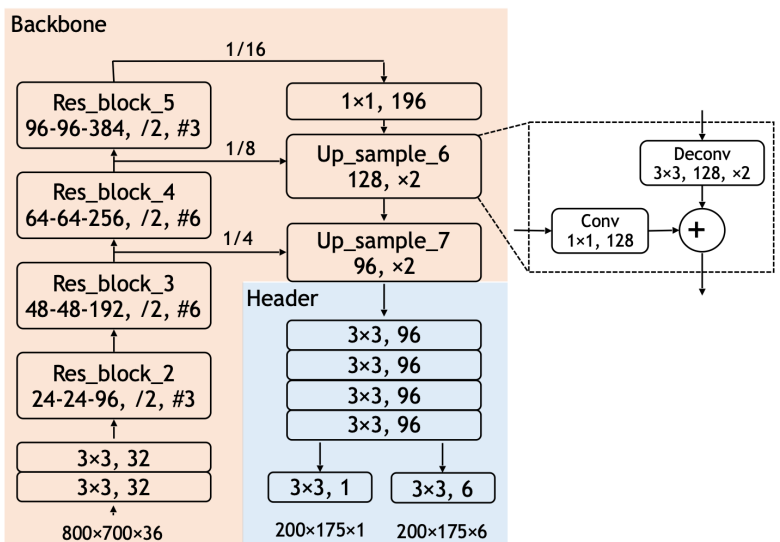
PIXOR

- Bird's eye view object detection.





Yang et al. PIXOR: Real-time 3D Object Detection from Point Clouds. CVPR 2018.

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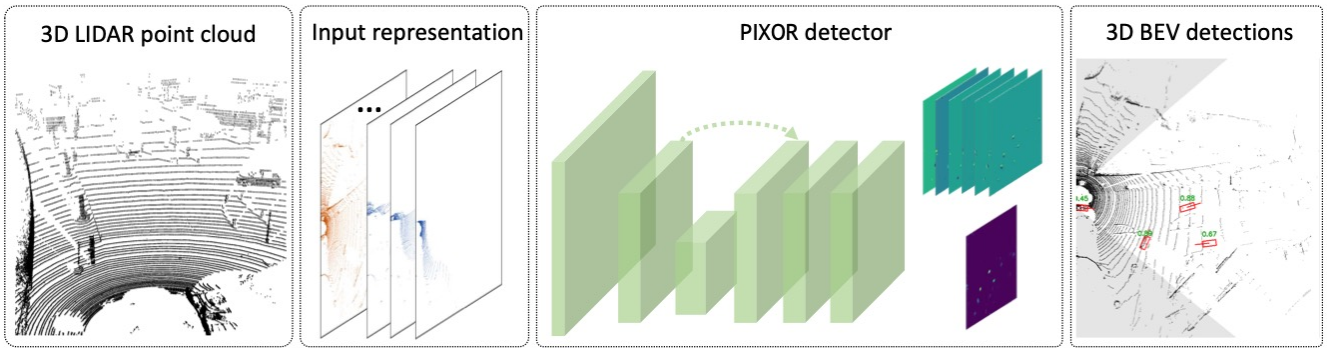
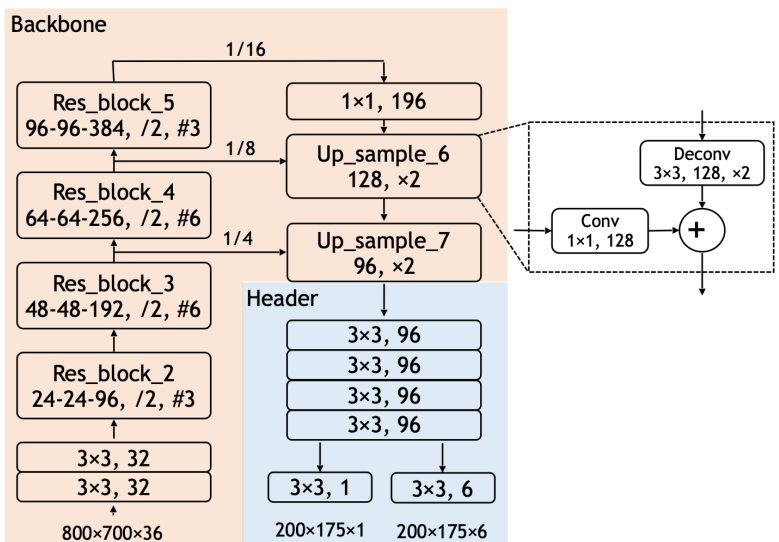
PIXOR

- Bird's eye view object detection.
- Used the ResNet + FPN network single-stage architecture.





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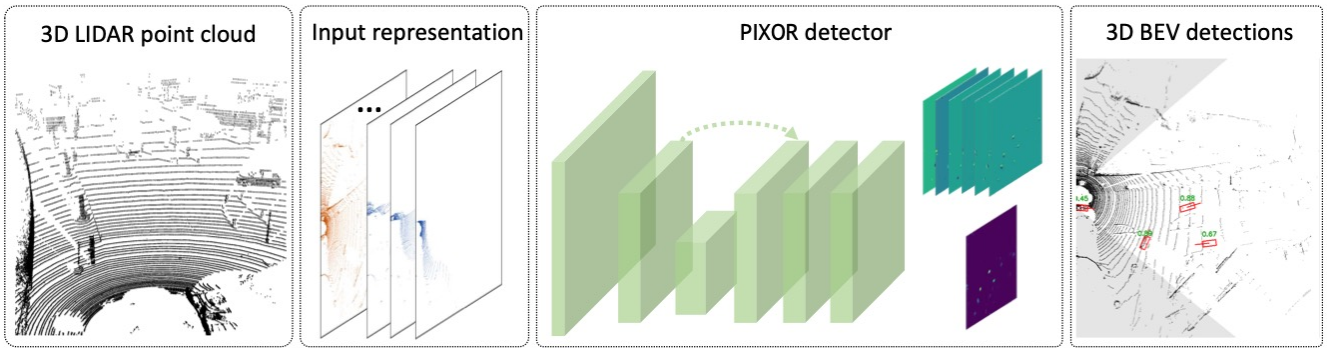

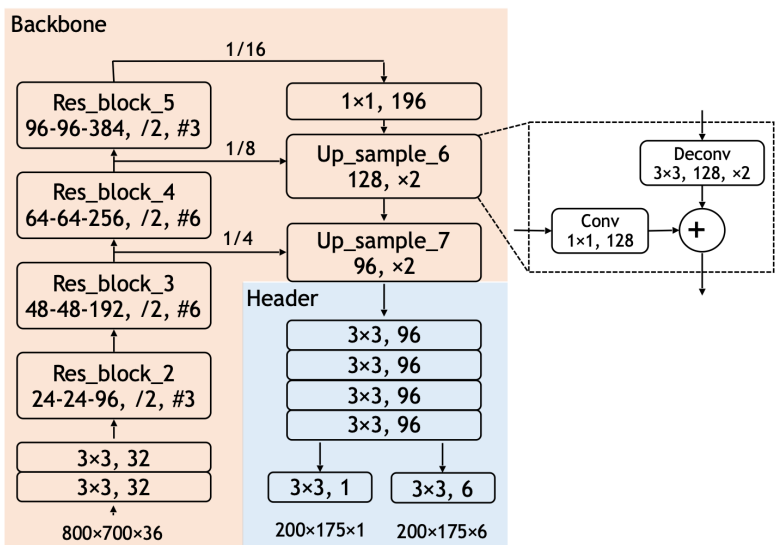

Yang et al. PIXOR: Real-time 3D Object Detection from Point Clouds. CVPR 2018.

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PIXOR

- Bird's eye view object detection.
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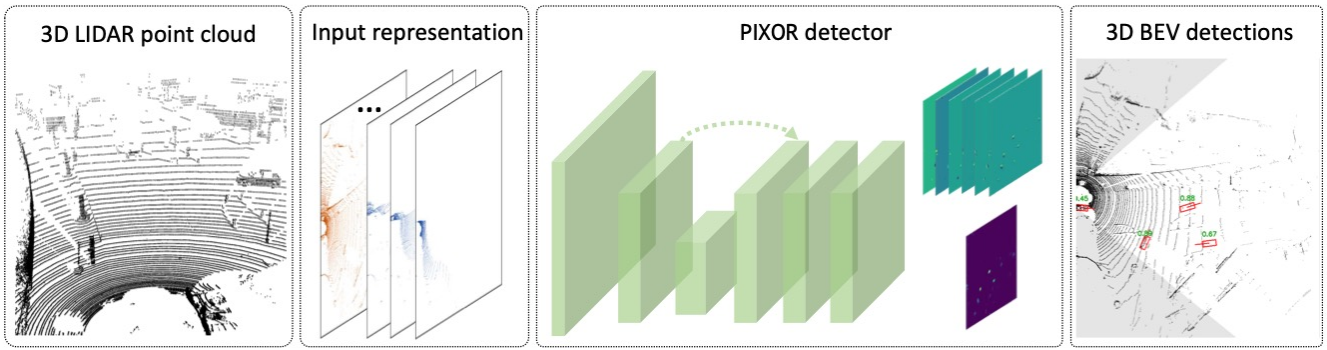
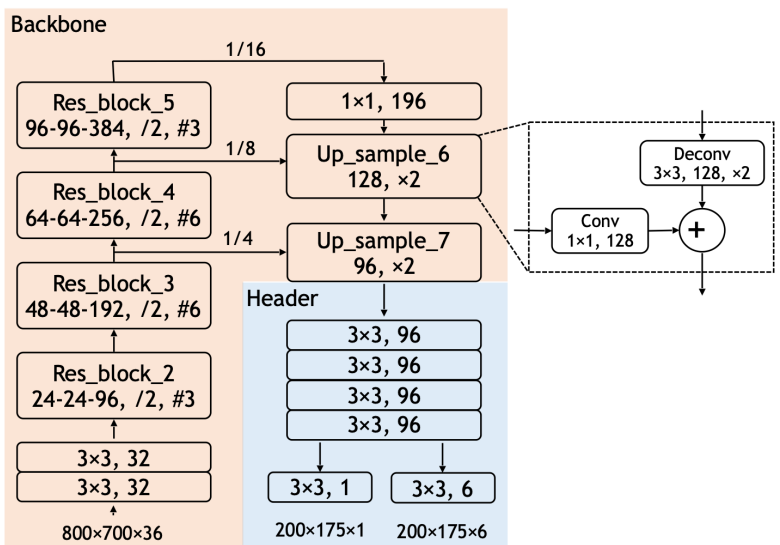
- Detection: Classification + regression cosθ,sinθ,dx,dy,w,l.
- First real-time 3D detection network.





Yang et al. PIXOR: Real-time 3D Object Detection from Point Clouds. CVPR 2018.

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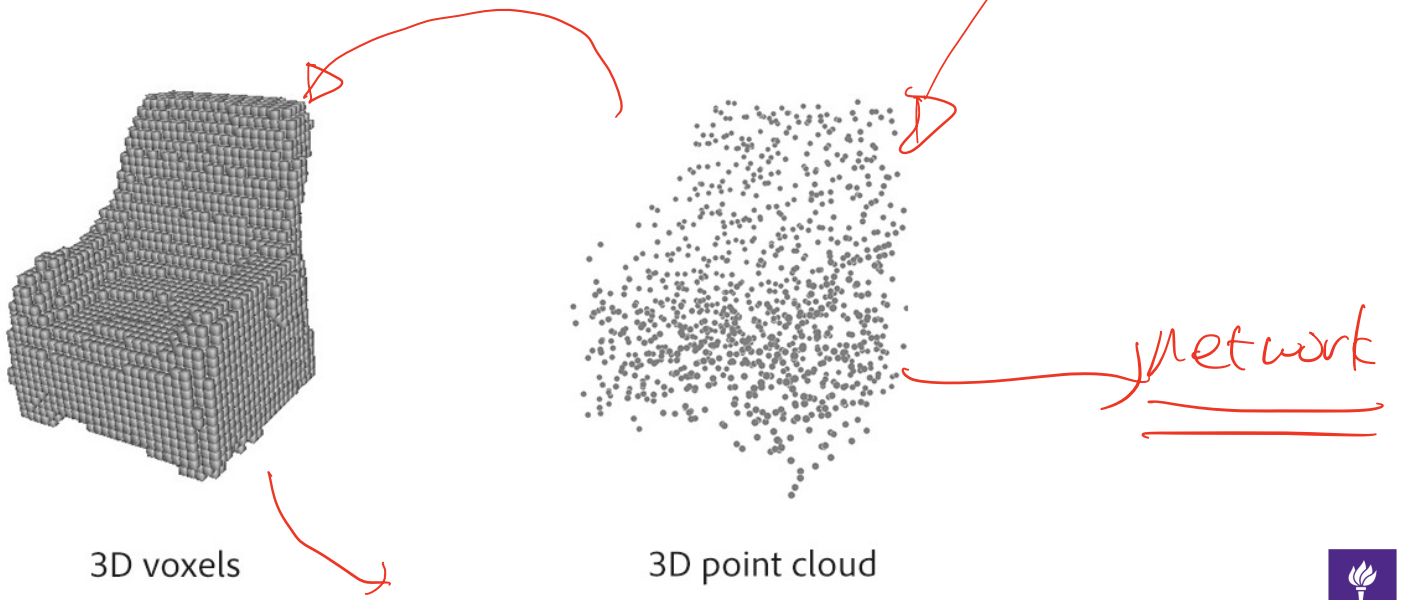
Point Cloud

samples of 3D world

- Point cloud is native for many 3D-depth sensors: RGBD sensor, LiDAR sensor, etc.



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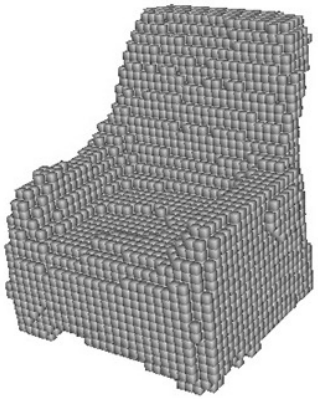
Point Cloud

- Point cloud is native for many 3D-depth sensors: RGBD sensor, LiDAR sensor, etc.
- List of 3D points: $[(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_N, y_N, z_N)]$

20000... or more.



3D voxels

3D point cloud

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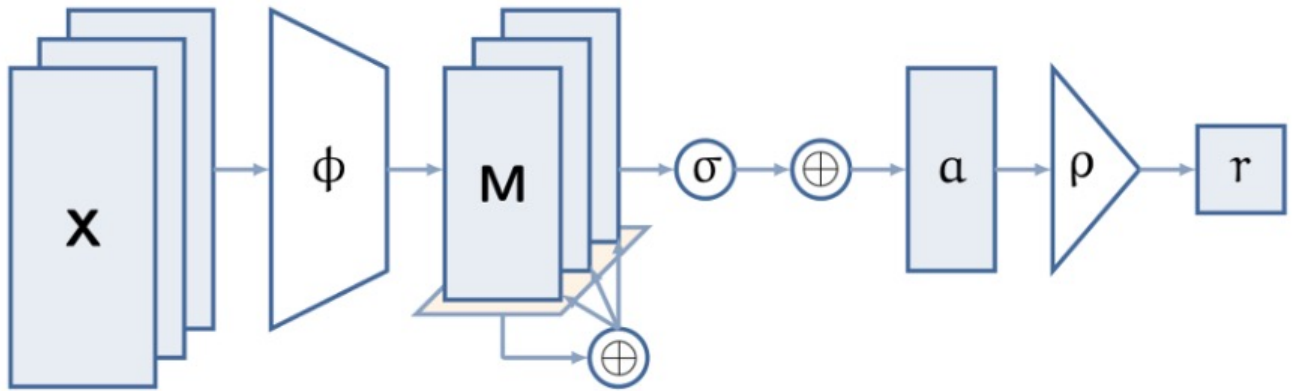
Permutation Invariance

- Point cloud is a set.



Zaheer et al. Deep Sets. NIPS 2017. https://argmax.ai/blog/setinvariance/

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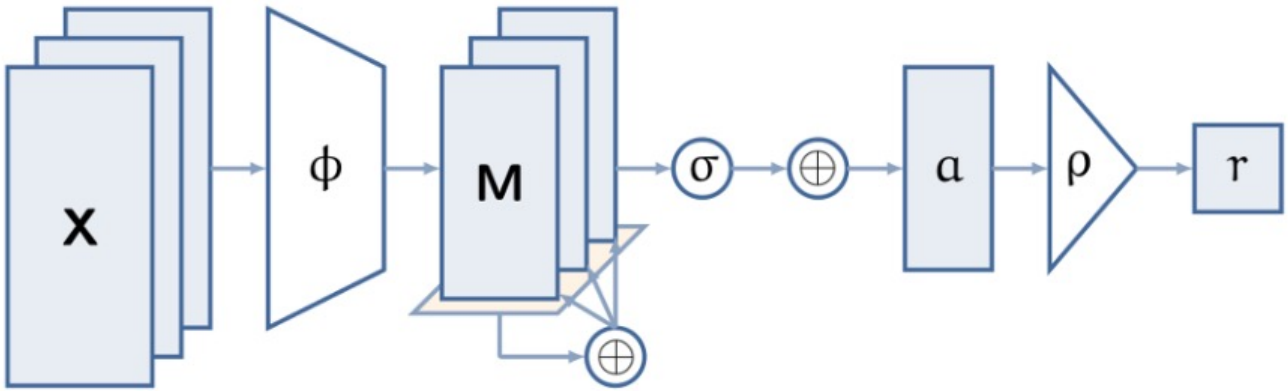
Permutation Invariance

- Point cloud is a set.
- Permutation does not affect the classification in the output.



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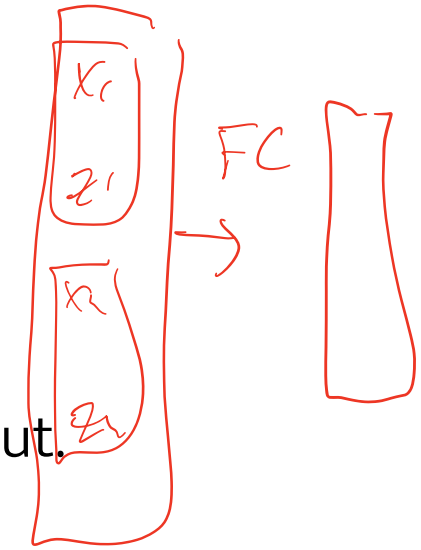
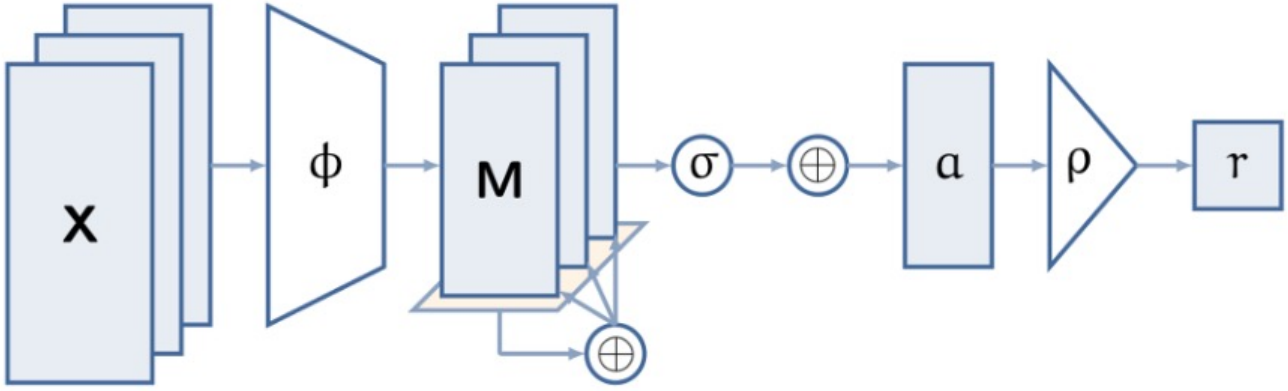


- Point cloud is a set.
- Permutation does not affect the classification in the output.
- What operations are permutation invariant?



Zaheer et al. Deep Sets. NIPS 2017. https://argmax.ai/blog/setinvariance/

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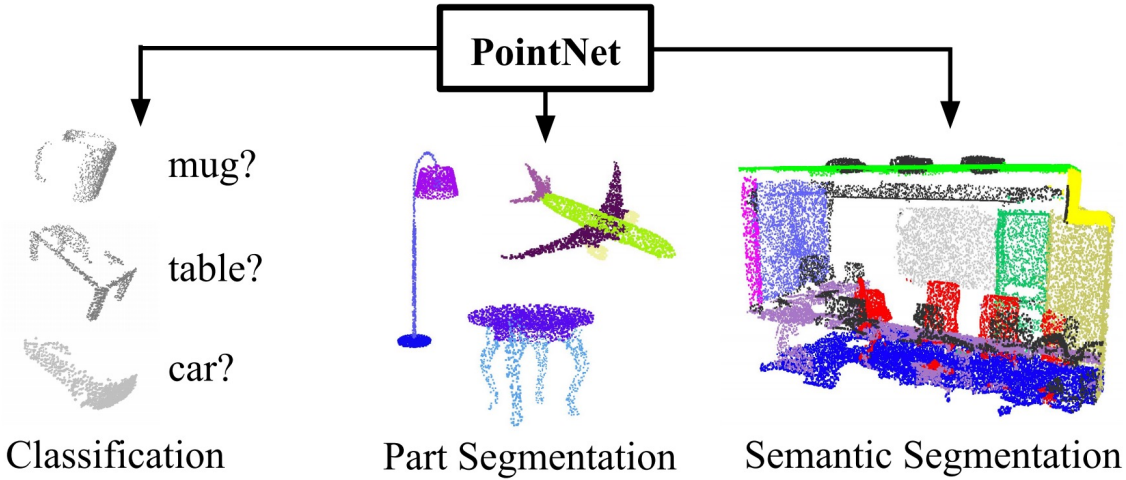
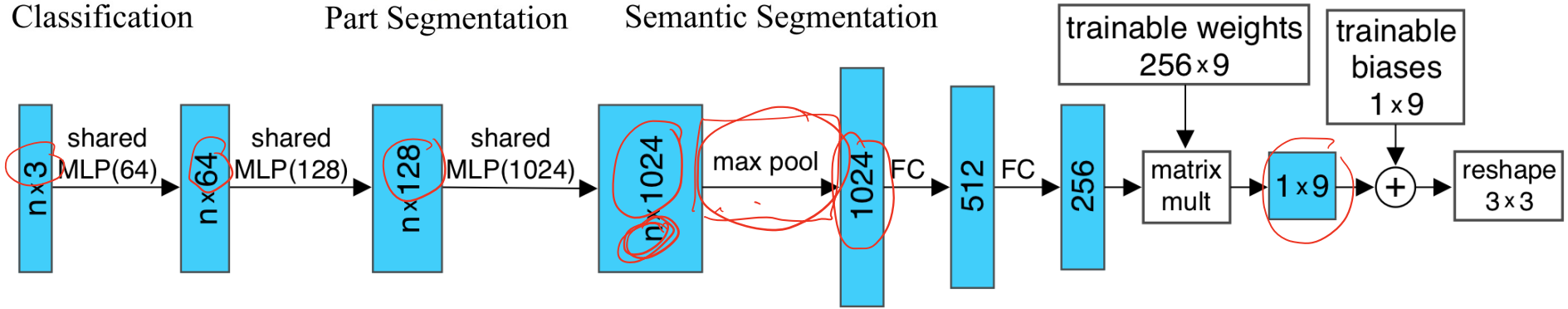
PointNet



- Apply an MLP on each point.
- Max pool the features across all points.



Qi et al., PointNet: Deep learning on point sets for 3D classification and segmentation, CVPR 2017.

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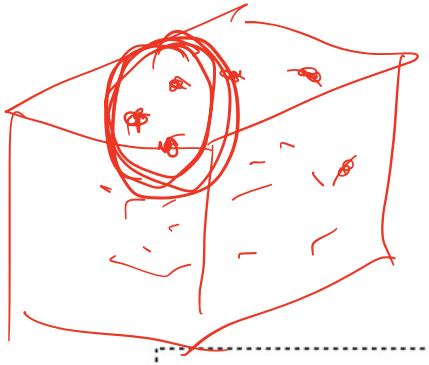
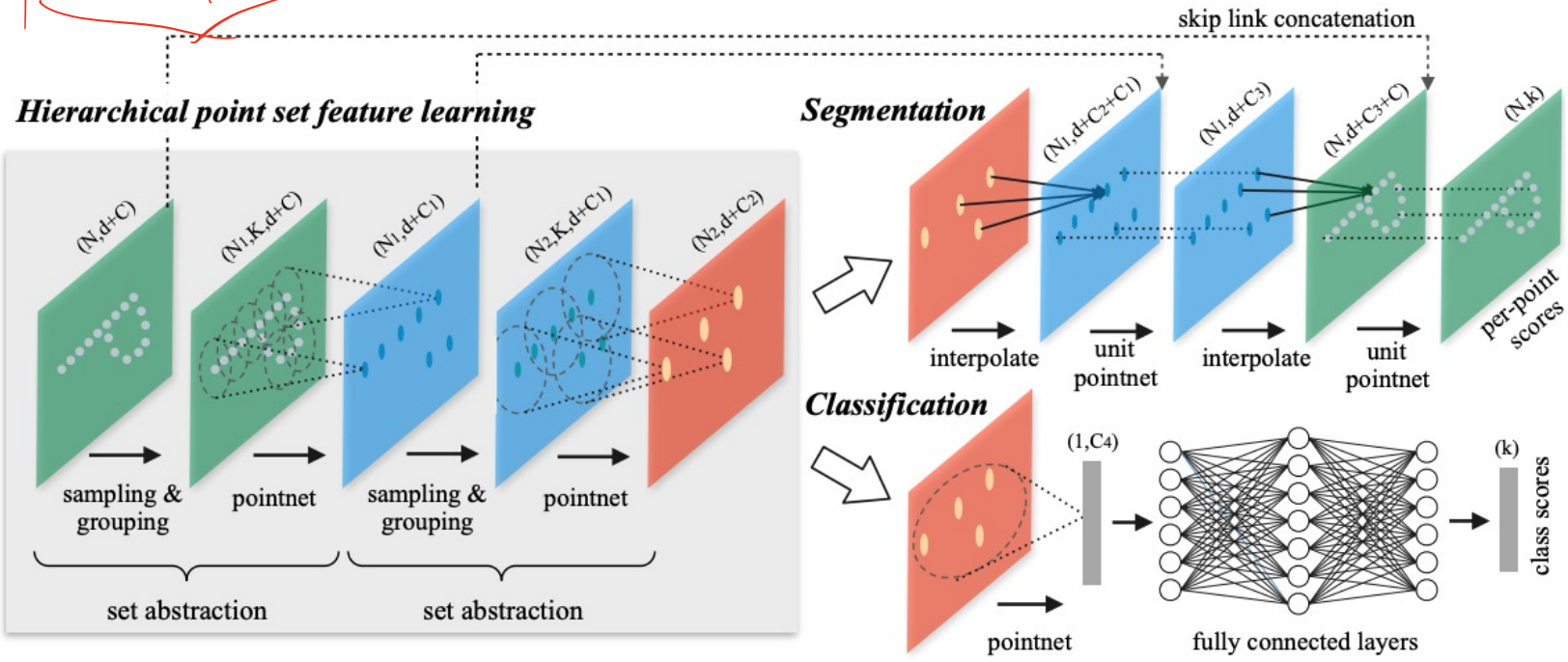


PointNet++



Qi et al., PointNet++: Deep hierarchical feature learning on point sets in a metric space, NIPS 2017.

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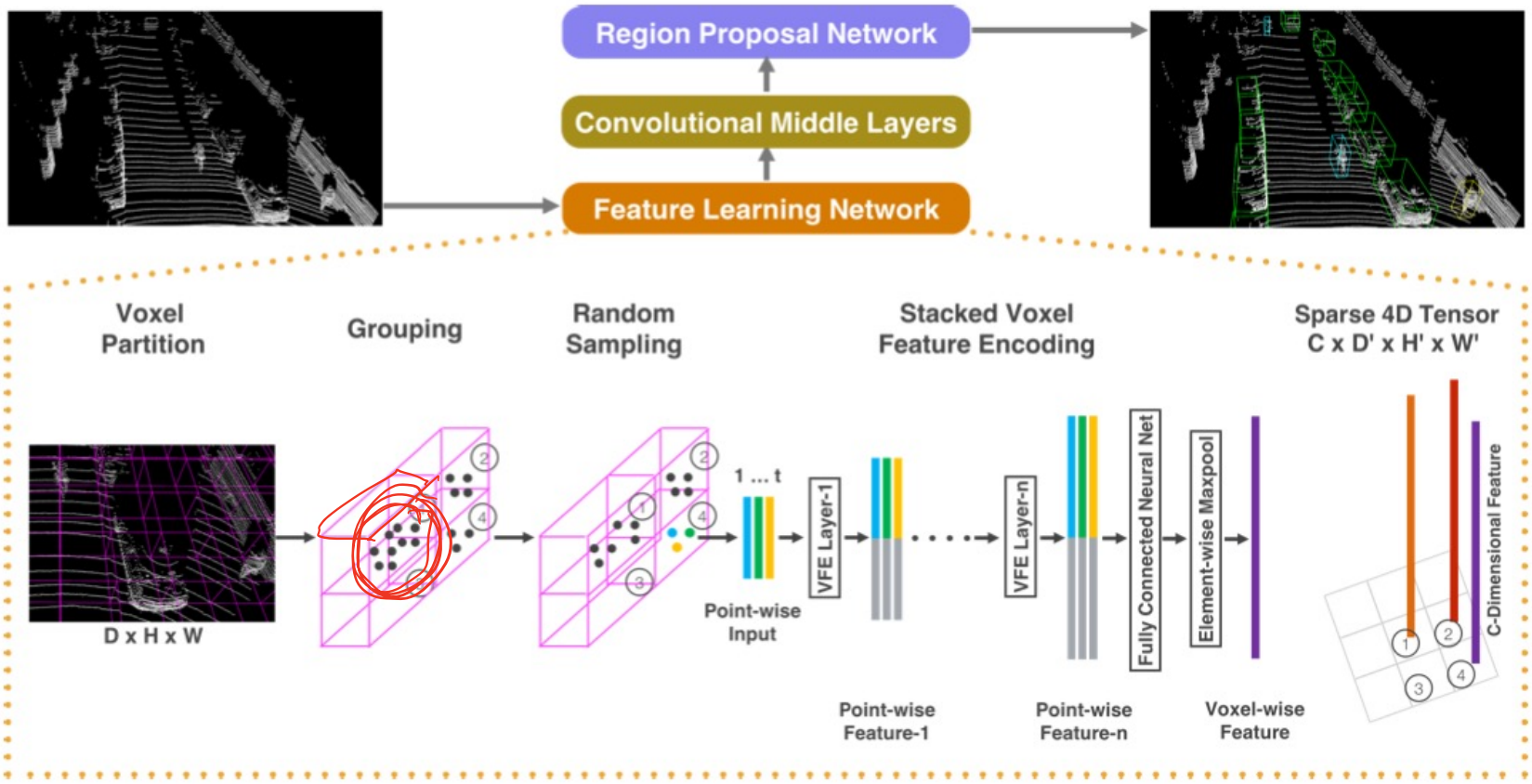
VoxelNet



Zhou and Tuzel. Voxelnet: End-to-end learning for point cloud based 3d object detection. CVPR 2018.

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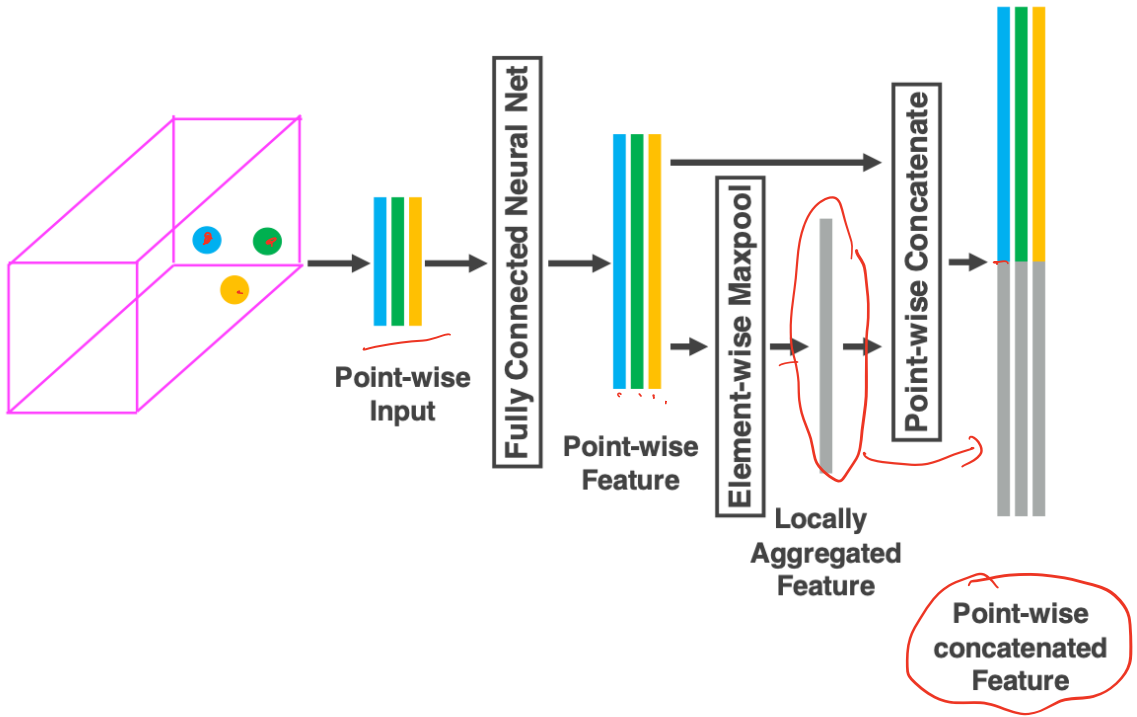
VoxelNet

- Zooming inside voxel feature encoding (VFE)



Zhou and Tuzel. Voxelnet: End-to-end learning for point cloud based 3d object detection. CVPR 2018.

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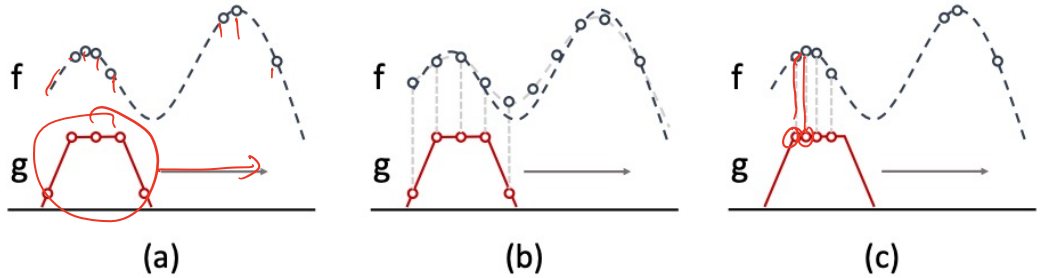
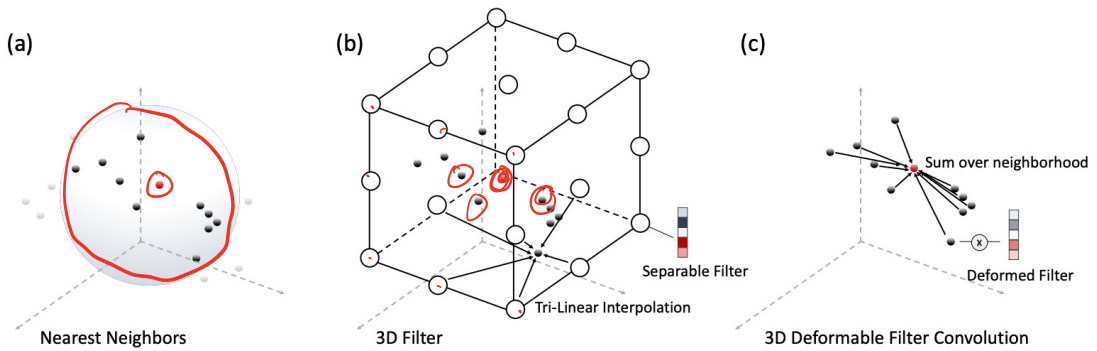
Deformable Convolution in Point Cloud

- Can we convolve a point cloud with a spatially defined kernel function?





Xiong et al. Deformable Filter Convolution for Point Cloud Reasoning. 2019.

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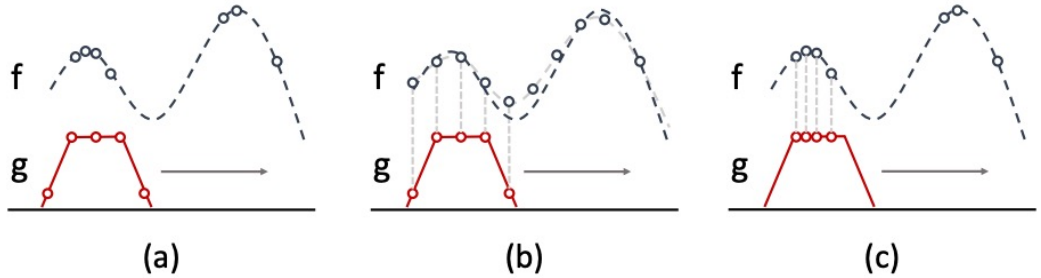
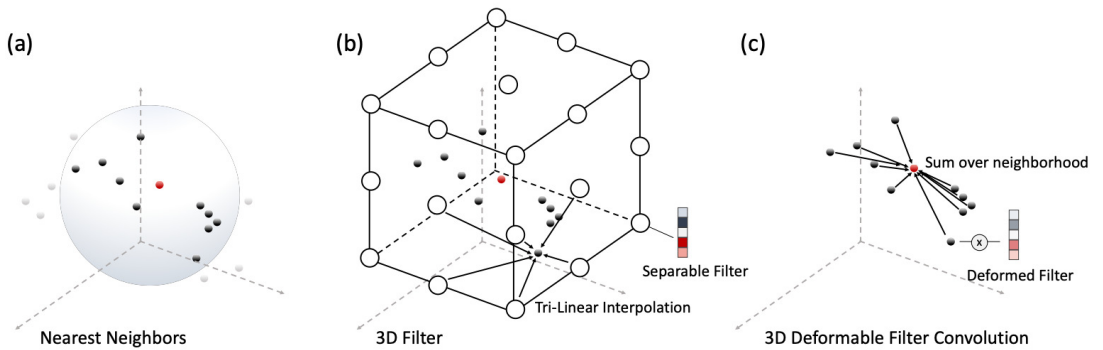
Deformable Convolution in Point Cloud

- Can we convolve a point cloud with a spatially defined kernel function?
- Resample the kernel at the point location.





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Deformable Convolution in Point Cloud

- Can we convolve a point cloud with a spatially defined kernel function?
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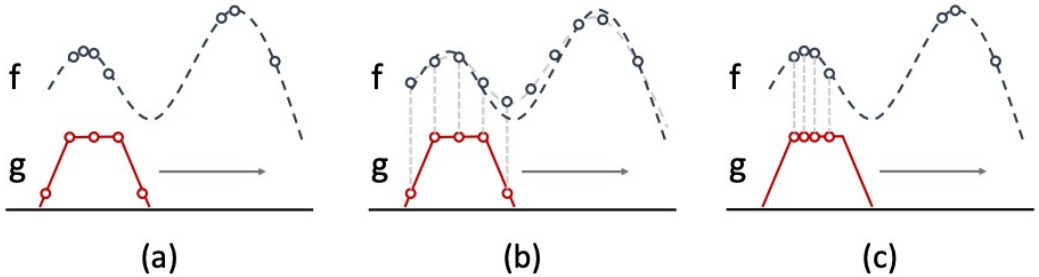
- Compute the weighted sum around a neighborhood.



Xiong et al. Deformable Filter Convolution for Point Cloud Reasoning. 2019.

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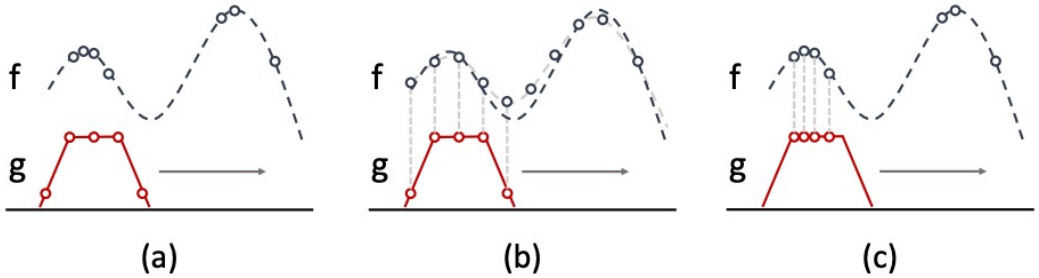
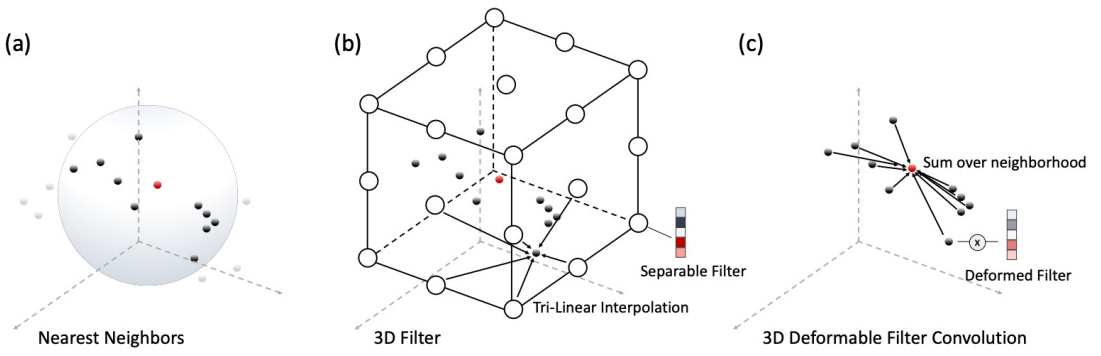
(x, y, z).

Deformable Convolution in Point Cloud

- Can we convolve a point cloud with a spatially defined kernel function?
- Resample the kernel at the point location.
- Compute the weighted sum around a neighborhood.
- Translational equivariance.





Xiong et al. Deformable Filter Convolution for Point Cloud Reasoning. 2019.

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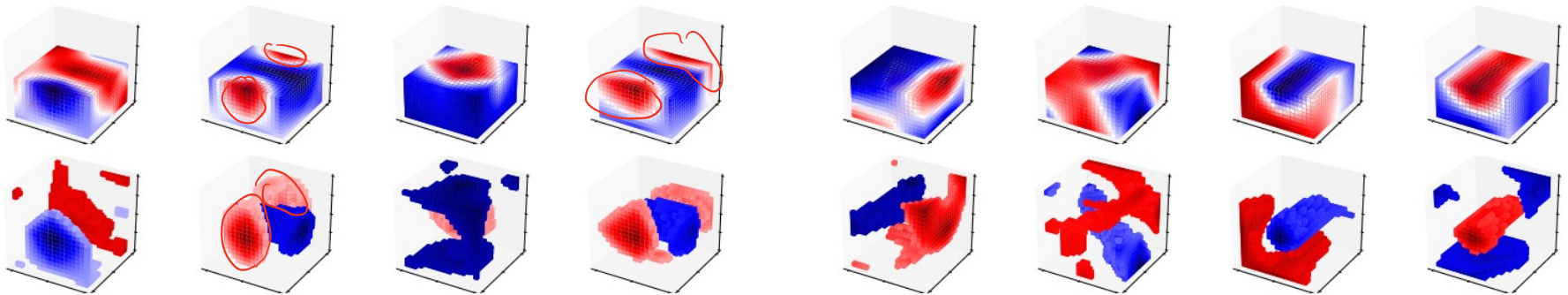
3D Filters

- Visualizing 3D convolution kernels.



Xiong et al. Deformable Filter Convolution for Point Cloud Reasoning. 2019.

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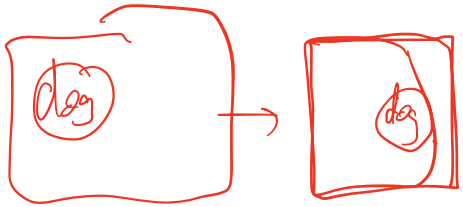
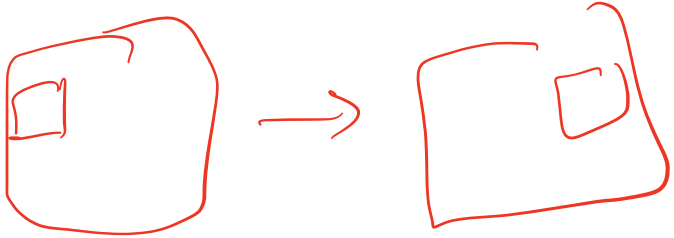
More on Equivariance



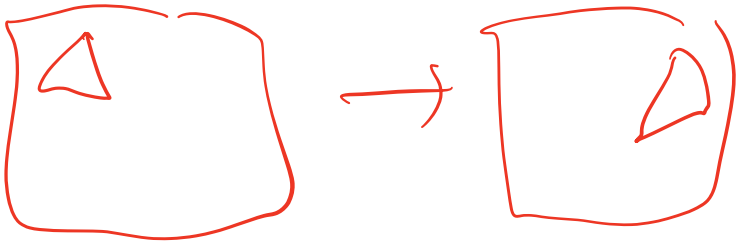
- Equivariance is a symmetry property on functions and transformations.

$$f(\mathcal{T}x) = \mathcal{T}'f(x)$$

translation



rotation



H x W x C.

rotate.

new feature map.

More on Equivariance

- Equivariance is a symmetry property on functions and transformations.

$$f(\mathcal{T}x) = \mathcal{T}' f(x)$$

- Useful property to constrain the hidden representation space.

data efficiency, ⟷ Compute efficiency (not always).

More on Equivariance

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- 3D convolution handles translational equivariance.
- What about rotation?

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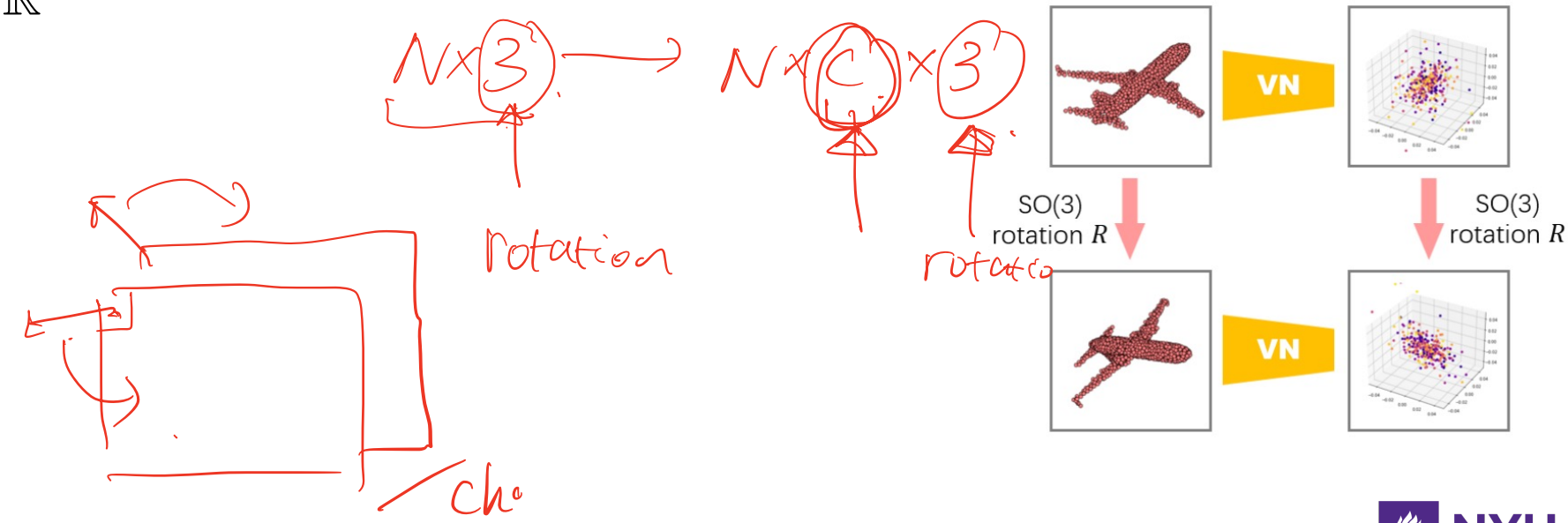
3D Rotational Equivariance

- For object centered point clouds, rotation are more important.
- Imagine every feature has a vector / arrow.

$V \in \mathbb{R}^{N \times C \times 3}$



Deng et al. Vector Neurons: A General Framework for SO(3)-Equivariant Networks. ICCV 2021.

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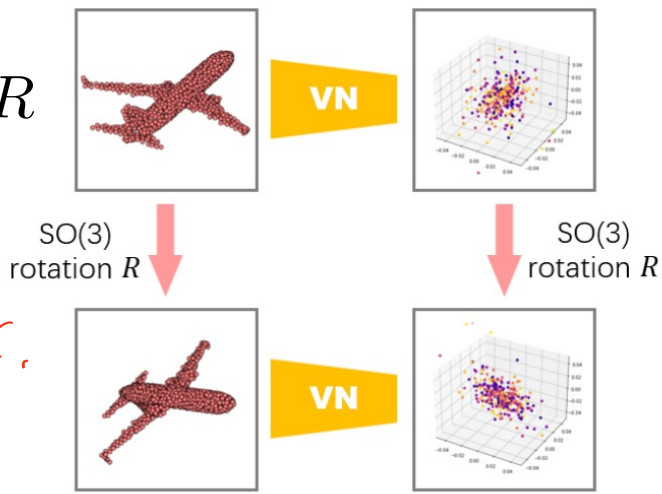
linear. layer.

$=$ $\neq$

rotation matrix.



3D Rotational Equivariance

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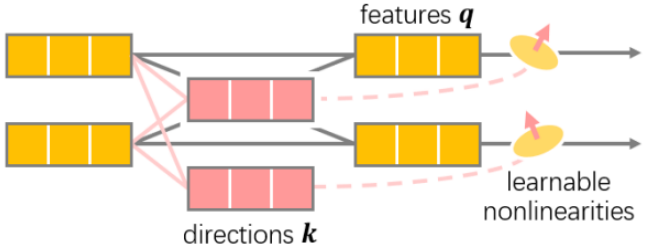
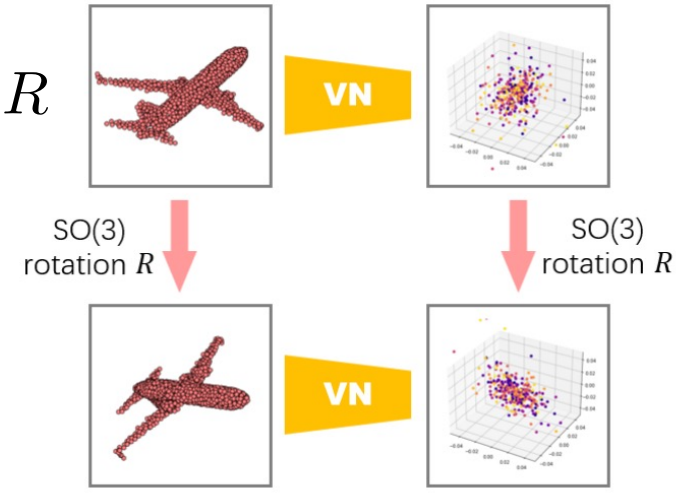
- Imagine every feature has a vector / arrow.
  $V \in \mathbb{R}^{N \times C \times 3}$
- Linear layers are already good $W(VR) = (WV)R$
- Need a different nonlinearity

$$\boldsymbol{q} = \mathbf{W}\boldsymbol{V}, \quad \boldsymbol{k} = \mathbf{U}\boldsymbol{V}$$

$$\boldsymbol{v}' = \begin{cases} \boldsymbol{q} & \text{if } \langle \boldsymbol{q}, \boldsymbol{k} \rangle \geqslant 0 \\ \boldsymbol{q} - \left\langle \boldsymbol{q}, \frac{\boldsymbol{k}}{\|\boldsymbol{k}\|} \right\rangle \frac{\boldsymbol{k}}{\|\boldsymbol{k}\|} & \text{otherwise,} \end{cases}$$





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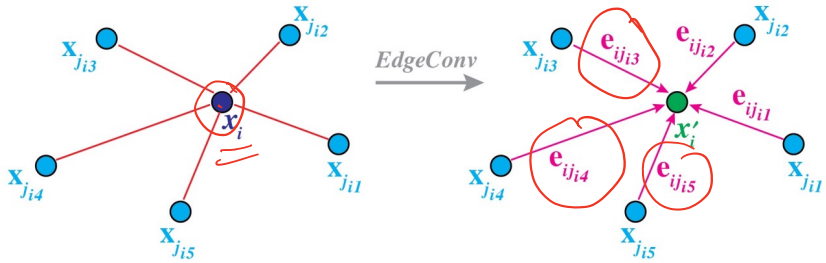
More General Equivariance

- Making VN also translational equivariant.

Edge. Conv → VN

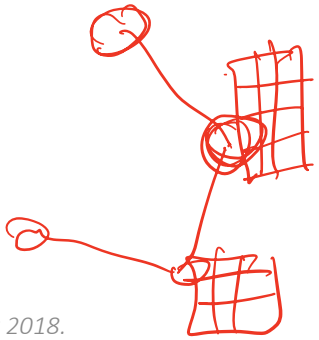
translational eq.

rot eq.



[Wang et al. 2019]

- For higher order equivariance (e.g. matrices, tensors), check out Tensor Field Networks.
- Convert features into spherical harmonics.



Wang et al. Dynamic Graph CNN for Learning on Point Clouds. TOG 2019.
Thomas et al. Tensor field networks: Rotation- and translation-equivariant neural networks for 3D point clouds. arXiv 2018.

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Challenges and Tradeoffs

- Computational and memory efficiency vs. training data efficiency

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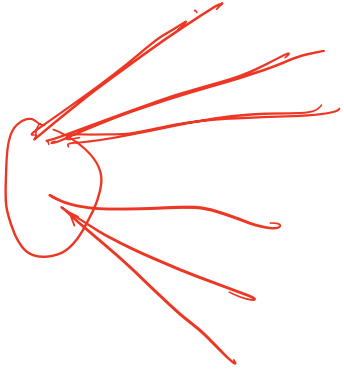
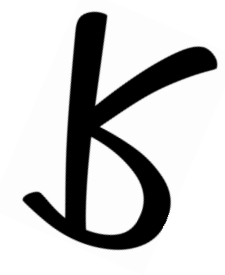
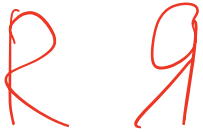
- Mental rotation: Rotate in the latent space may not be as simple as a geometric transformation. The skill may still be experience driven.







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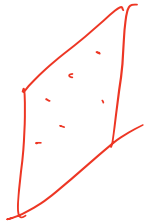

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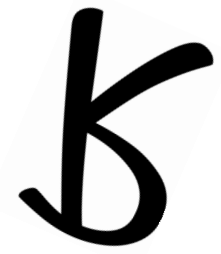

- Neuroscience evidence: No spatial equivariance in connections. The LGN performs “pre-pre-training” from simulating retina waves [Blankenship & Feller, 2009].

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Multi-Sensor Fusion

- LiDAR is precise in depth perception, but the point cloud format is sparse and non-uniform (dense around the ego-car and sparse in long distance.)

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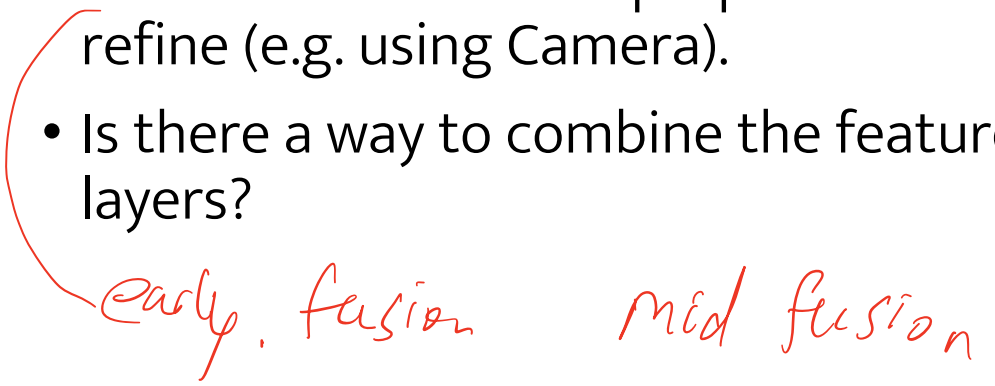
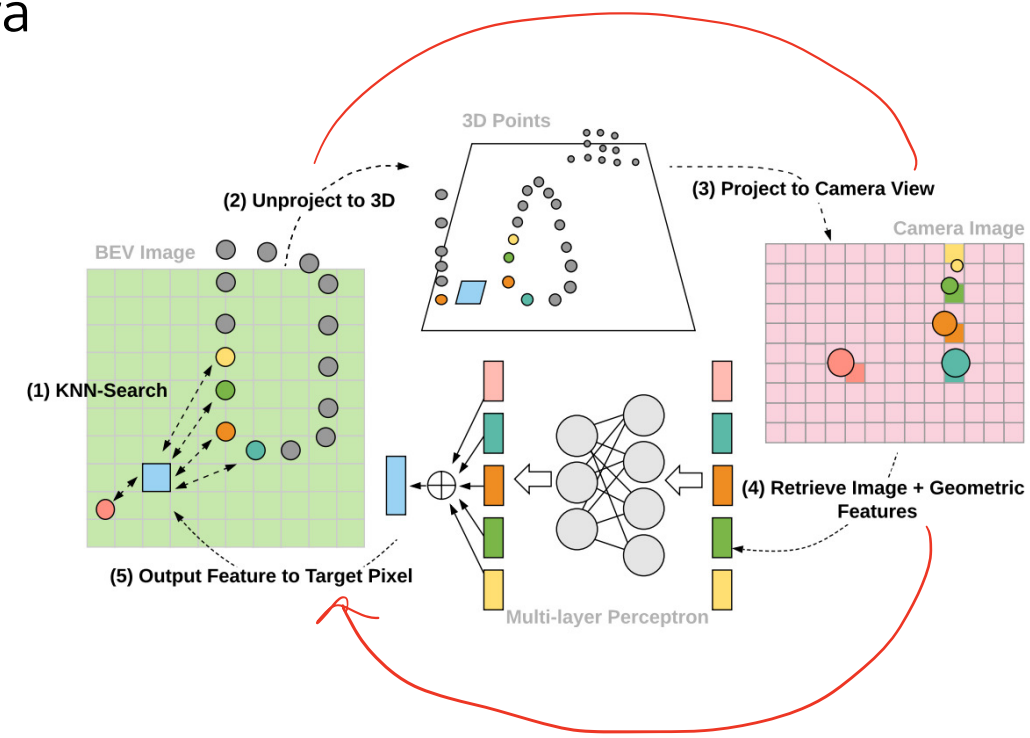
- Late fusion: Generate proposals from one branch (e.g. LiDAR) and refine (e.g. using Camera).
- Is there a way to combine the features from both modality in lower layers?



Camera-LiDAR Projection

- Unproject LiDAR points to camera view (i.e. Range View)
- Query the closest camera RGB features for each LiDAR point.
- For empty space in BEV, we can interpolate from neighboring points using kNN.
- Continuous Fusion: $h_i = \sum_j MLP([f_j, x_j - x_i])$.



Liang et al. Deep Continuous Fusion for Multi-Sensor 3D Object Detection. ECCV 2018.

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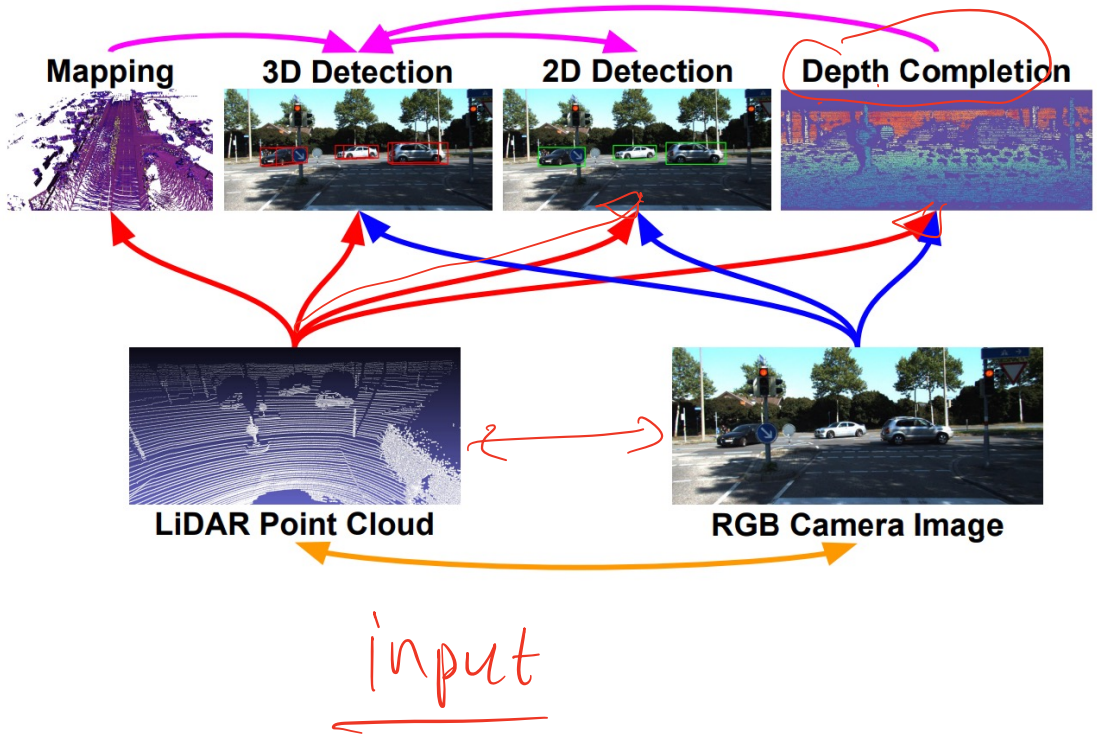
Supervised Dense Depth

- Drawback of continuous fusion: Sparse LiDAR can cause the fusion process to be less accurate. Relies on kNN.



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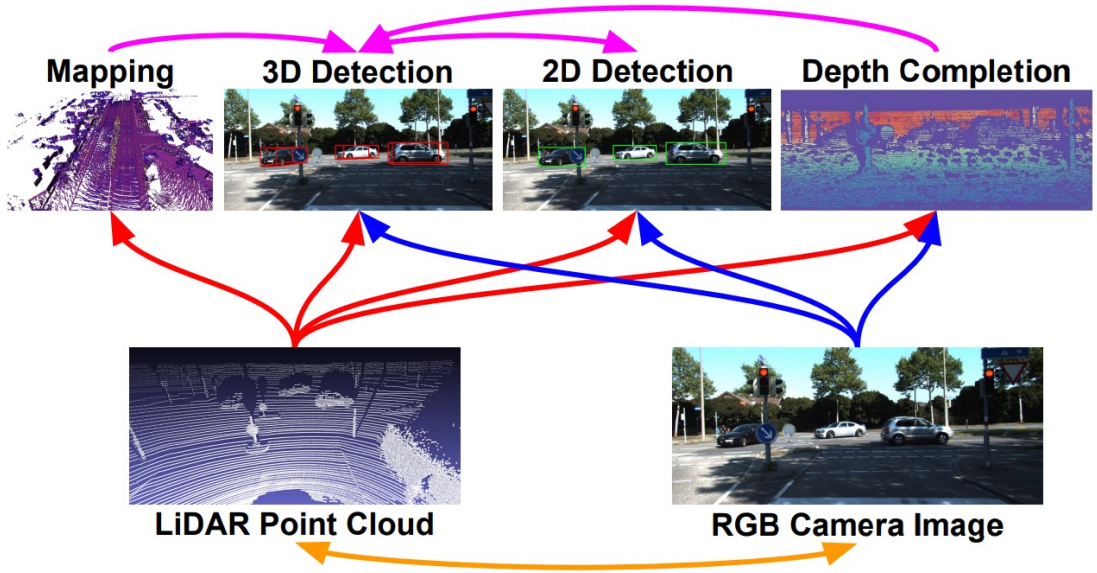

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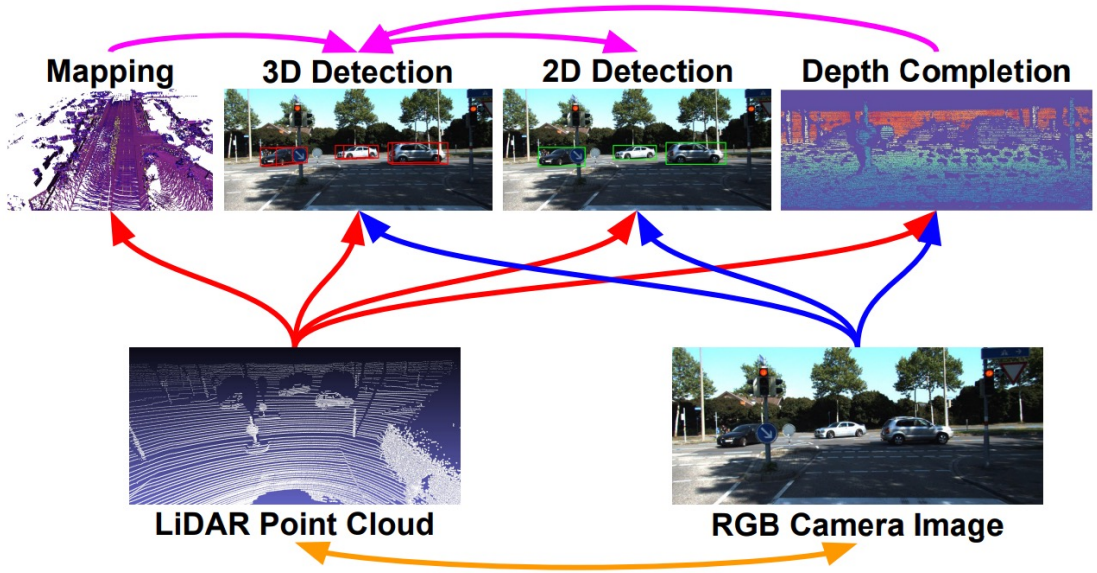
Supervised Dense Depth

- Drawback of continuous fusion: Sparse LiDAR can cause the fusion process to be less accurate. Relies on kNN.
- Why not predict a dense depth to pair with the camera image?
- Depth completion module is supervised by sparse LiDAR and is used for dense fusion.



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3D Perception

- With the ease of use of automatic differentiation libraries, we can compose a computation graph in millions of ways.

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3D Perception

- With the ease of use of automatic differentiation libraries, we can compose a computation graph in millions of ways.
- We can design layers and operators to accomodate different types of inputs and outputs. 3D, point cloud, sparse data, etc.

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3D Perception

- With the ease of use of automatic differentiation libraries, we can compose a computation graph in millions of ways.
- We can design layers and operators to accomodate different types of inputs and outputs. 3D, point cloud, sparse data, etc.
- We can fuse different modalities together too, by leveraging geometric relationships.

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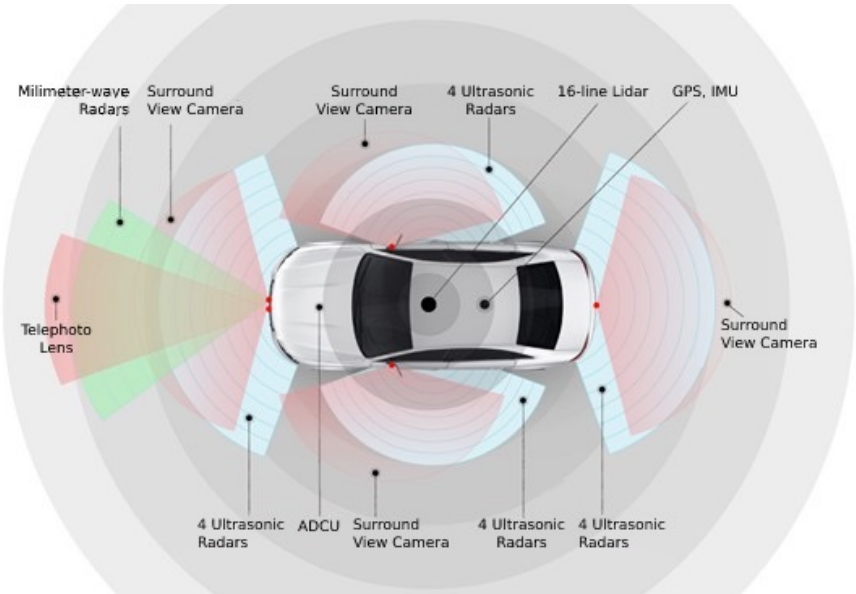
2D to 3D

- Not all embodied agents have the luxury to have a full set of sensors.
- Can we infer the geometric structure with 2D perception?



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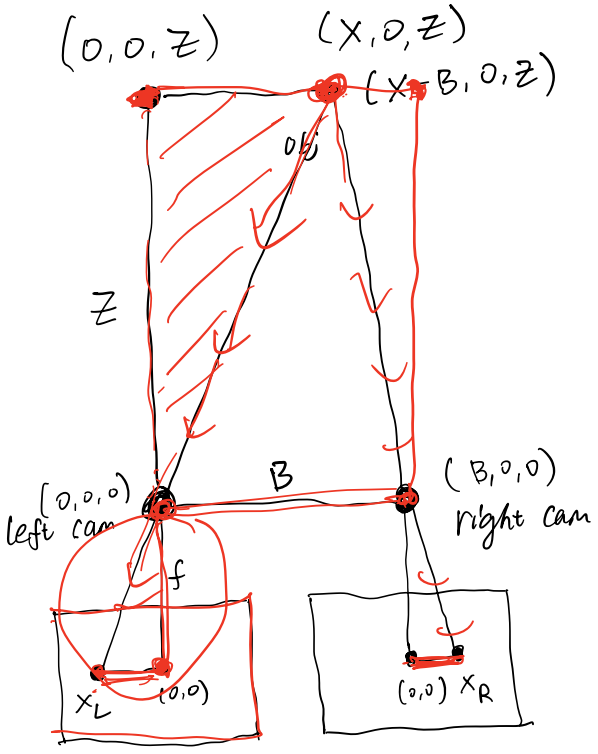
$(x, y) \rightarrow z$

Classic Vision on Depth and Disparity

- One source of depth is from the displacement of pixels in a stereo setup.
- But we need to estimate disparity.



$$x_L = \frac{Xf}{z}$$

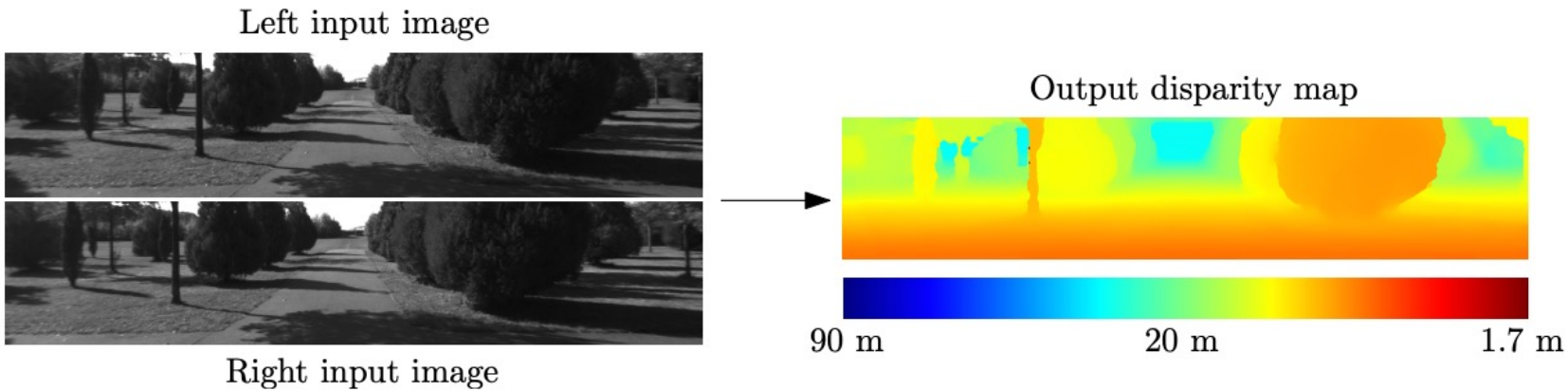
$$x_R = \frac{(X-B)f}{z}$$

$$\delta = x_L - x_R = \frac{[X-(X-B)]f}{z} = \frac{Bf}{z}$$

$$z = \frac{Bf}{\delta}$$

From 2D to 3D: Depth Network

- A network that can output disparity.
- Using LiDAR or depth camera as groundtruth supervision.



Zbontar & LeCun. Computing the Stereo Matching Cost with a Convolutional Neural Network. CVPR 2015.

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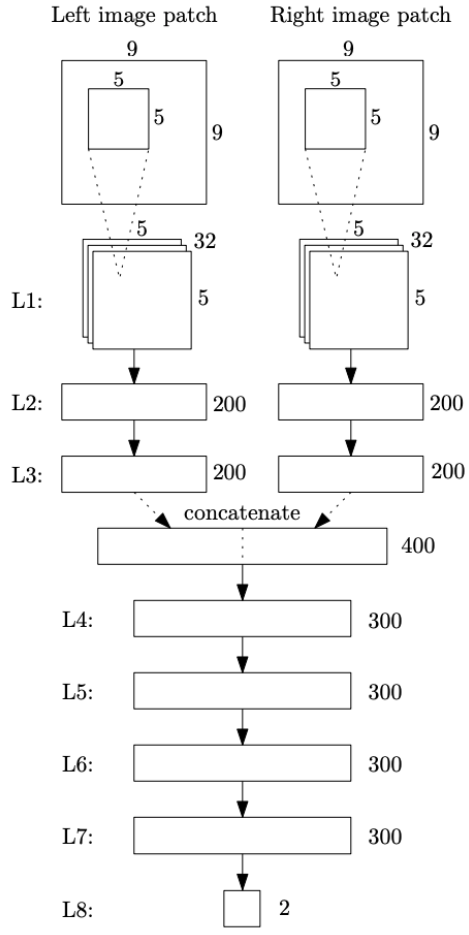
The Energy-Based Approach

- The energy penalize matching with high cost (unary), and when neighboring pixels have disparity differences greater or equal to one (pairwise).
- Cost network: Train with binary classification

Energy $$E(D) = \sum_{\mathbf{p}} \Big(C^4_{\text{CBCA}}(\mathbf{p}, D(\mathbf{p})) + \sum_{\mathbf{q} \in \mathcal{N}_\mathbf{p}} P_1 \times 1\{|D(\mathbf{p}) - D(\mathbf{q})| = 1\} + \sum_{\mathbf{q} \in \mathcal{N}_\mathbf{p}} P_2 \times 1\{|D(\mathbf{p}) - D(\mathbf{q})| > 1\} \Big),$$

$$D(\mathbf{p}) = \underset{d}{\operatorname{argmin}}\, C(\mathbf{p}, d).$$



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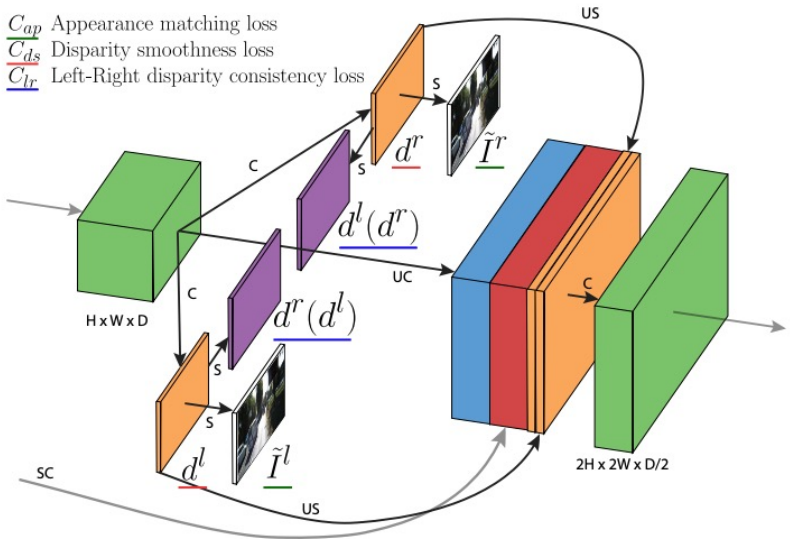
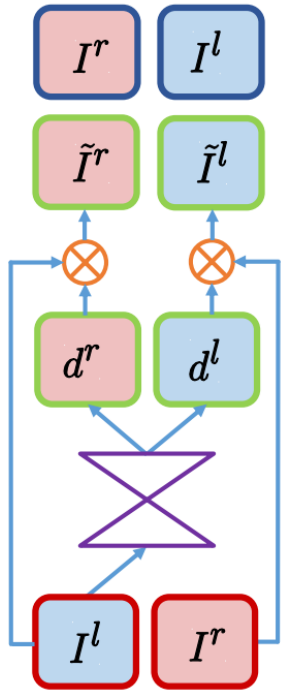
Self-Supervised Depth

- Appearance matching loss

$$C_{ap}^{l} = \frac{1}{N}\sum_{i,j}\alpha\frac{1-\text{SSIM}(I_{ij}^{l},\tilde{I}_{ij}^{l})}{2} + (1-\alpha)\left\|I_{ij}^{l}-\tilde{I}_{ij}^{l}\right\|.$$





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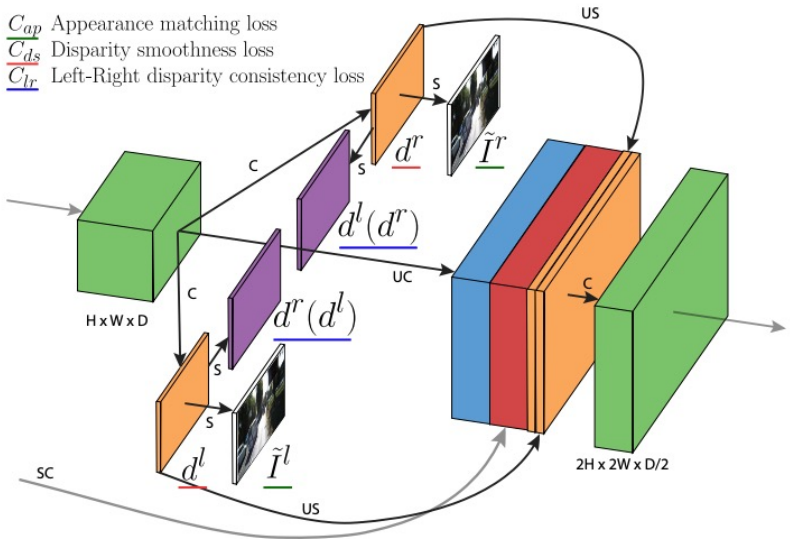
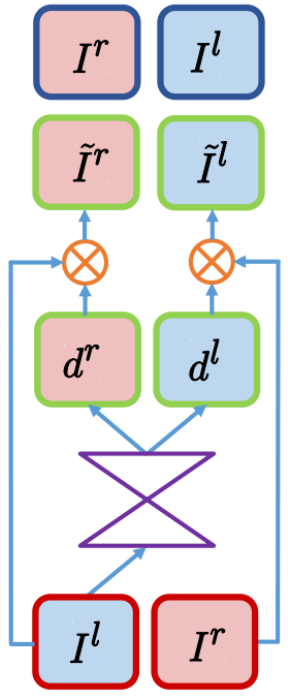
- Disparity smoothness loss

$$C_{ds}^{l} = \frac{1}{N}\sum_{i,j} \left|\partial_x d_{ij}^{l}\right| e^{-\left\|\partial_x I_{ij}^{l}\right\|} + \left|\partial_y d_{ij}^{l}\right| e^{-\left\|\partial_y I_{ij}^{l}\right\|}.$$





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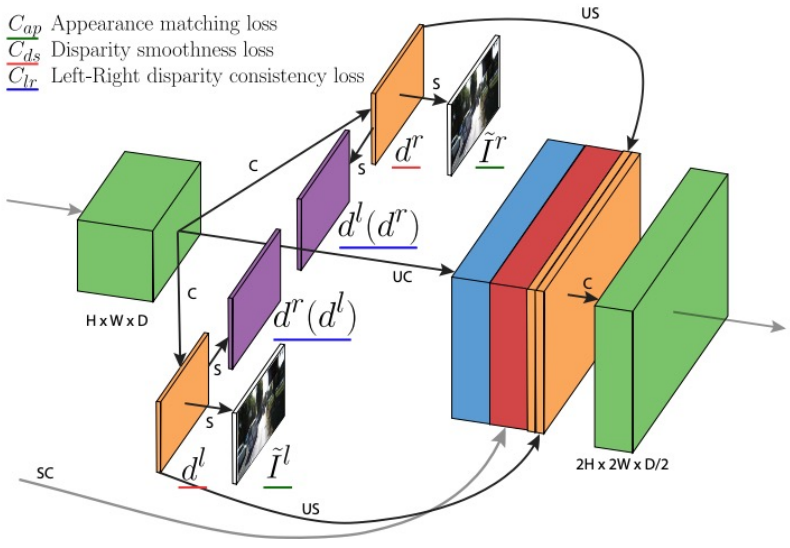
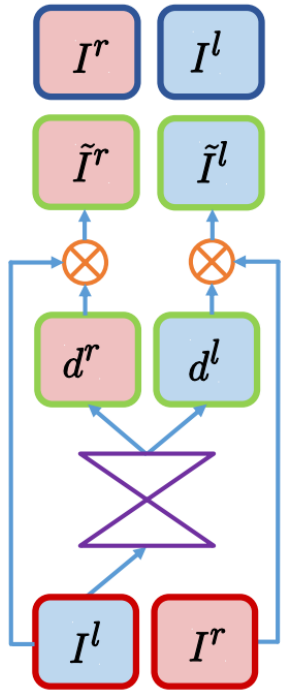
- Left-right disparity consistency loss

$$C_{lr}^{l} = \frac{1}{N}\sum_{i,j}\left|d_{ij}^{l}-d_{ij+d_{ij}^{l}}^{r}\right|.$$





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Motion, Optical Flow

- Another task that takes into a pair of image is to estimate the motion of pixels across two consecutive video frames.

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$$\boxed{I_x u + I_y v + I_t = 0.}$$

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Classical Approach

- Under-constrained system

$$I_x u + I_y v + I_t = 0.$$

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Classical Approach

- Under-constrained system
- Use a local patch and assume smooth motion

$$I_x u + I_y v + I_t = 0.$$

$$\mathbf{Au} = \mathbf{b}$$

$$\begin{pmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{N^2}) & I_y(\mathbf{p}_{N^2}) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} I_t(\mathbf{p}_1) \\ \vdots \\ I_t(\mathbf{p}_{N^2}) \end{pmatrix}$$

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Classical Approach

- Under-constrained system
- Use a local patch and assume smooth motion
- Rigid, contains many assumptions

$$I_x u + I_y v + I_t = 0.$$

$$\mathbf{A}\mathbf{u} = \mathbf{b}$$

$$\begin{pmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{N^2}) & I_y(\mathbf{p}_{N^2}) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} I_t(\mathbf{p}_1) \\ \vdots \\ I_t(\mathbf{p}_{N^2}) \end{pmatrix}$$

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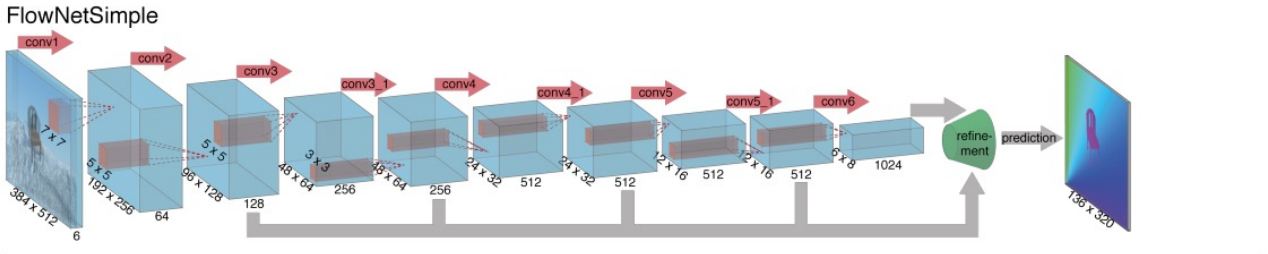
Correlation Volume Approach

- Simple Approach: Concatenate the two images together.



Fischer et al. FlowNet: Learning Optical Flow with Convolutional Networks. ICCV 2015.

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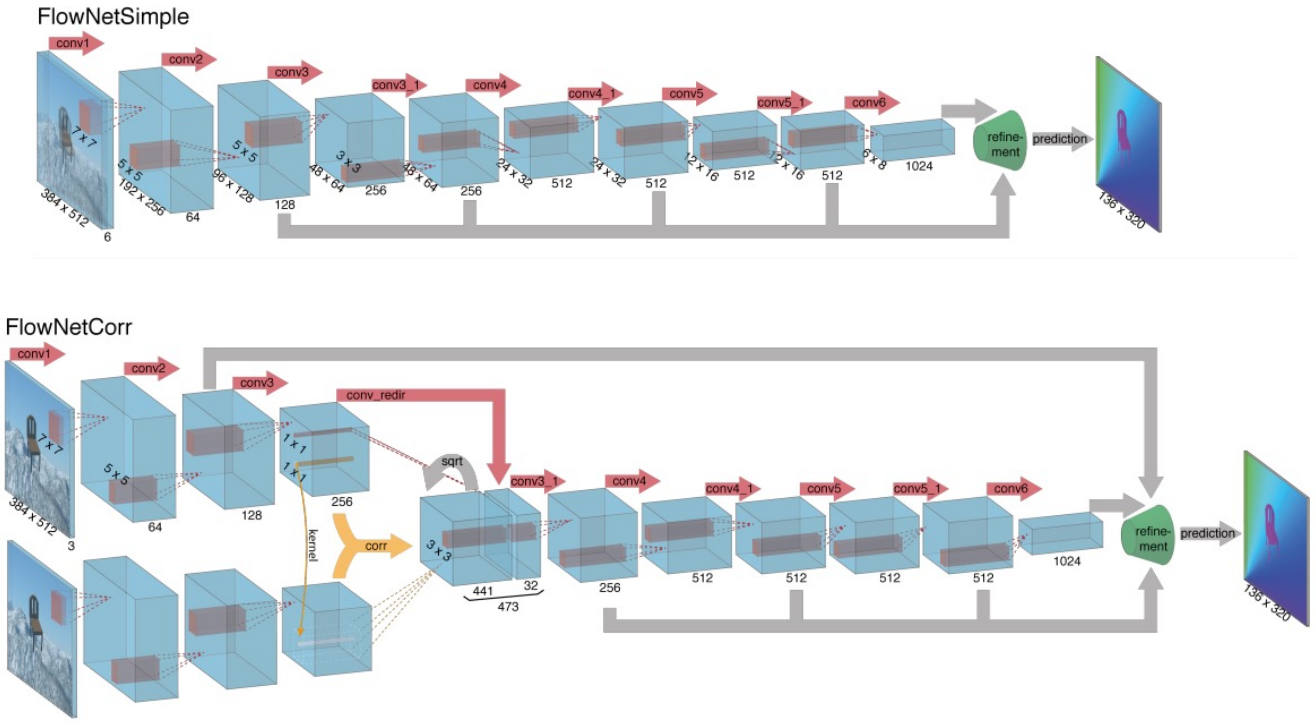
Correlation Volume Approach

- Simple Approach: Concatenate the two images together.
- Correlation: Extract some levels of features, and convolve one feature on top of another.



Fischer et al. FlowNet: Learning Optical Flow with Convolutional Networks. ICCV 2015.

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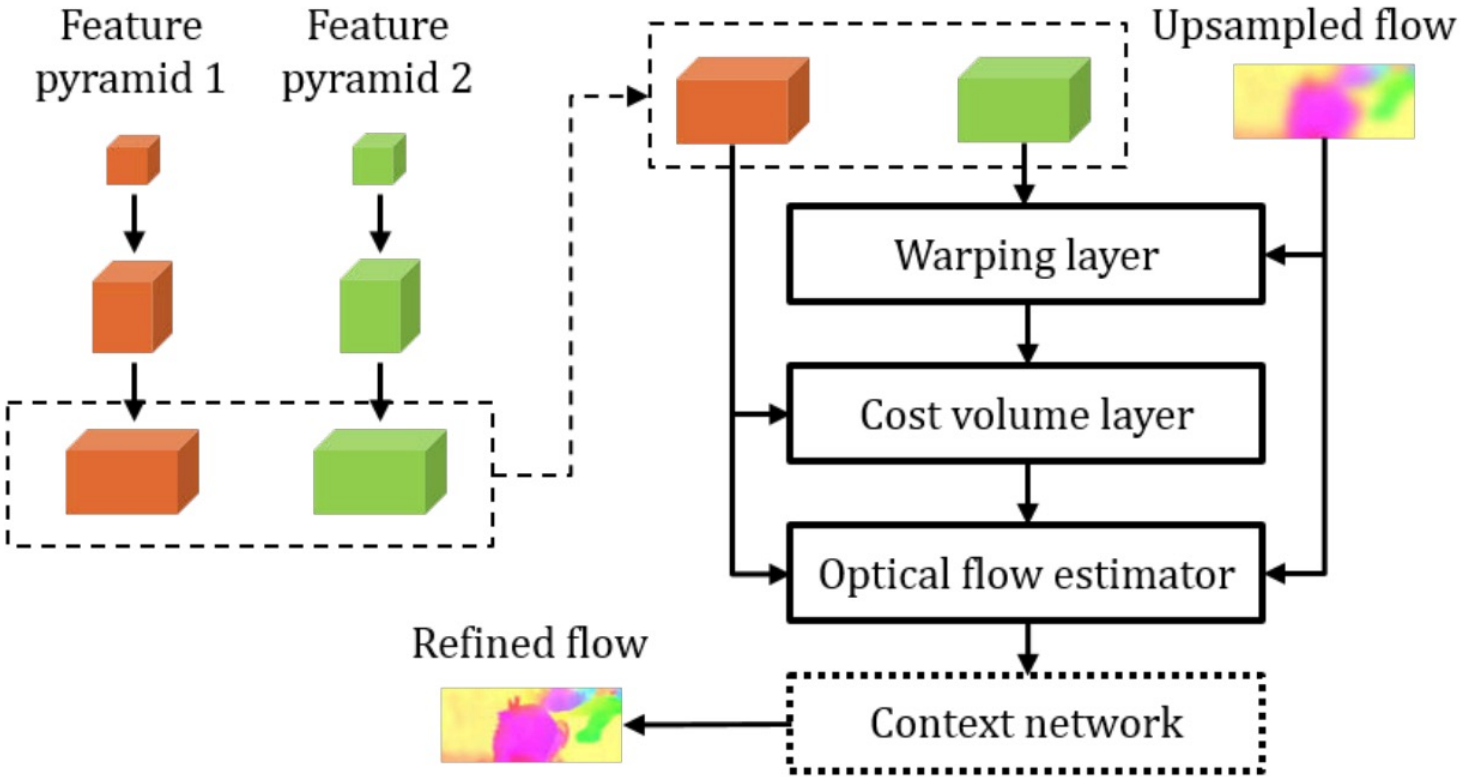
Iterative Refining through Feature Pyramid



Sun et al. PWC-Net: CNNs for Optical Flow Using Pyramid, Warping, and Cost Volume.

NYU

Unsupervised Flow

- Photometric Consistency (Appearance)

Jonschkowski et al. What Matters in Unsupervised Optical Flow. ECCV 2020

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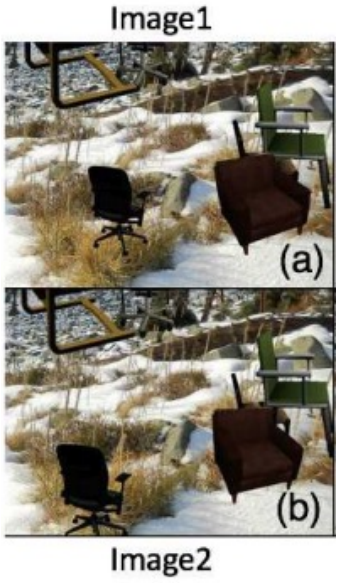
Unsupervised Flow

- Photometric Consistency (Appearance)
- Occlusion Estimation
  - Forward-backward consistency



Wang et al., 2018

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- Photometric Consistency (Appearance)
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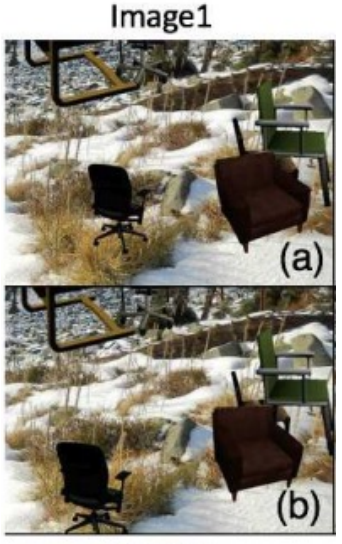
- Smoothness



Wang et al., 2018

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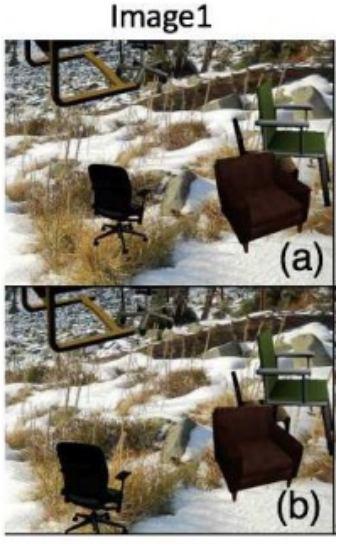

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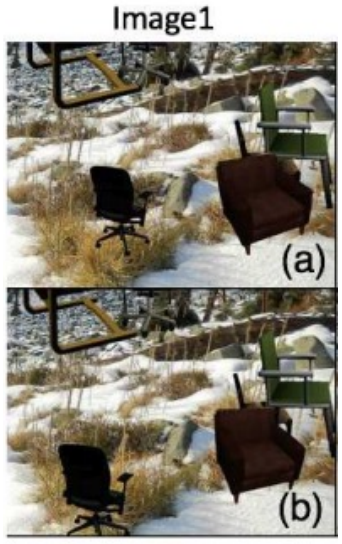
- Smoothness
- Self-supervision: Ensure consistent flow at different augmentation (e.g. crops)
- Can 3D information help us reason about motion?



Wang et al., 2018

Jonschkowski et al. What Matters in Unsupervised Optical Flow. ECCV 2020

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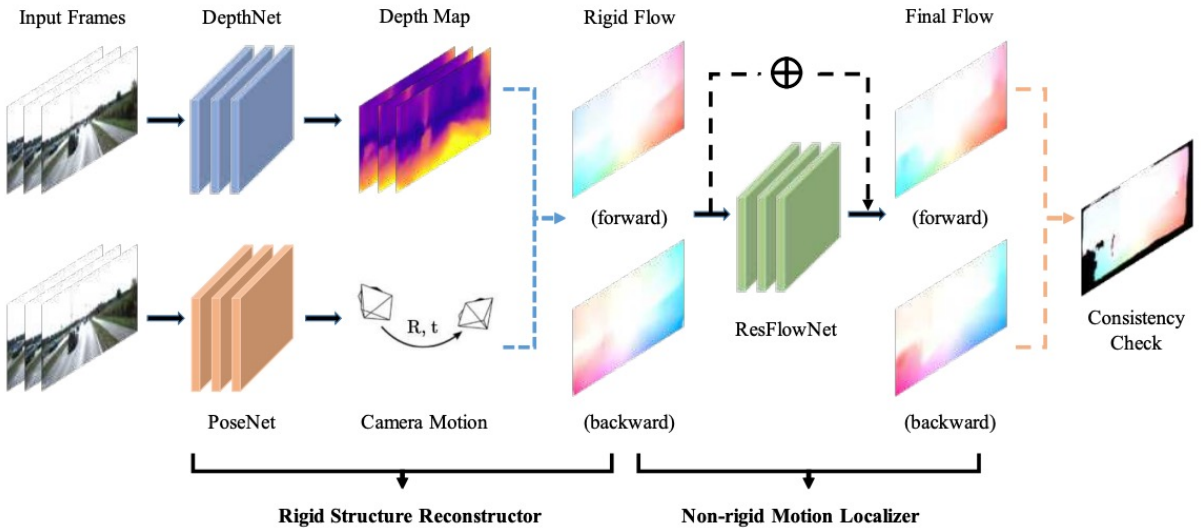
Depth, Flow, and Pose Movement

- The static objects follow rigid flow: determined by camera motion and depth.

$$f^{rig}_{t\mapsto s}(p_t) = KT_{t\mapsto s}D_t(p_t)K^{-1}p_t - p_t.$$



Yin & Shi. GeoNet: Unsupervised Learning of Dense Depth, Optical Flow and Camera Pose. CVPR 2018.

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Training Losses

- Appearance Loss (Warping):

$$\mathcal{L}_{rw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{rig})}{2} + (1 - \alpha) \| I_t - \tilde{I}_s^{rig} \|_1.$$

$$\mathcal{L}_{fw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{full})}{2} + (1 - \alpha) \| I_t - \tilde{I}_s^{full} \|_1.$$

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$$\mathcal{L} = \sum_{p_t} [\delta(p_t)] \cdot \|\Delta f_{t \mapsto s}^{full}(p_t)\|_1.$$

$$\delta(p_t) = \|\Delta f_{t \mapsto s}^{full}(p_t)\|_2 < \max\{\alpha, \beta \|f_{t \mapsto s}^{full}(p_t)\|_2\}.$$

Yin & Shi. GeoNet: Unsupervised Learning of Dense Depth, Optical Flow and Camera Pose. CVPR 2018.

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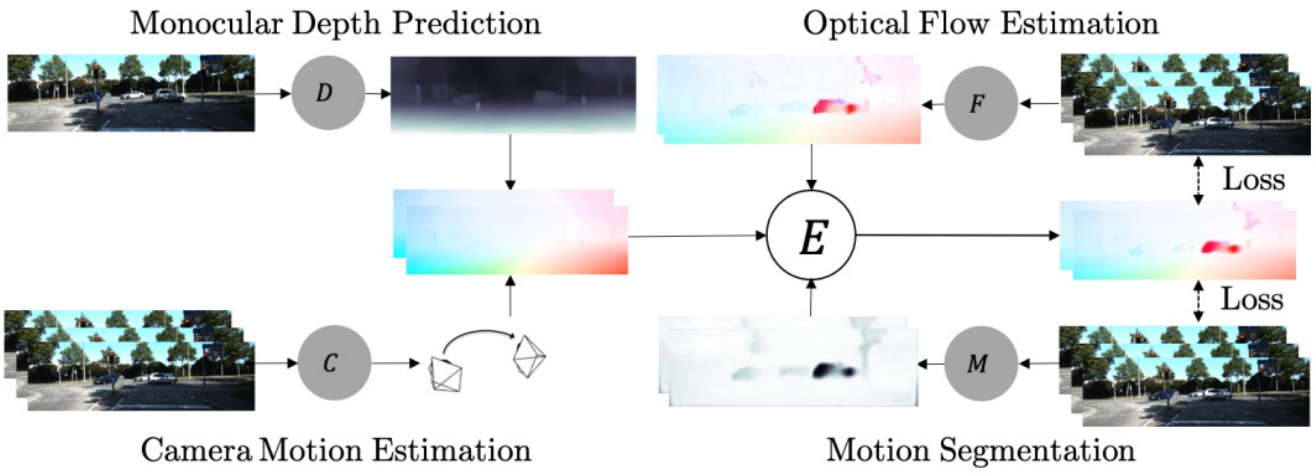
Summary

- Leverage cross correlation structure for spatial similarity matching.



Ranjan et al. Competitive Collaboration: Joint Unsupervised Learning of Depth, Camera, Motion, Optical Flow and Motion Segmentation. CVPR 2019

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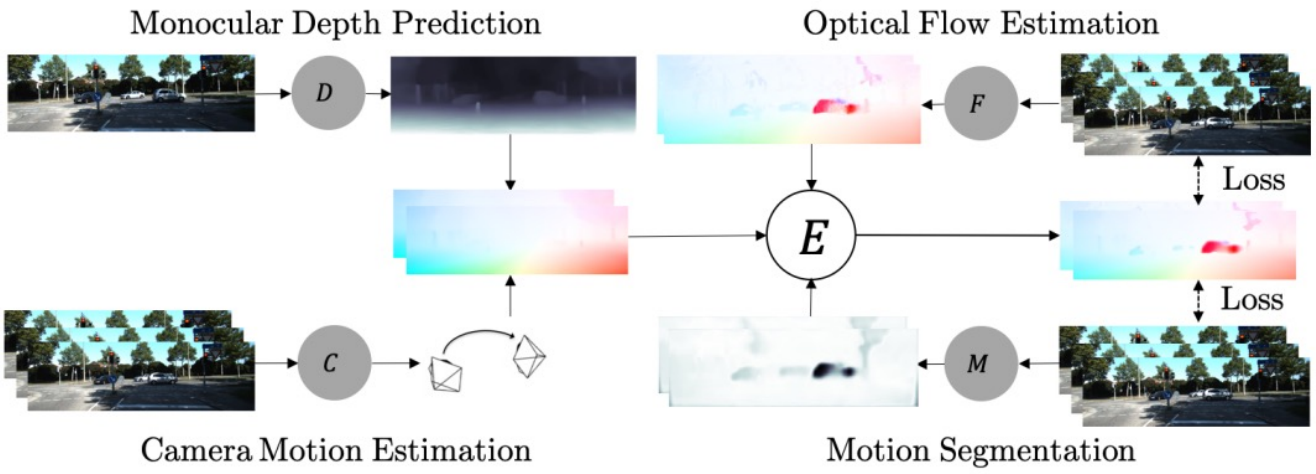
Summary

- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.



Ranjan et al. Competitive Collaboration: Joint Unsupervised Learning of Depth, Camera, Motion, Optical Flow and Motion Segmentation. CVPR 2019

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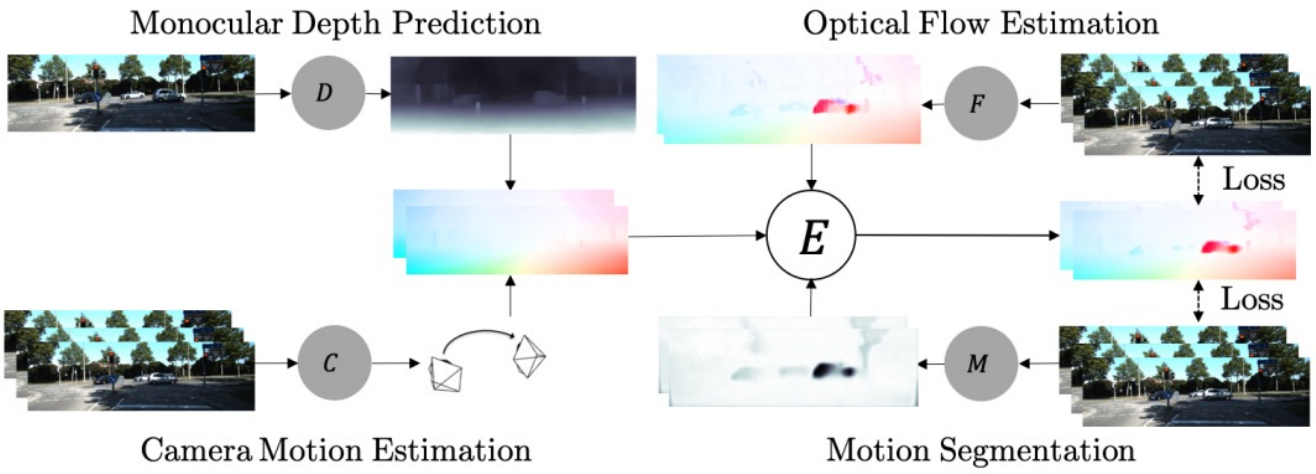
Summary

- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.
- Can form triangulation for self-supervision.



Ranjan et al. Competitive Collaboration: Joint Unsupervised Learning of Depth, Camera, Motion, Optical Flow and Motion Segmentation. CVPR 2019

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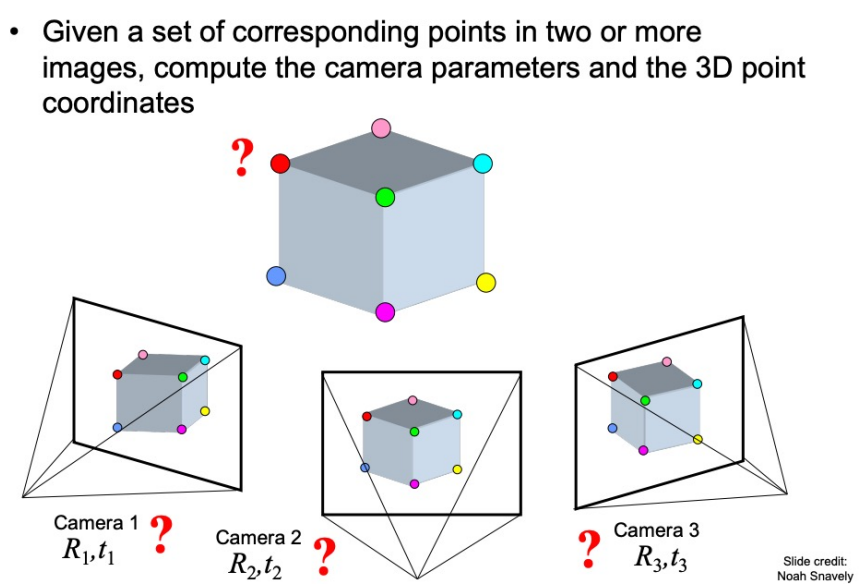
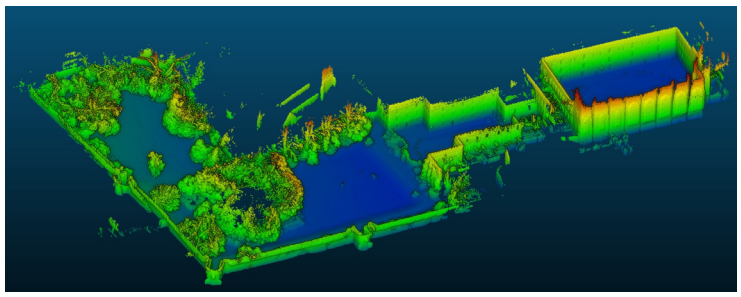
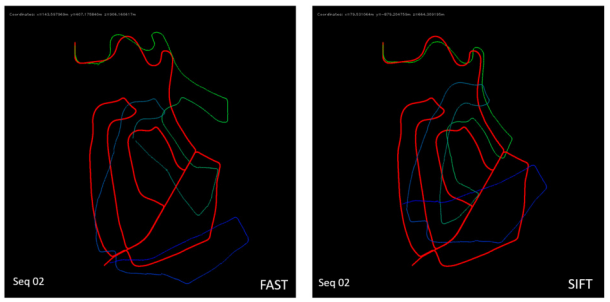
Classical Mapping

- Estimating 3D structure and location from 2D observations.
- Simultaneous Localization and Mapping.
- Common Techniques: Extended Kalman Filter, GraphSLAM







Garg & Jain

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Common Drawbacks

- Probabilistic inference can take long to compute, and mapping takes a large memory storage.

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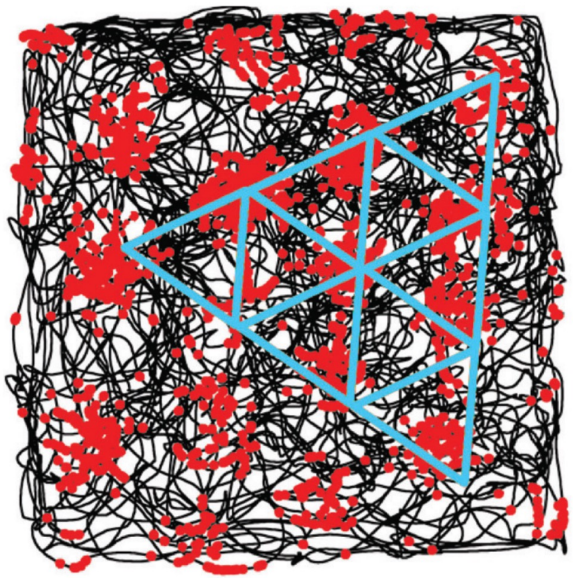
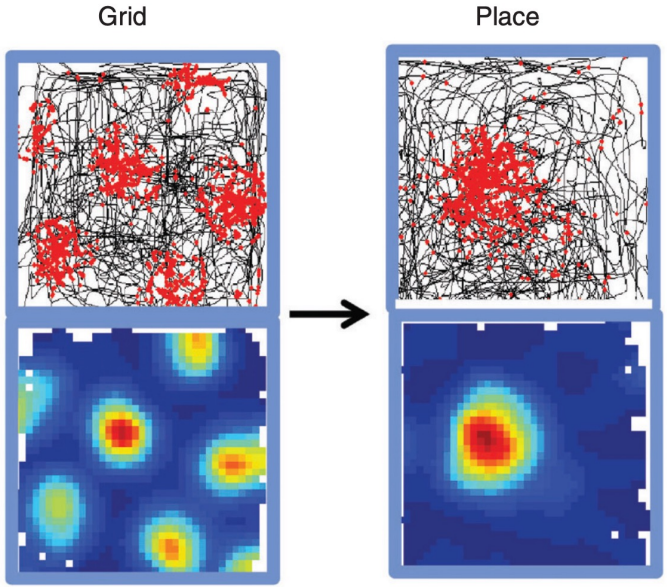
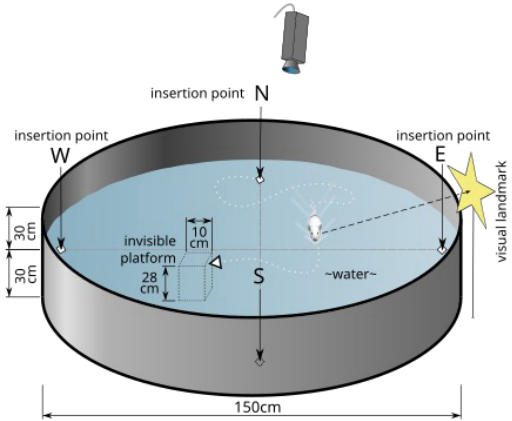
- Great for 3D reconstruction but downstream tasks may not need a full precision explicit map.
- May not fully understand dynamic objects (averaging across multiple scans).
- Is there a more end-to-end version?

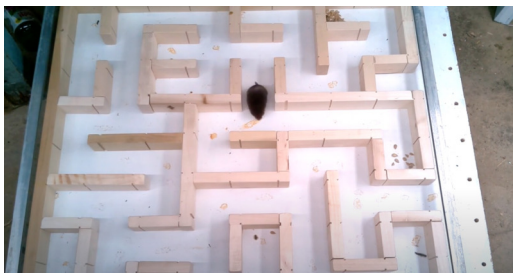
Mapping in the Brain: Grid and Place Cells







Morris water maze



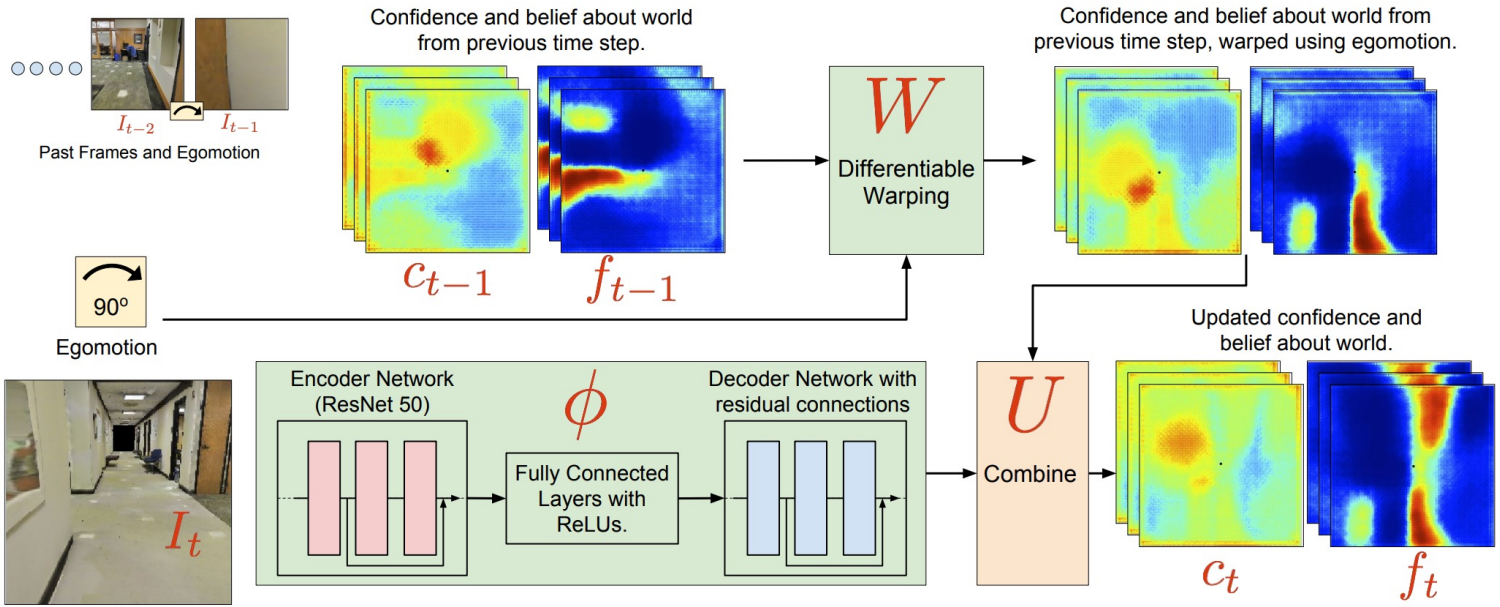
Matthias Wandel, 2018

May-Britt Moser, David C. Rowland, and Edvard I. Moser. Place Cells, Grid Cells, and Memory.

NYU

Neural Mapping

- Can we learn a mapping representation?
- Metric space, top-down warping (known egomotion).



Gupta et al. Cognitive Mapping and Planning for Visual Navigation. IJCV 2017.

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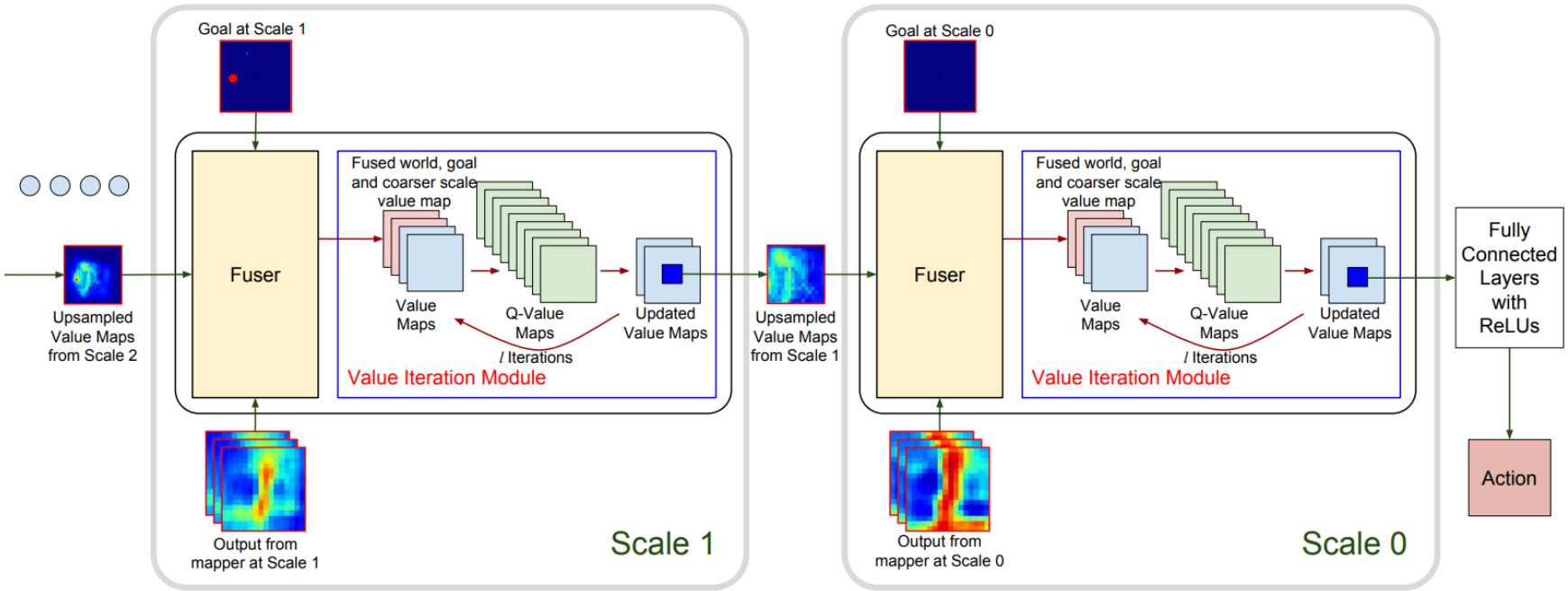
Hierarchical Planning

- How do we use the learned map (allocentric) feature of the world?



Gupta et al. Cognitive Mapping and Planning for Visual Navigation. IJCV 2017.

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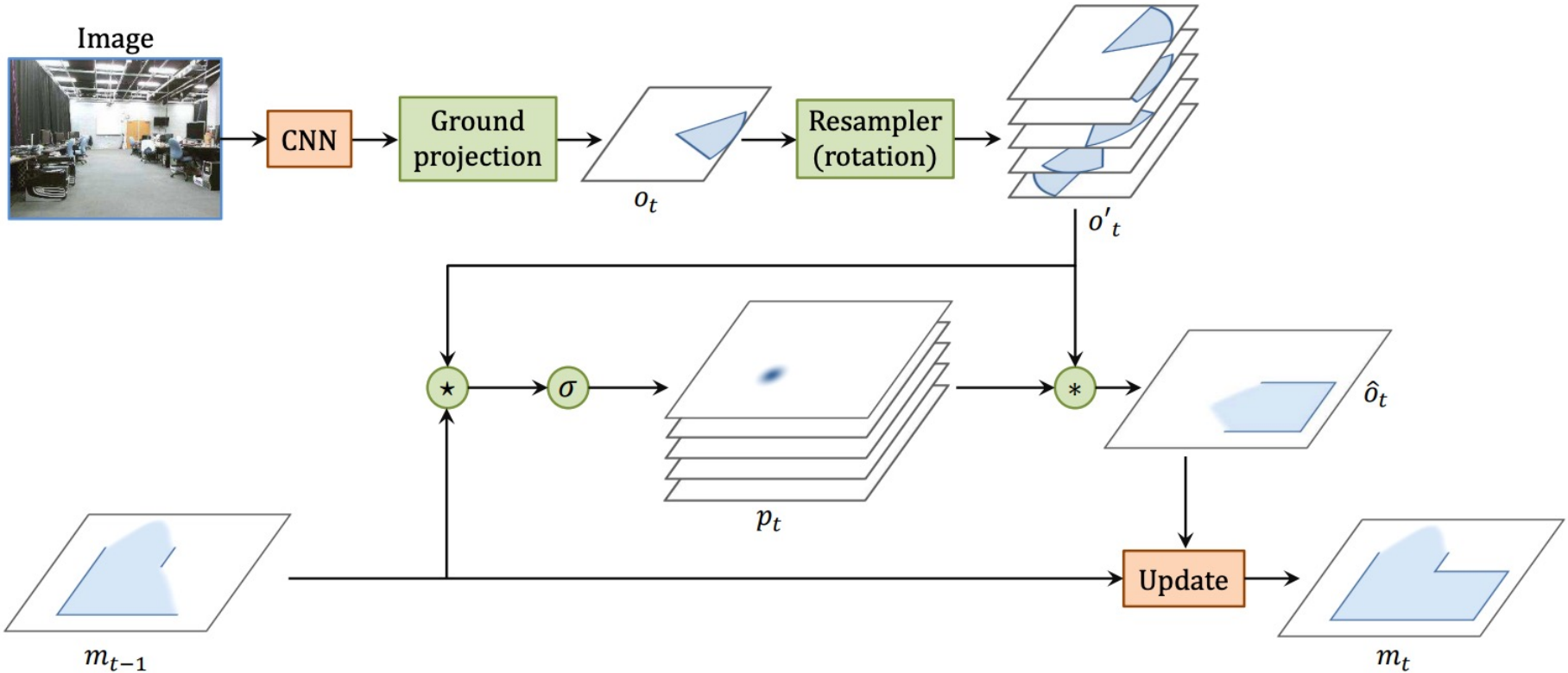
Simultaneous Localization and Registration



Henriques & Vedaldi. MapNet: An Allocentric Spatial Memory for Mapping Environments. CVPR 2018.

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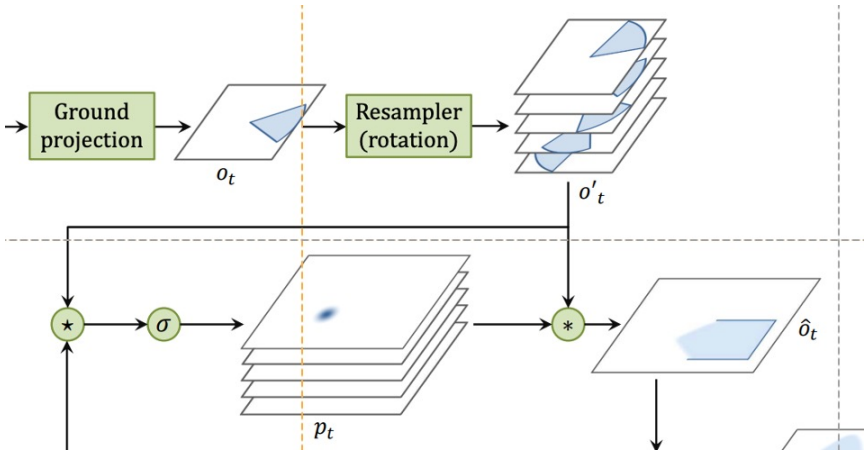
Simultaneous Localization and Registration

- The observations o_t are transformed into a stack o'_t by applying a rotation resampler.

$$o'_{ijkl} = [R(o, 2\pi l/r)]_{ijk}.$$



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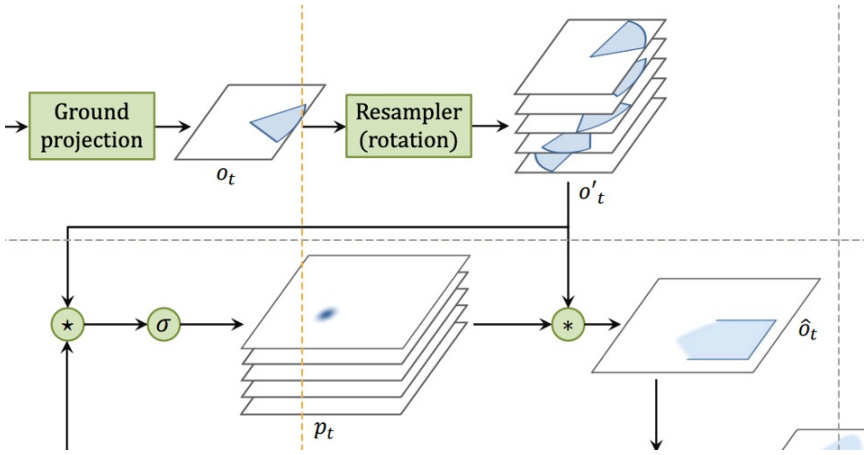

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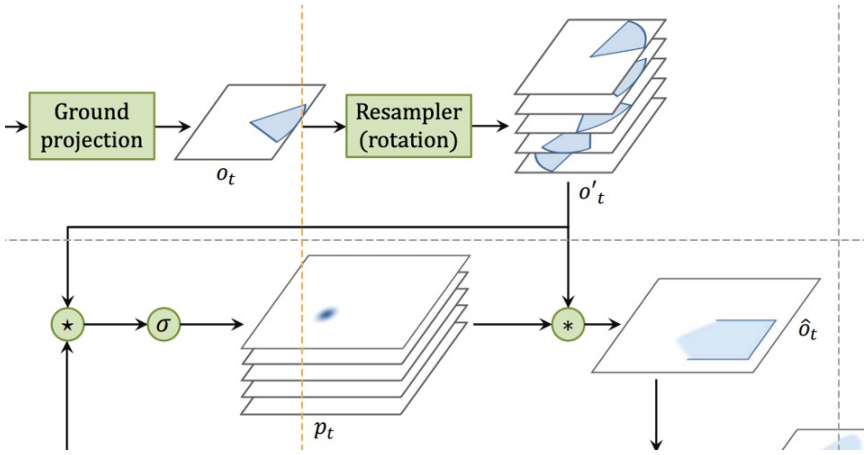

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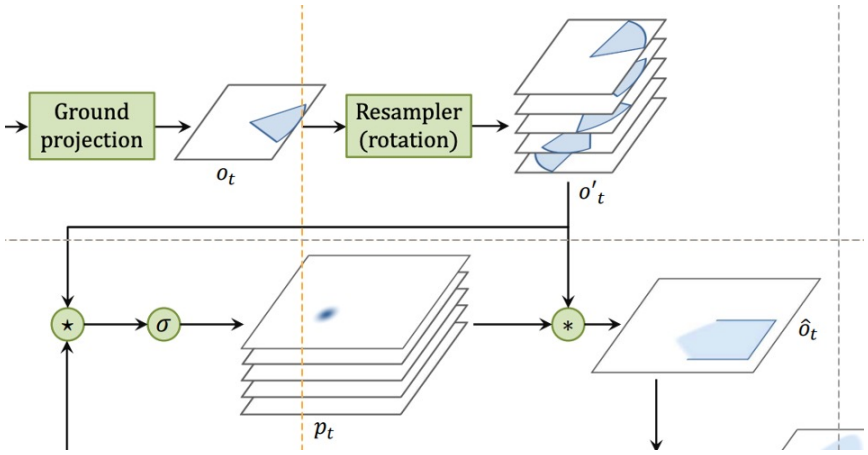

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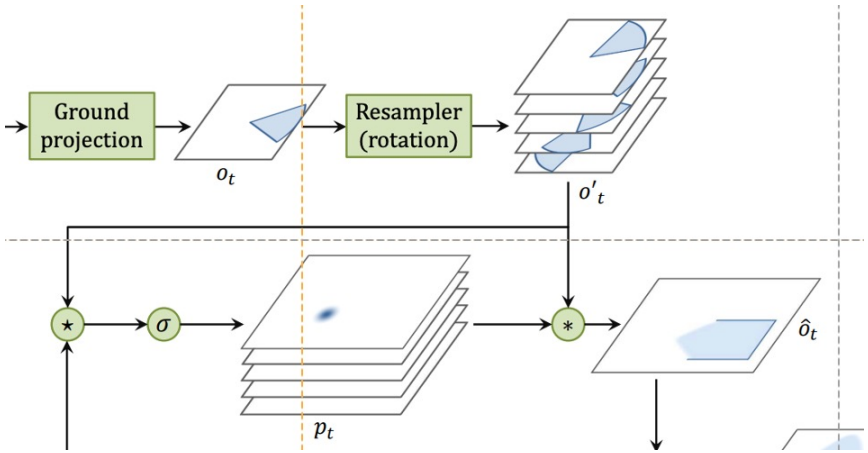

Loss:

$$\mathcal{L}(p) = -\log \sum_t p_{H_t W_t R_t t}.$$

Henriques & Vedaldi. MapNet: An Allocentric Spatial Memory for Mapping Environments. CVPR 2018.

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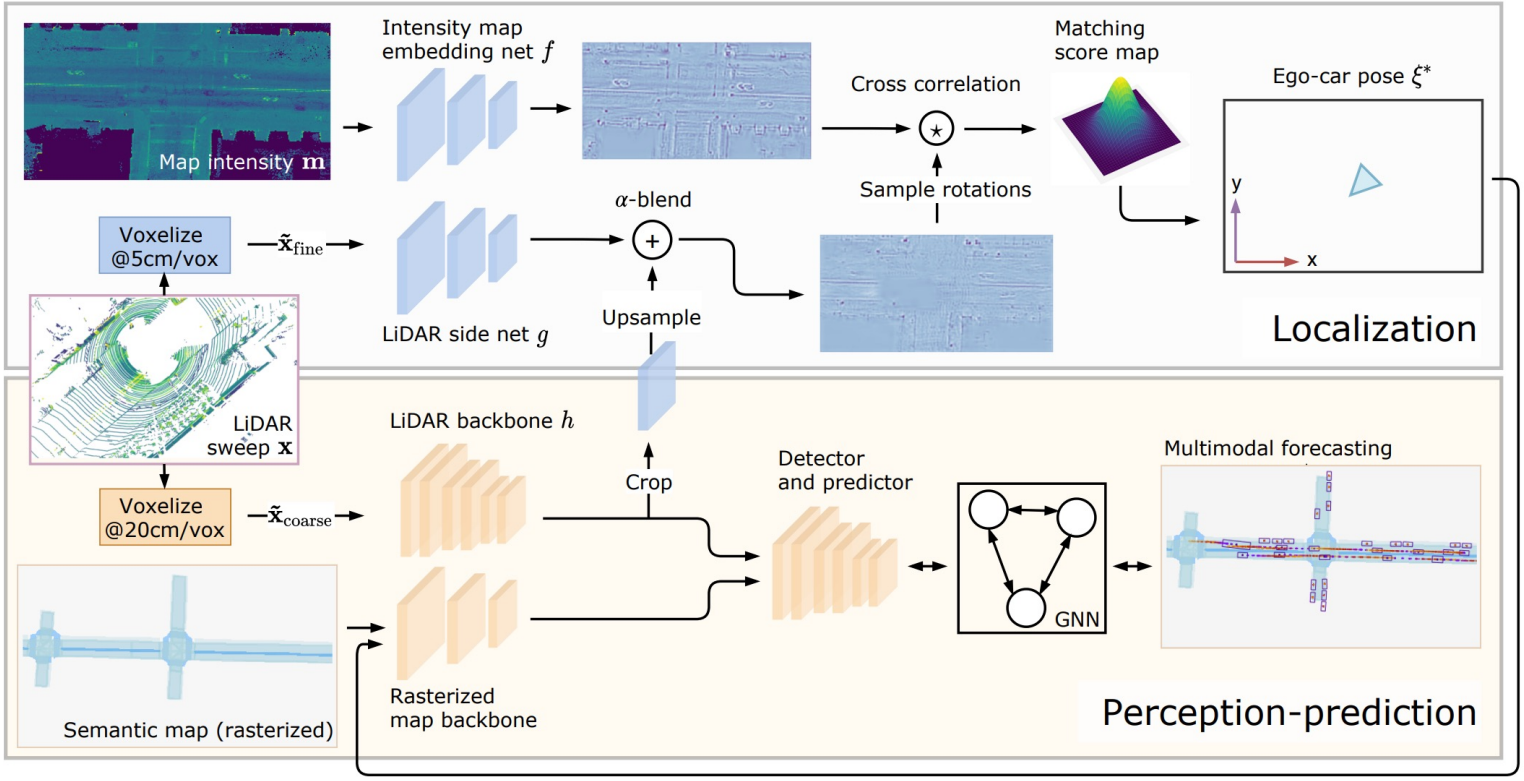
Joint Localization, Perception and Prediction



Philips et al. Deep Multi-Task Learning for Joint Localization, Perception, and Prediction CVPR 2021.

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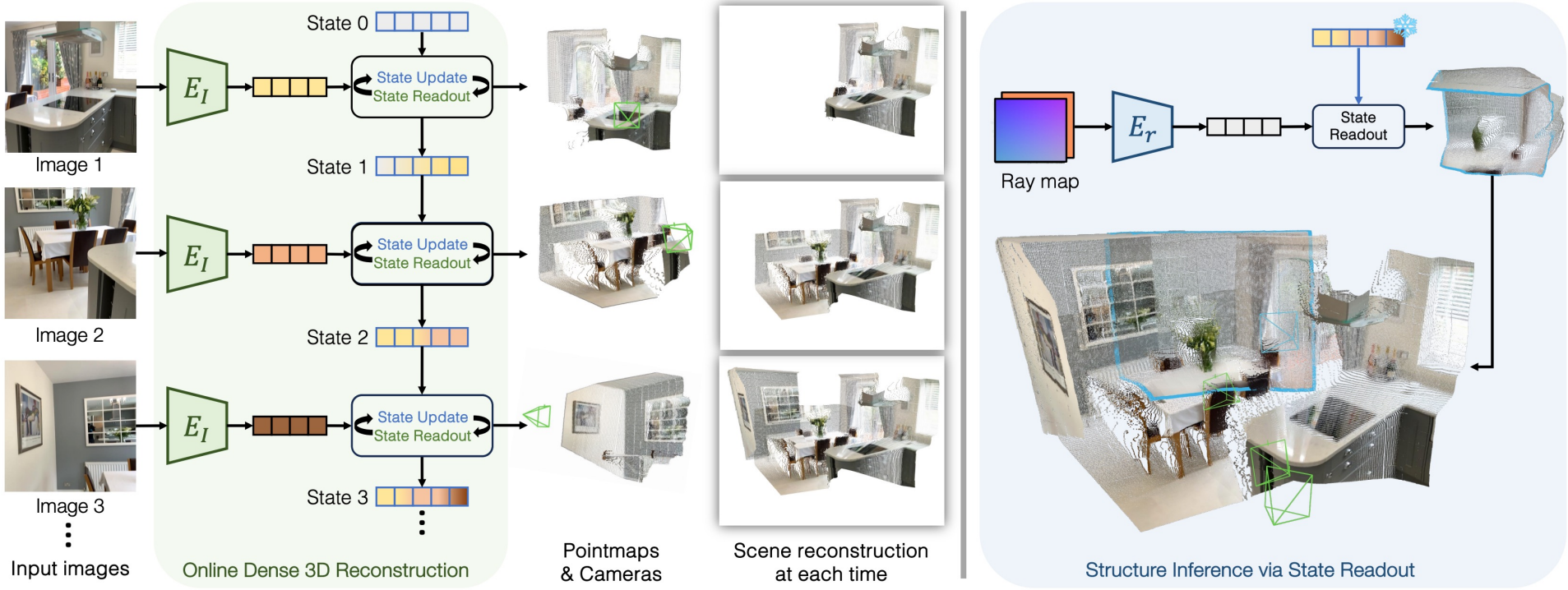
Continuous 3D Perception and Mapping



Wang et al. Continuous 3D Perception Model with Persistent State. arXiv 2025.

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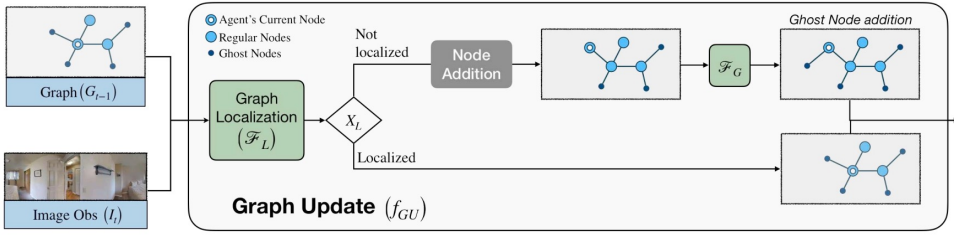
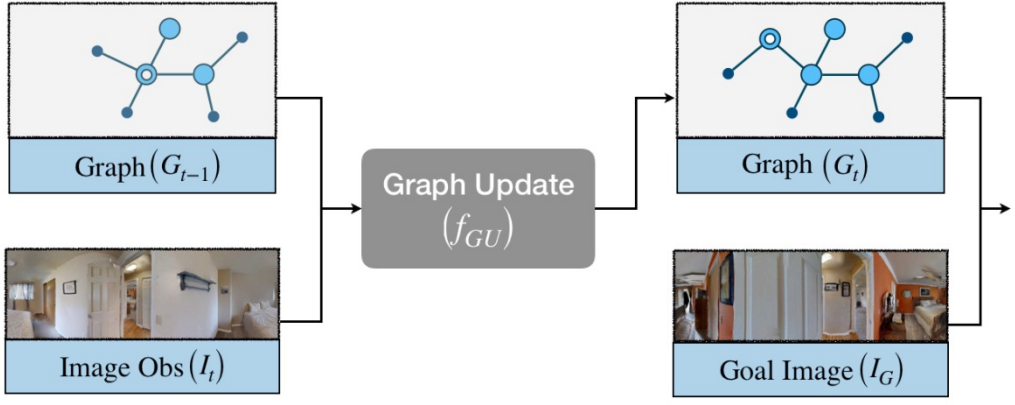
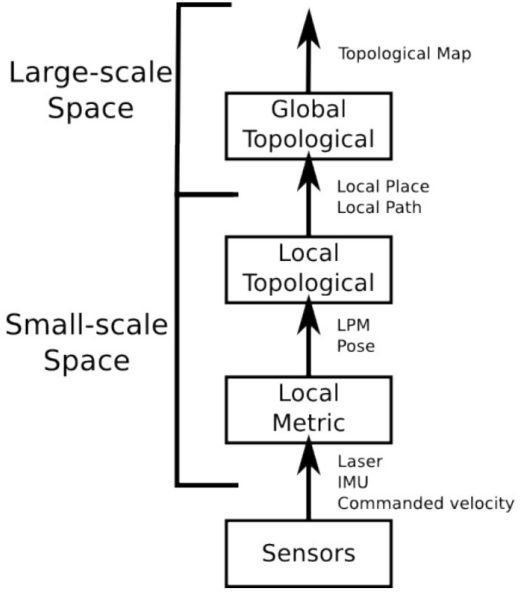
Topological Mapping

- High-level graph representation
- Each node contains more summarized information
- Enables global planning







Johnson, 2018

Johnson. Topological Mapping and Navigation in Real-World Environments. 2018.
Chaplot et al. Neural Topological SLAM for Visual Navigation. CVPR 2020.

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Summary - 3D Vision

- Covers 3D, equivariance, motion, depth, and mapping.

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- Can be made unsupervised
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- Design joint learning frameworks.
- Using geometric transformation to ground representations.
- Useful for planning (a few weeks from now).

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